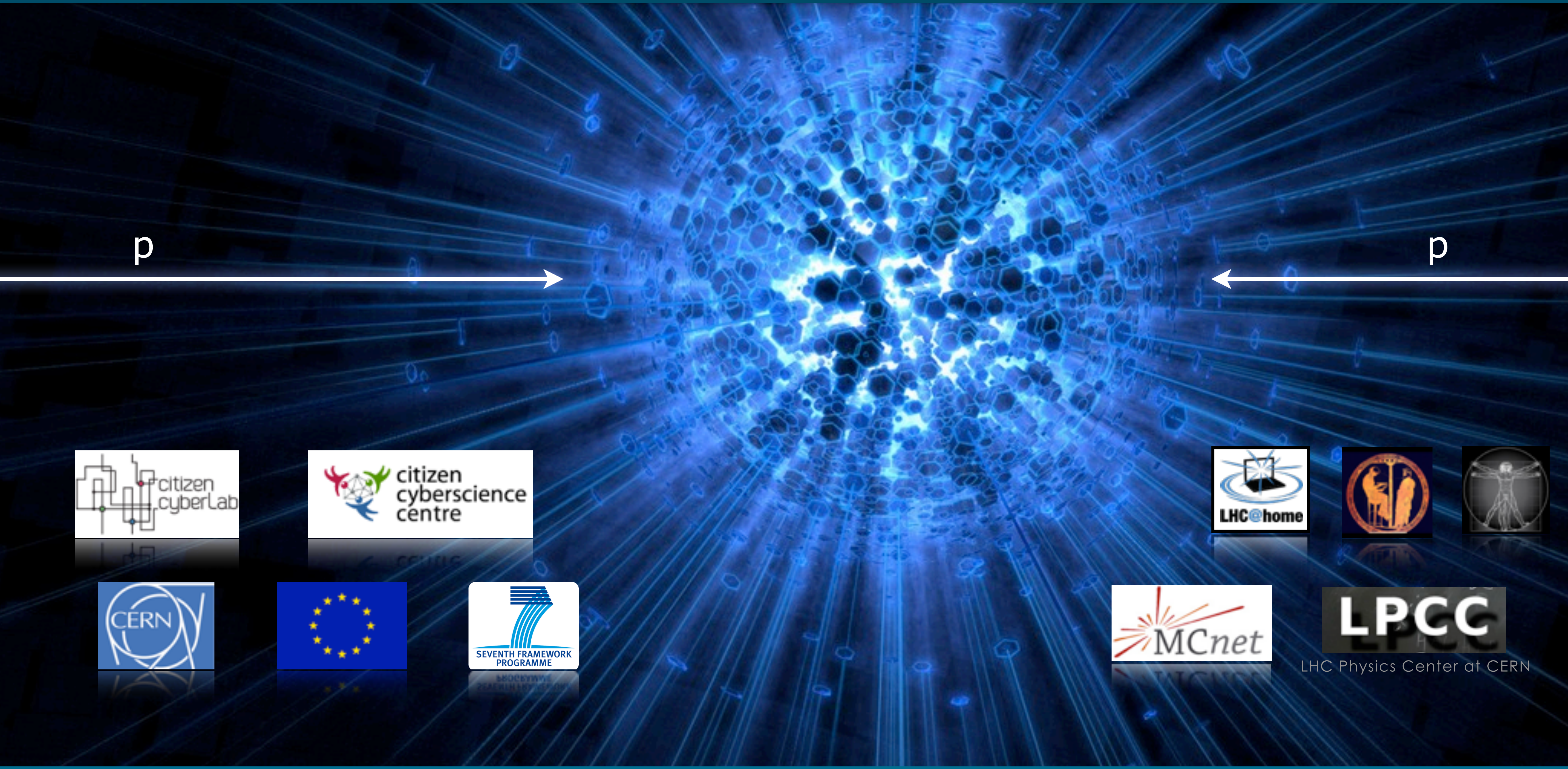
Event Generator Physics

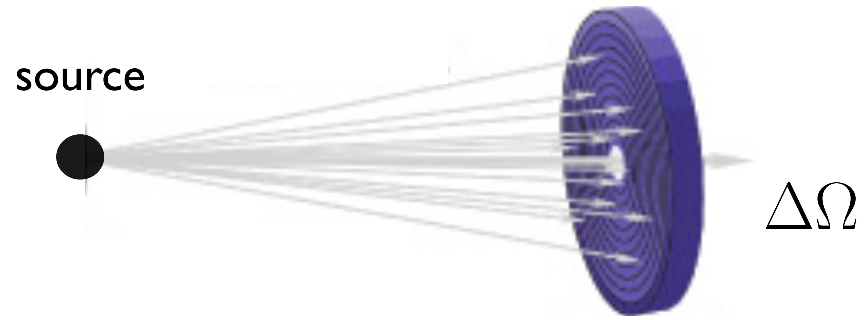
Peter Skands (CERN Theoretical Physics Dept)



Ecole Joliot Curie
Frejus, France, September - October 2013

Lecture 1 / 2

Scattering Experiments



LHC detector
Cosmic-Ray detector
Neutrino detector
X-ray telescope
…

→ Integrate differential cross sections over specific phase-space regions

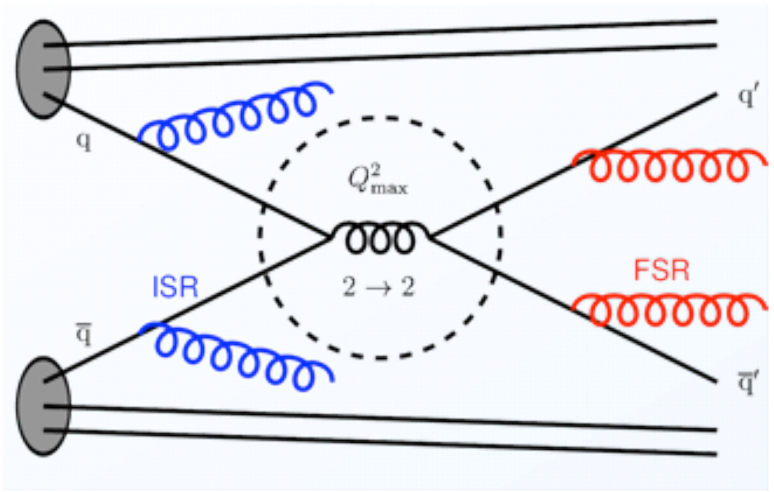
Predicted number of counts
= integral over solid angle

$$N_{\mathrm{count}}(\Delta\Omega) \propto \int_{\Delta\Omega} \mathrm{d}\Omega \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$

Lots of dimensions?
Complicated integrands?
→ Use Monte Carlo

In particle physics:
Integrate over all quantum histories
(+ interferences)

General-Purpose Event Generators



Calculate Everything ≈ solve QCD → requires compromise!

Improve lowest-order perturbation theory,
by including the 'most significant' corrections
→ complete events (can evaluate any observable you want)

The Workhorses

PYTHIA : Successor to JETSET (begun in 1978). Originated in hadronization studies: Lund String.
HERWIG : Successor to EARWIG (begun in 1984). Originated in coherence studies: angular ordering.
SHERPA : Begun in 2000. Originated in "matching" of matrix elements to showers: CKKW-L.
+ MORE SPECIALIZED: ALPGEN, MADGRAPH, ARIADNE, VINCIA, WHIZARD, MC@NLO, POWHEG, …

Factorization

Why is Fixed Order QCD not enough?
: It requires all resolved scales >> Λ_{QCD} **AND** no large hierarchies

Trivially untrue for QCD

We're colliding, and observing, hadrons → small scales

We want to consider high-scale processes → large scale differences

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \sum_{a,b}\sum_{f}\int_{\hat{X}_f} f_a(x_a,Q_i^2)f_b(x_b,Q_i^2)\frac{\mathrm{d}\hat{\sigma}_{ab\to f}(x_a,x_b,f,Q_i^2,Q_f^2)}{\mathrm{d}\hat{X}_f} D(\hat{X}_f \to X, Q_i^2, Q_f^2)$$

PDFs: needed to compute inclusive cross sections

FFs: needed to compute (semi-)exclusive cross sections

Resummed pQCD: All resolved scales >> Λ_{QCD} **AND** X Infrared Safe

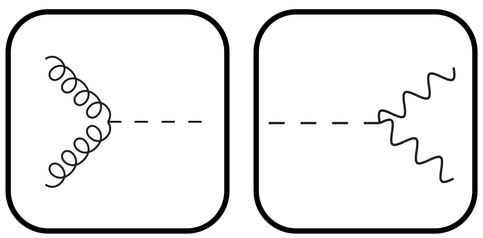
*) pQCD = perturbative QCD

Divide and Conquer

Factorization → Split the problem into many (nested) pieces

+ Quantum mechanics → Probabilities → Random Numbers

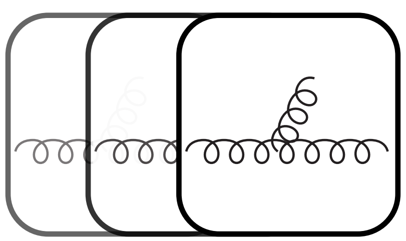
$$\mathcal{P}_{\rm event} = \mathcal{P}_{\rm hard} \otimes \mathcal{P}_{\rm dec} \otimes \mathcal{P}_{\rm ISR} \otimes \mathcal{P}_{\rm FSR} \otimes \mathcal{P}_{\rm MPI} \otimes \mathcal{P}_{\rm Had} \otimes \ldots$$



Hard Process & Decays:

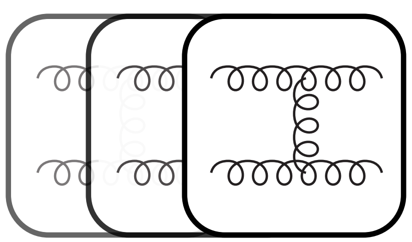
Use (N)LO matrix elements

→ Sets "hard" resolution scale for process: $Q_{\rm MAX}$



Initial- & Final-State Radiation (ISR & FSR):

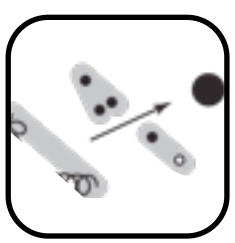
Altarelli-Parisi equations → differential evolution, dP/dQ^2, as function of resolution scale; run from $Q_{\rm MAX}$ to ~ 1 GeV (More later)



MPI (Multi-Parton Interactions)

Additional (soft) parton-parton interactions: LO matrix elements

→ Additional (soft) "Underlying-Event" activity



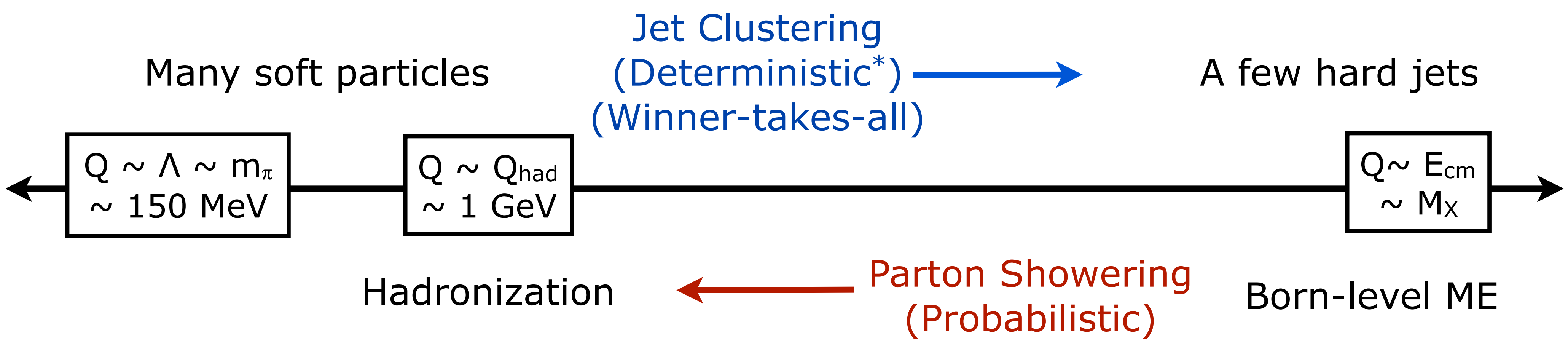
Hadronization

Non-perturbative model of color-singlet parton systems → hadrons

Jets vs Showers

Jet clustering algorithms

Map event from low resolution scale (i.e., with many partons/hadrons, most of which are soft) to a higher resolution scale (with fewer, hard, IR-safe, jets)



Parton shower algorithms

Map a few hard partons to many softer ones

Probabilistic → closer to nature. Not uniquely invertible by any jet algorithm*

(* See "Qjets" for a probabilistic jet algorithm, arXiv:1201.1914)

(* See "Sector Showers" for a deterministic shower, arXiv:1109.3608)

(PYTHIA)

PYTHIA anno 1978

(then called JETSET)

LU TP 78-18
November, 1978

A Monte Carlo Program for Quark Jet Generation

T. Sjöstrand, B. Söderberg

A Monte Carlo computer program is presented, that simulates the fragmentation of a fast parton into a jet of mesons. It uses an iterative scaling scheme and is compatible with the jet model of Field and Feynman.

Note:
Field-Feynman was an early fragmentation model
Now superseded by the String (in PYTHIA) and Cluster (in HERWIG & SHERPA) models.

```
      SUBROUTINE JETGEN(N)
      COMMON /JET/ K(100,2), P(100,5)
      COMMON /PAR/ PUD, PS1, SIGMA, CX2, EBEG, WFIN, IFLBEG
      COMMON /DATA1/ MESO(9,2), CMIX(6,2), PMAS(19)
      IFLSGN=(10-IFLBEG)/5
      W=2.*EBEG
      I=0
      IPD=0
C 1 FLAVOUR AND PT FOR FIRST QUARK
      IFL1=IABS(IFLBEG)
      PT1=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI1=6.2832*RANF(0)
      PX1=PT1*COS(PHI1)
      PY1=PT1*SIN(PHI1)
  100 I=I+1
C 2 FLAVOUR AND PT FOR NEXT ANTIQUARK
      IFL2=1+INT(RANF(0)/PUD)
      PT2=SIGMA*SQRT(-ALOG(RANF(0)))
      PHI2=6.2832*RANF(0)
      PX2=PT2*COS(PHI2)
      PY2=PT2*SIN(PHI2)
C 3 MESON FORMED, SPIN ADDED AND FLAVOUR MIXED
      K(I,1)=MESO(3*(IFL1-1)+IFL2,IFLSGN)
      ISPIN=INT(PS1+RANF(0))
      K(I,2)=1+9*ISPIN+K(I,1)
      IF(K(I,1).LE.6) GOTO 110
      TMIX=RANF(0)
      KM=K(I,1)-6+3*ISPIN
      K(I,2)=8+9*ISPIN+INT(TMIX+CMIX(KM,1))+INT(TMIX+CMIX(KM,2))
C 4 MESON MASS FROM TABLE, PT FROM CONSTITUENTS
  110 P(I,5)=PMAS(K(I,2))
      P(I,1)=PX1+PX2
      P(I,2)=PY1+PY2
      PMTS=P(I,1)**2+P(I,2)**2+P(I,5)**2
C 5 RANDOM CHOICE OF X=(E+PZ)MESON/(E+PZ)AVAILABLE GIVES E AND PZ
      X=RANF(0)
      IF(RANF(0).LT.CX2) X=1.-X**(1./3.)
      P(I,3)=(X*W-PMTS/(X*W))/2.
      P(I,4)=(X*W+PMTS/(X*W))/2.
C 6 IF UNSTABLE, DECAY CHAIN INTO STABLE PARTICLES
  120 IPD=IPD+1
      IF(K(IPD,2).GE.8) CALL DECAY(IPD,I)
      IF(IPD.LT.I.AND.I.LE.96) GOTO 120
C 7 FLAVOUR AND PT OF QUARK FORMED IN PAIR WITH ANTIQUARK ABOVE
      IFL1=IFL2
      PX1=-PX2
      PY1=-PY2
C 8 IF ENOUGH E+PZ LEFT, GO TO 2
      W=(1.-X)*W
      IF(W.GT.WFIN.AND.I.LE.95) GOTO 100
      N=I
      RETURN
      END
```

(PYTHIA)

PYTHIA anno 2013

(now called PYTHIA 8)

LU TP 07-28 (CPC 178 (2008) 852)
October, 2007

A Brief Introduction to PYTHIA 8.1

T. Sjöstrand, S. Mrenna, P. Skands

The Pythia program is a standard tool for the generation of high-energy collisions, comprising a coherent set of physics models for the evolution from a few-body hard process to a complex multihadronic final state. It contains a library of hard processes and models for initial- and final-state parton showers, multiple parton-parton interactions, beam remnants, string fragmentation and particle decays. It also has a set of utilities and interfaces to external programs. […]

~ 100,000 lines of C++

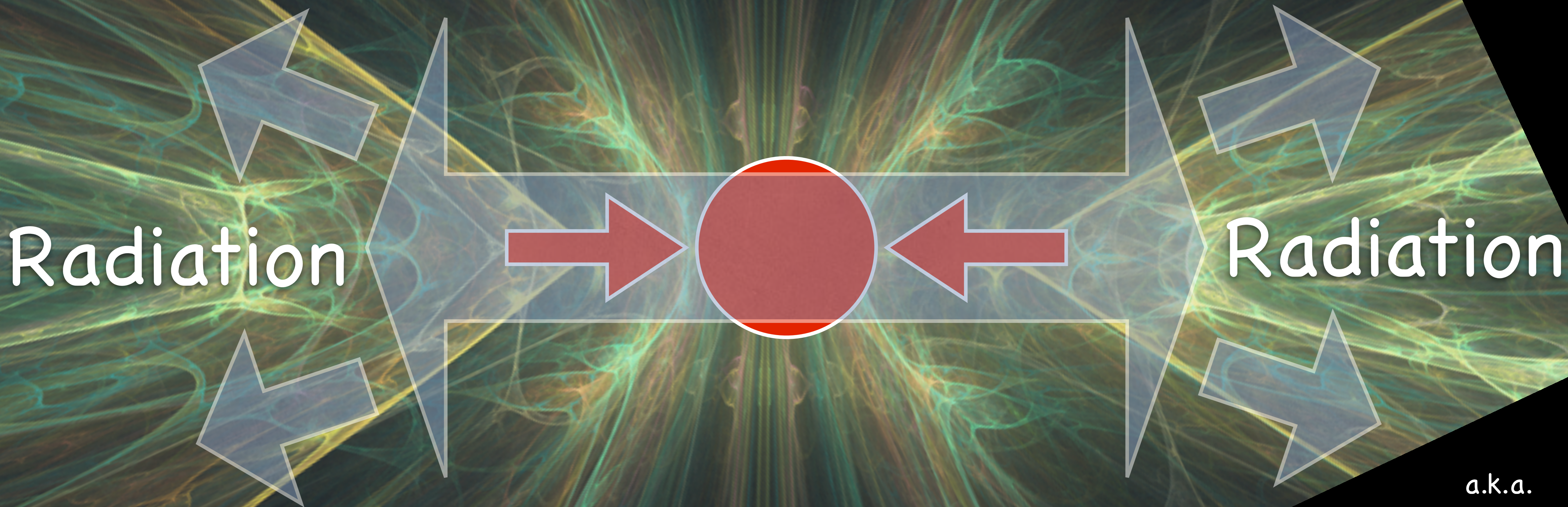
What a modern MC generator has inside:

- Hard Processes (internal, interfaced, or via Les Houches events)
- BSM (internal or via interfaces)
- PDFs (internal or via interfaces)
- Showers (internal or inherited)
- Multiple parton interactions
- Beam Remnants
- String Fragmentation
- Decays (internal or via interfaces)
- Examples and Tutorial
- Online HTML / PHP Manual
- Utilities and interfaces to external programs

(some) Physics

cf. equivalent-photon approximation
Weiszäcker, Williams
~ 1934

Charges Stopped or kicked

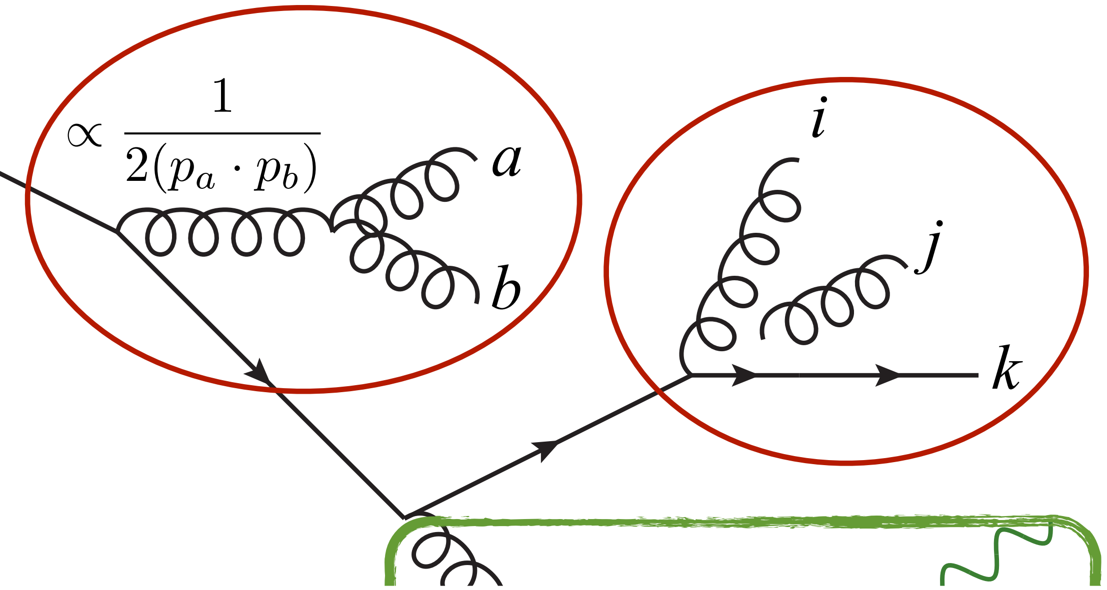


a.k.a.
Bremsstrahlung
Synchrotron Radiation

The harder they stop, the harder the fluctations that continue to become radiation

Jets ≈ Fractals

- **Most bremsstrahlung** is driven by divergent propagators → simple structure
- **Amplitudes factorize in singular limits** (→ universal "conformal" or "fractal" structure)



Partons ab → "collinear":

P(z) = Altarelli-Parisi splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\to} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Gluon j → "soft":

Coherence → Parton j really emitted by (i,k) "colour antenna"

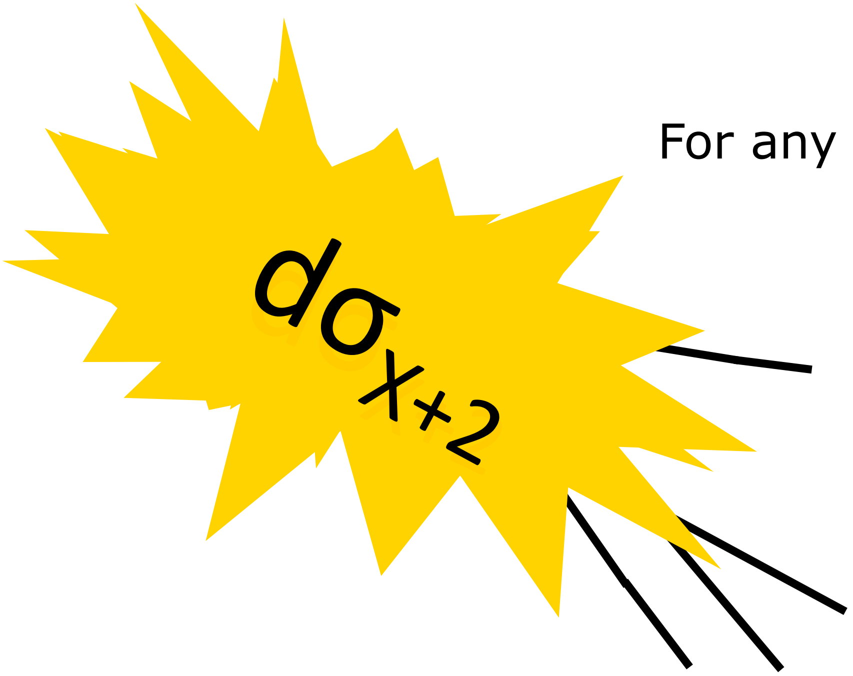
$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

+ scaling violation: $g_s^2 \to 4\pi\alpha_s(Q^2)$

Can apply this many times
→ nested factorizations

See: PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Bremsstrahlung



For any basic process $d\sigma_X$ = ✓ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2 g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad ✓$$

$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad ✓$$

$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad ...$$

Factorization in Soft and Collinear Limits

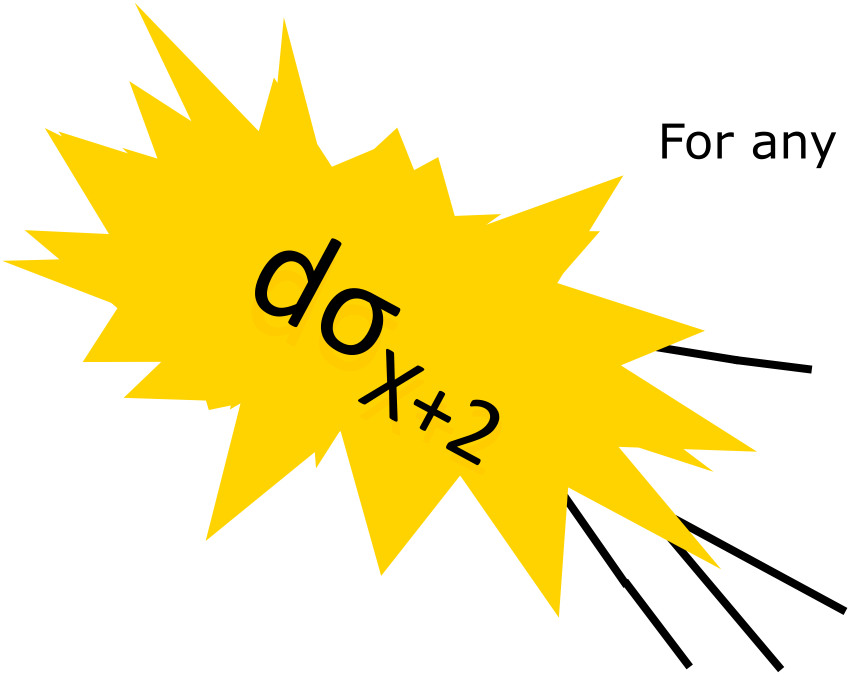
P(z) : "Altarelli-Parisi Splitting Functions" (more later)

$$|M(\ldots,p_i,p_j\ldots)|^2 \overset{i||j}{\to} g_s^2 \mathcal{C} \frac{P(z)}{s_{ij}} |M(\ldots,p_i+p_j,\ldots)|^2$$

$$|M(\ldots,p_i,p_j,p_k\ldots)|^2 \overset{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{2s_{ik}}{s_{ij}s_{jk}} |M(\ldots,p_i,p_k,\ldots)|^2$$

"Soft Eikonal" : generalizes to Dipole/Antenna Functions (more later)

Bremsstrahlung



For any basic process $d\sigma_X$ = ✓ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2 g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad ✓$$

$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad ✓$$

$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Singularities: mandated by gauge theory
Non-singular terms: process-dependent

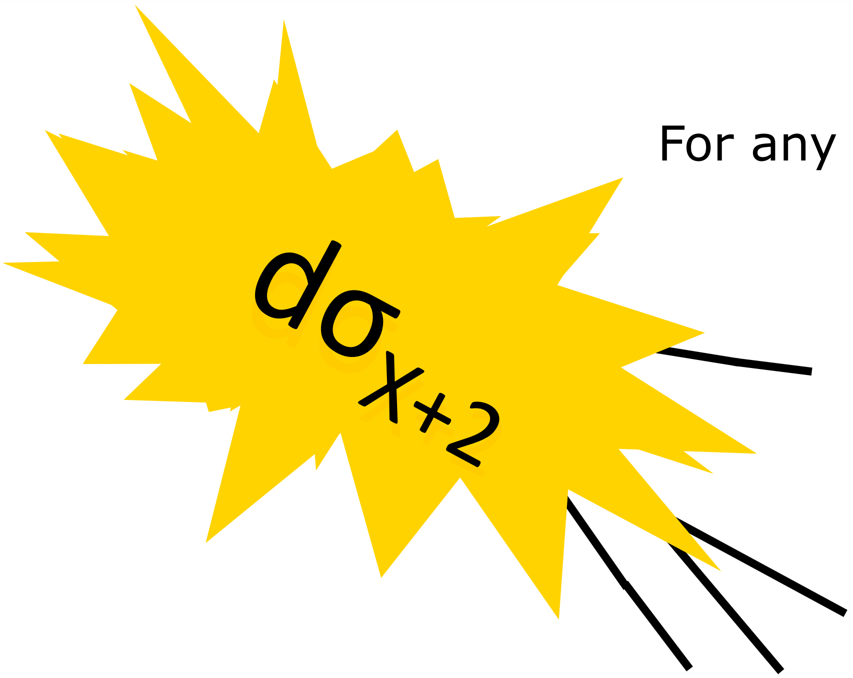
SOFT COLLINEAR

$$\frac{|\mathcal{M}(Z^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(Z^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]$$

$$\frac{|\mathcal{M}(H^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(H^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right]$$

SOFT COLLINEAR+F

Bremsstrahlung



For any basic process $d\sigma_X = \checkmark$ (calculated process by process)

$$d\sigma_{X+1} \sim N_C 2 g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \qquad \checkmark$$

$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \qquad \checkmark$$

$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \qquad \ldots$$

Iterated factorization

Gives us a universal approximation to ∞-order tree-level cross sections.

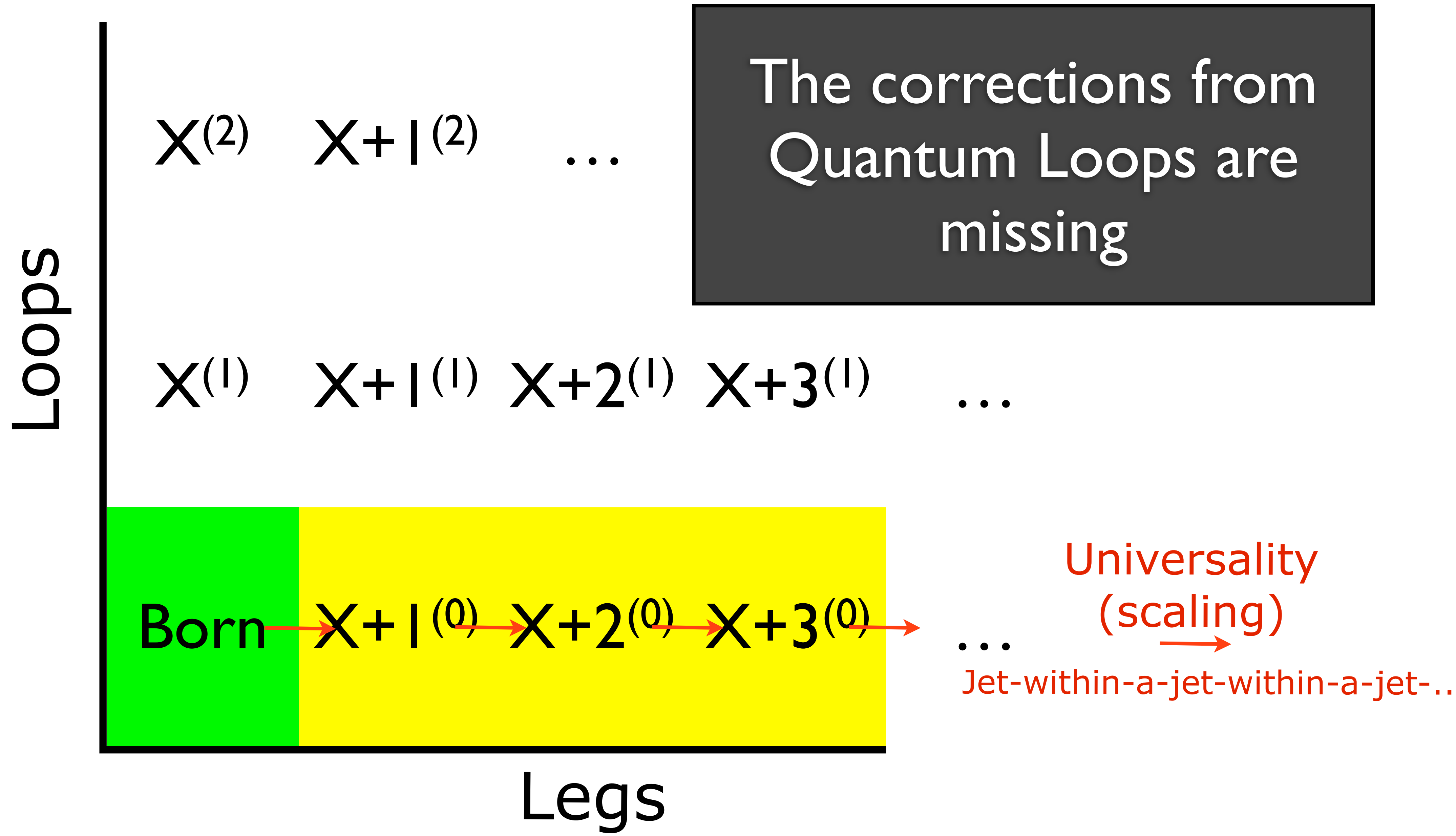
Exact in singular (strongly ordered) limit.

Finite terms (non-universal) → Uncertainties for non-singular (hard) radiation

But something is not right … Total σ would be infinite …

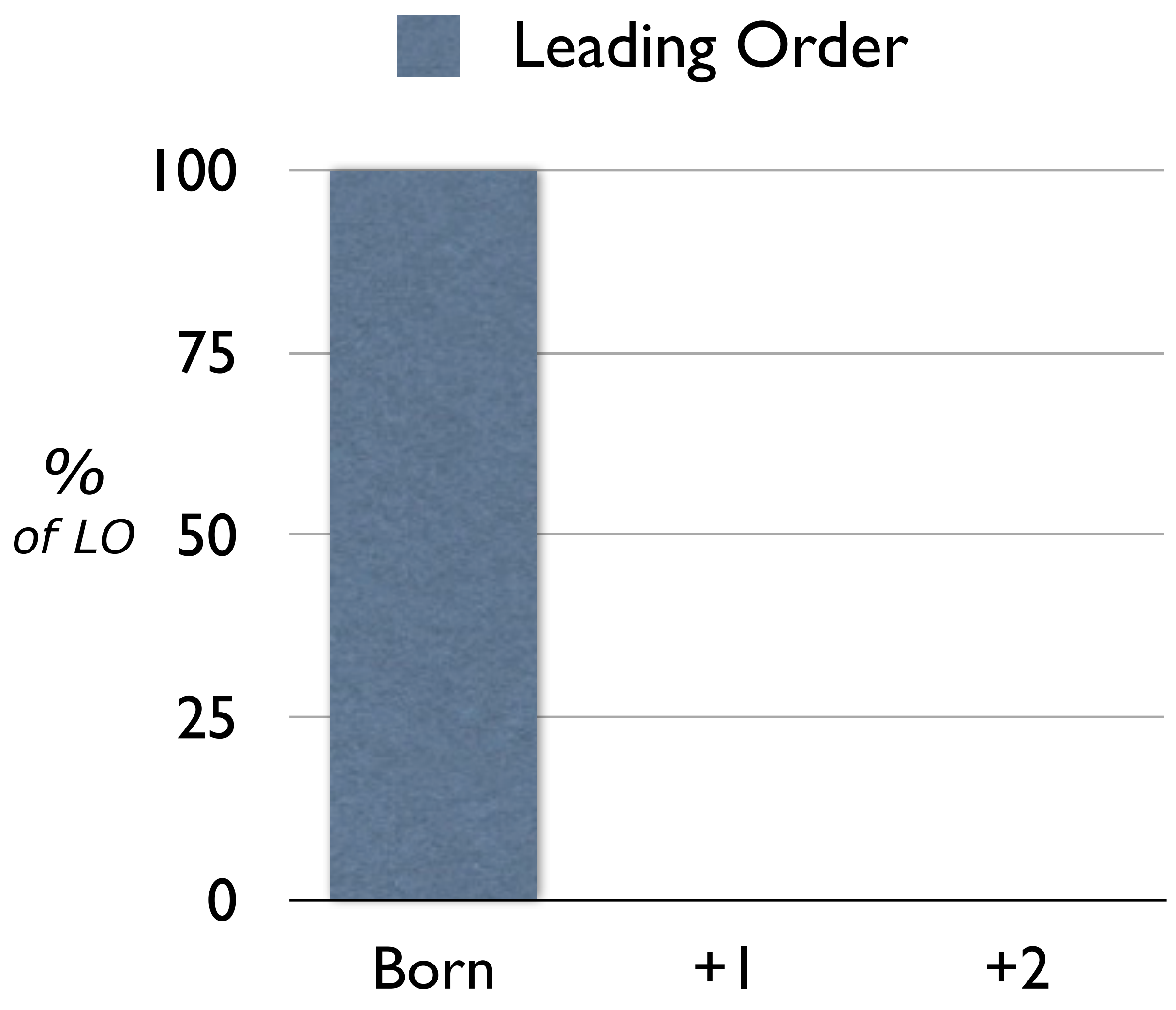
Loops and Legs

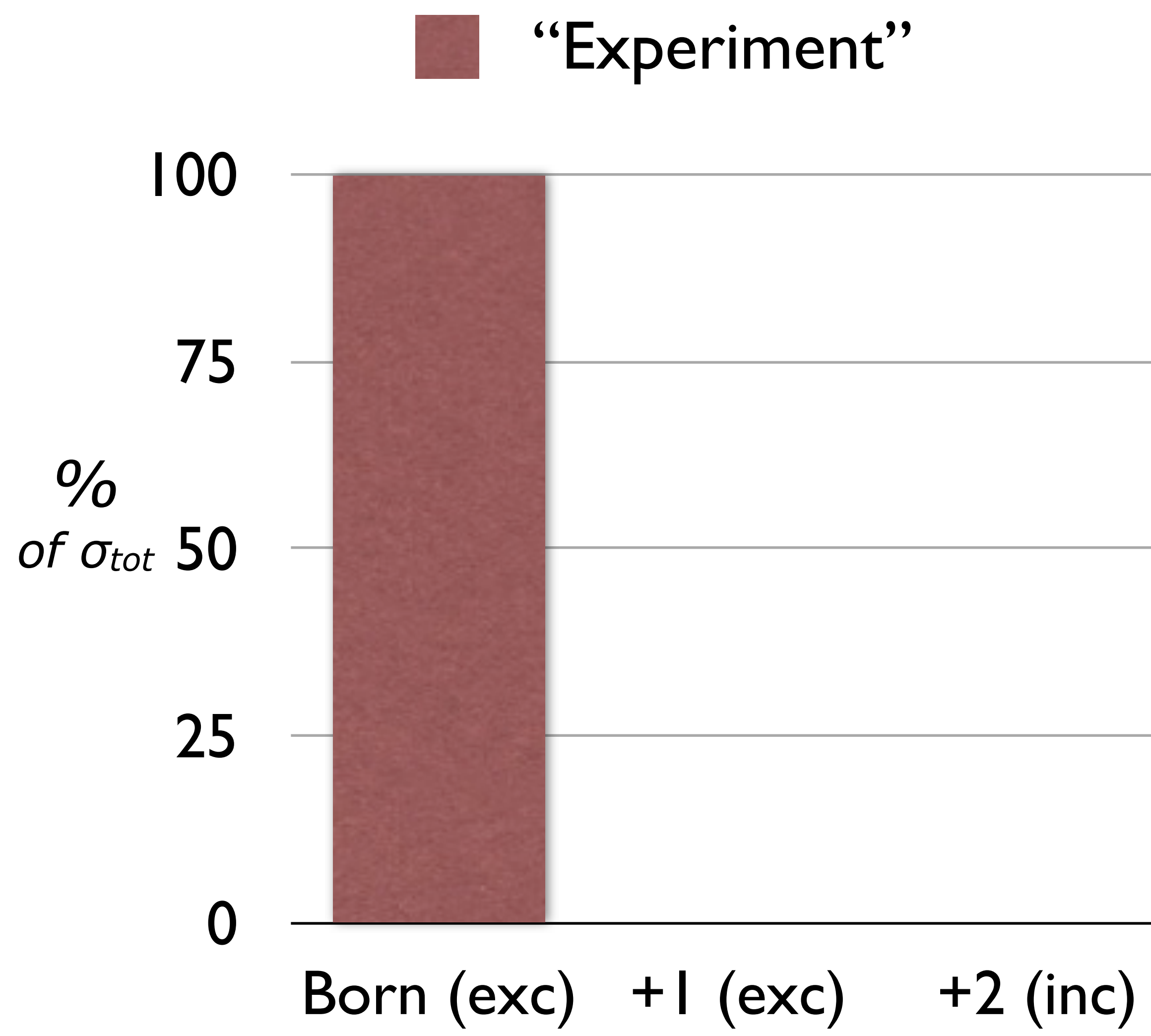
Coefficients of the Perturbative Series



Evolution

$$Q \sim Q_X$$



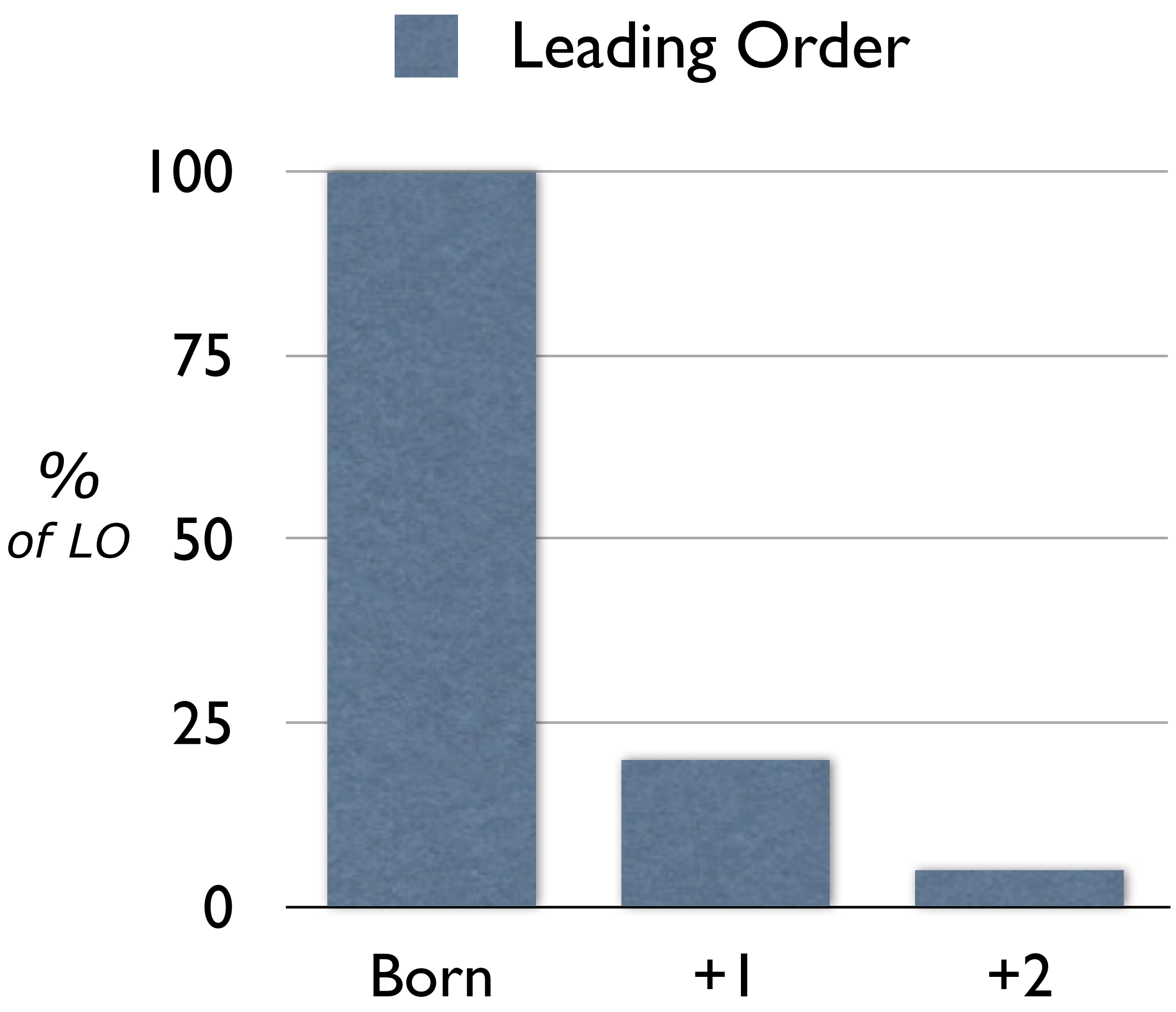


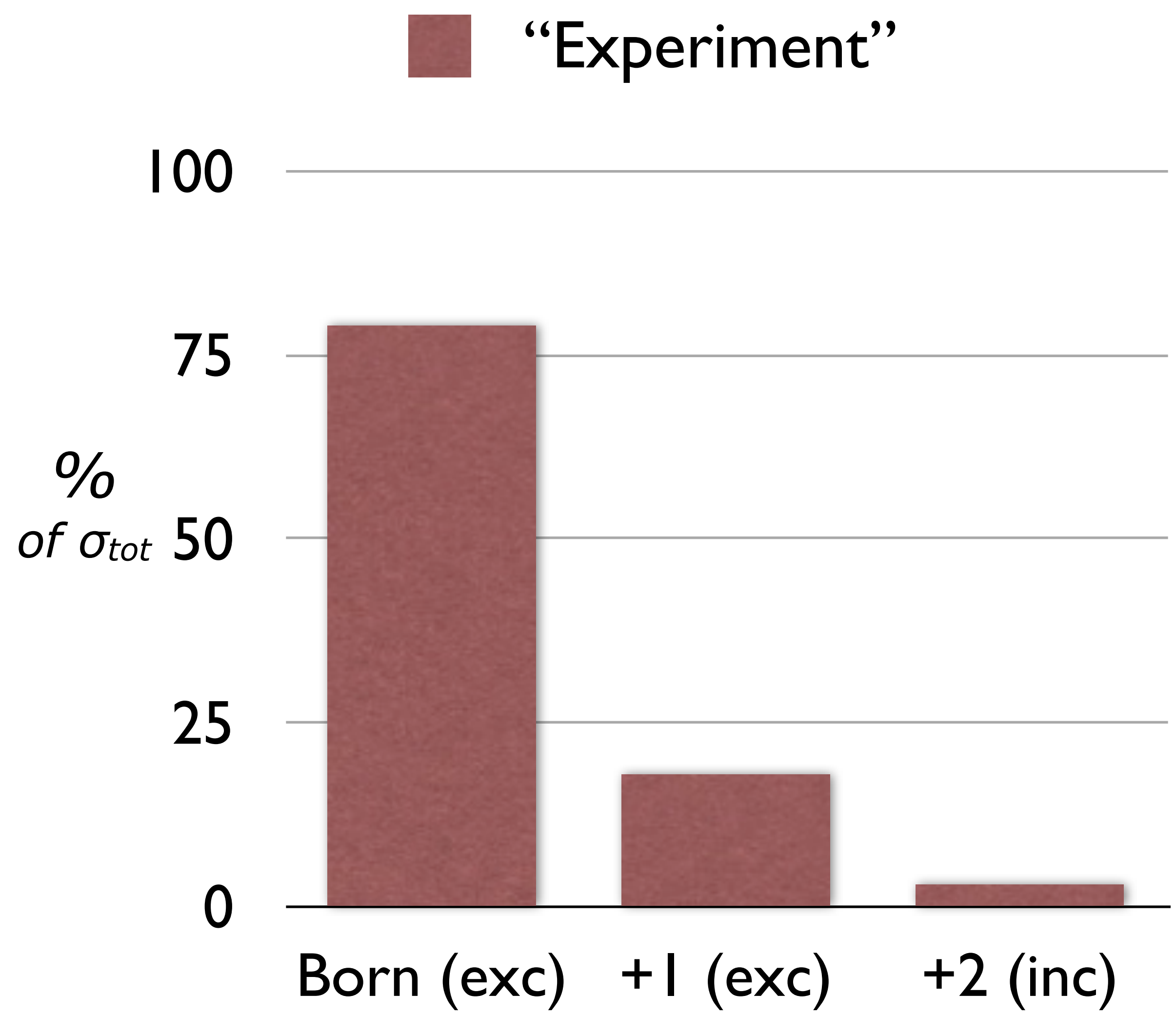
Exclusive = n and only n jets

Inclusive = n or more jets

Evolution

$$Q \sim \frac{Q_X}{\text{``A few''}}$$



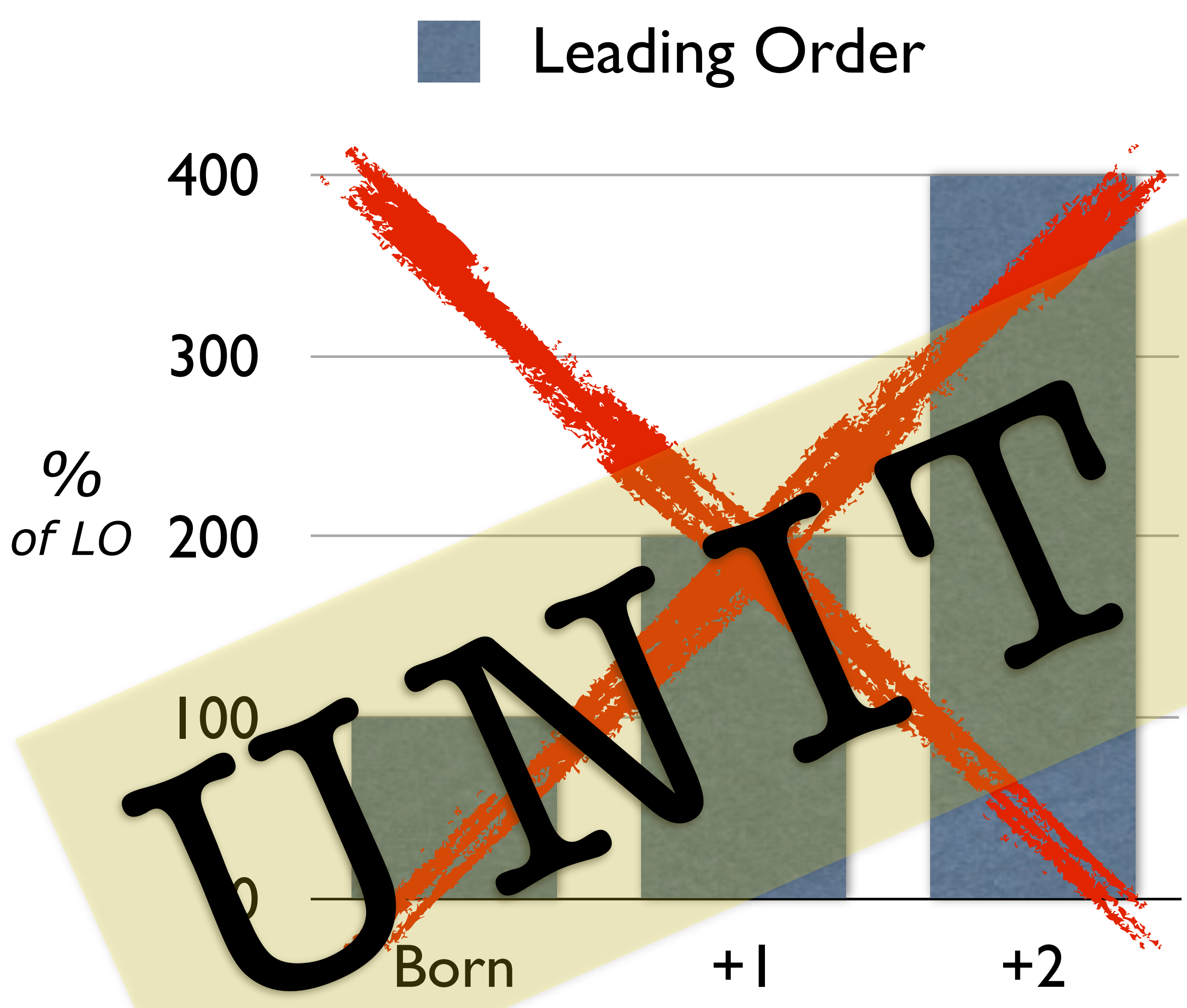


Exclusive = n and only n jets

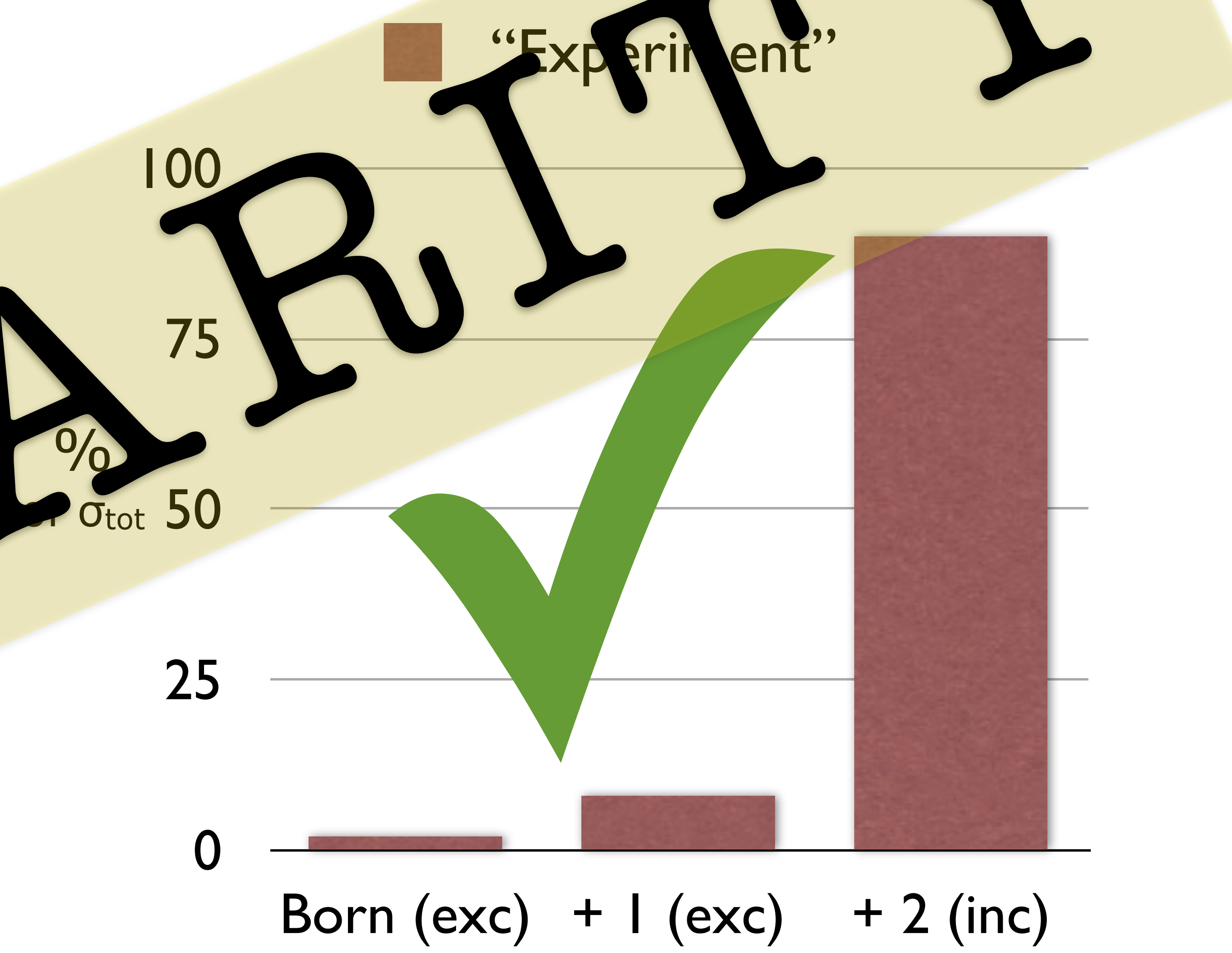
Inclusive = n or more jets

Evolution

$$Q \ll Q_X$$



Cross Section Diverges



Cross Section Remains = Born (IR safe)
Number of Partons Diverges (IR unsafe)

Unitarity → Evolution

Unitarity

Kinoshita-Lee-Nauenberg:
(sum over degenerate quantum states = finite)

$$\text{Loop} = -\,\text{Int(Tree)} + \text{F}$$

Parton Showers neglect F

→ *Leading-Logarithmic (LL) Approximation*

Imposed by Event *evolution*:

When (X) branches to (X+1):
Gain one (X+1). Loose one (X).

→ *evolution equation with kernel* $\frac{d\sigma_{X+1}}{d\sigma_X}$

Evolve in some measure of *resolution*
~ hardness, 1/time … ~ fractal scale

→ includes both real (tree) and virtual (loop) corrections

- Interpretation: the structure evolves! (example: X = 2-jets)
  - Take a jet algorithm, with resolution measure "Q", apply it to your events
  - At a very crude resolution, you find that everything is 2-jets

Evolution Equations

What we need is a differential equation

Boundary condition: a few partons defined at a high scale (Q_F)

Then evolves (or "runs") that parton system down to a low scale (the hadronization cutoff ~ 1 GeV) → It's an evolution equation in Q_F

Close analogue: nuclear decay

Evolve an unstable nucleus. Check if it decays + follow chains of decays.

Decay constant

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = c_N$$

Probability to remain undecayed in the time interval $[t_1,t_2]$

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \,\mathrm{d}t\right) = \exp\left(-c_N\,\Delta t\right)$$

$$= 1 - c_N \Delta t + \mathcal{O}(c_N^2)$$

Decay probability per unit time

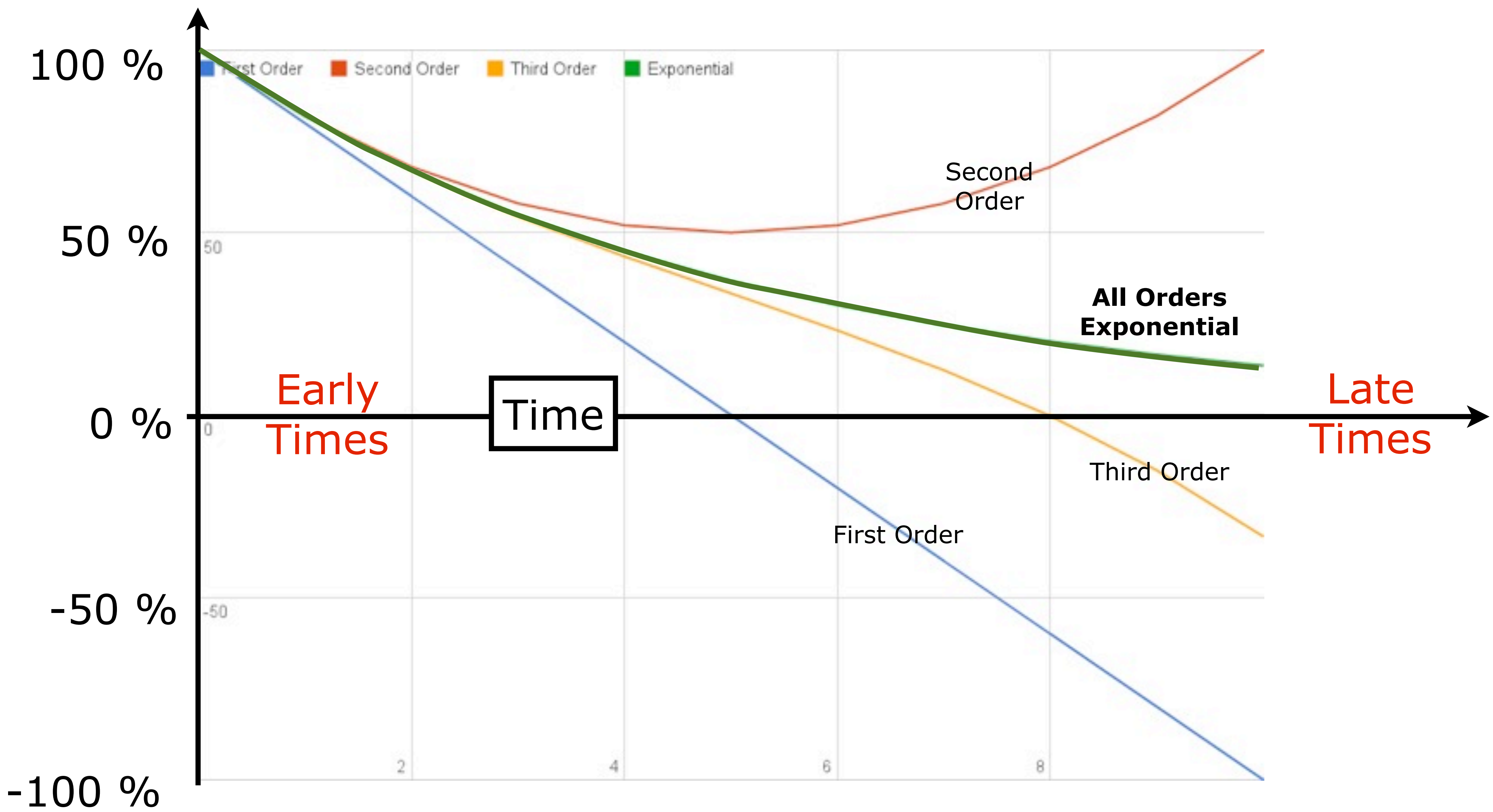
$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N\,\Delta(t_1,t)$$

(requires that the nucleus did not already decay)

$\Delta(t_1,t_2)$: "Sudakov Factor"

Nuclear Decay

Nuclei remaining undecayed after time t $= \Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \mathrm{d}t\,\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$



The Sudakov Factor

In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time t

Probability to remain undecayed in the time interval $[t_1,t_2]$

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \, \mathrm{d}t\right) \;=\; \exp\left(-c_N \, \Delta t\right)$$

The Sudakov factor for a parton system counts:

The probability that the parton system doesn't evolve (branch) when we run the factorization scale (~1/time) from a high to a low scale

Evolution probability per unit "time"

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N \, \Delta(t_1,t)$$

(replace t by shower evolution scale)

(replace c_N by proper shower evolution kernels)

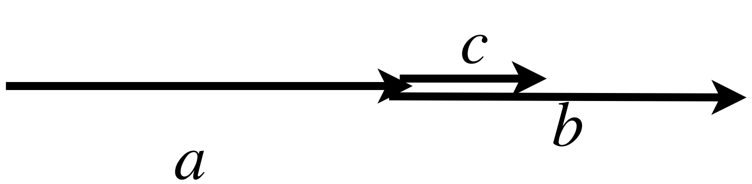
What's the evolution kernel?

Altarelli-Parisi splitting functions

Can be derived (*in the collinear limit*) from requiring invariance of the physical result with respect to Q_F → RGE

Altarelli-Parisi
(E.g., PYTHIA)

$$\mathrm{d}\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)\,\mathrm{d}t\,\mathrm{d}z\ .$$



$p_b = z\, p_a$

$p_c = (1\text{-}z)\, p_a$

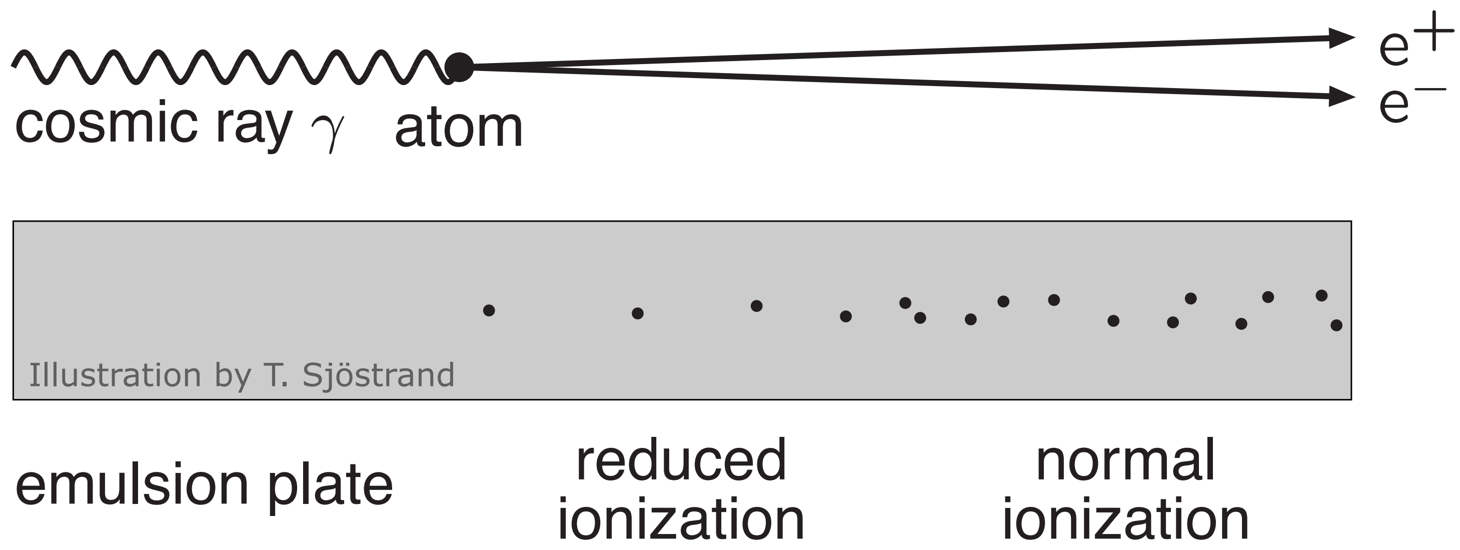
$$\begin{aligned}
P_{\mathrm{q}\to\mathrm{qg}}(z) &= C_F \frac{1+z^2}{1-z}\ , \\
P_{\mathrm{g}\to\mathrm{gg}}(z) &= N_C \frac{(1-z(1-z))^2}{z(1-z)}\ , \\
P_{\mathrm{g}\to\mathrm{q}\bar{\mathrm{q}}}(z) &= T_R\,(z^2+(1-z)^2)\ , \\
P_{\mathrm{q}\to\mathrm{q}\gamma}(z) &= e_{\mathrm{q}}^2 \frac{1+z^2}{1-z}\ , \\
P_{\ell\to\ell\gamma}(z) &= e_\ell^2 \frac{1+z^2}{1-z}\ ,
\end{aligned}$$

$$\mathrm{d}t = \frac{\mathrm{d}Q^2}{Q^2} = \mathrm{d}\ln Q^2$$

… with Q^2 some measure of "hardness"
= event/jet resolution
measuring parton virtualities / formation time / …

Coherence

QED: Chudakov effect (mid-fifties)



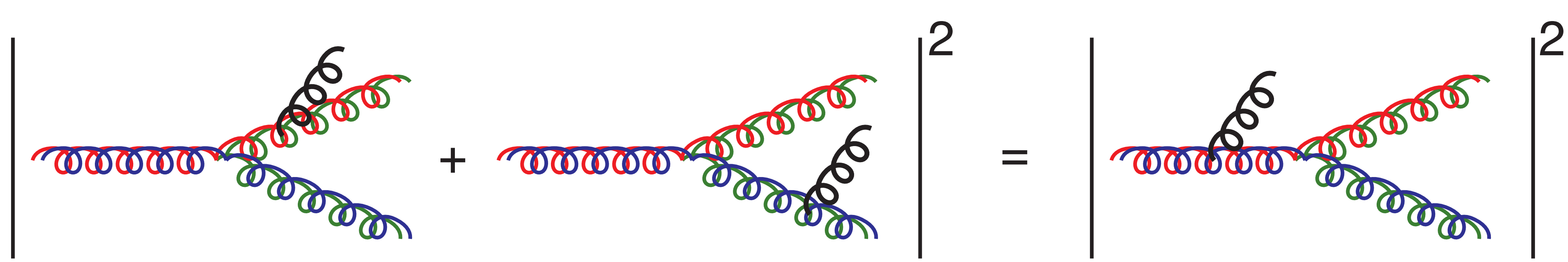
Approximations to Coherence:

Angular Ordering (HERWIG)

Angular Vetos (PYTHIA)

Coherent Dipoles/Antennae (ARIADNE, Catani-Seymour, VINCIA)

QCD: colour coherence for **soft** gluon emission



→ an example of an interference effect that can be treated probabilistically

More interference effects can be included by matching to full matrix elements

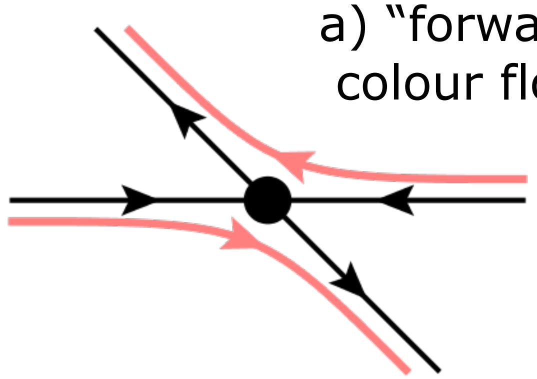
Example taken from: Ritzmann, Kosower, PS, PLB718 (2013) 1345

Example: quark-quark scattering in hadron collisions

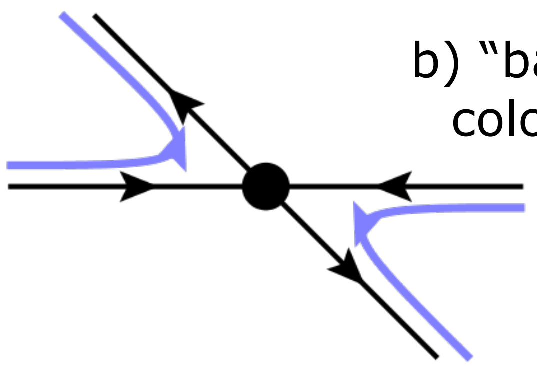
Consider one specific phase-space point (eg scattering at 45°)

2 possible colour flows: a and b

a) "forward" colour flow



b) "backward" colour flow



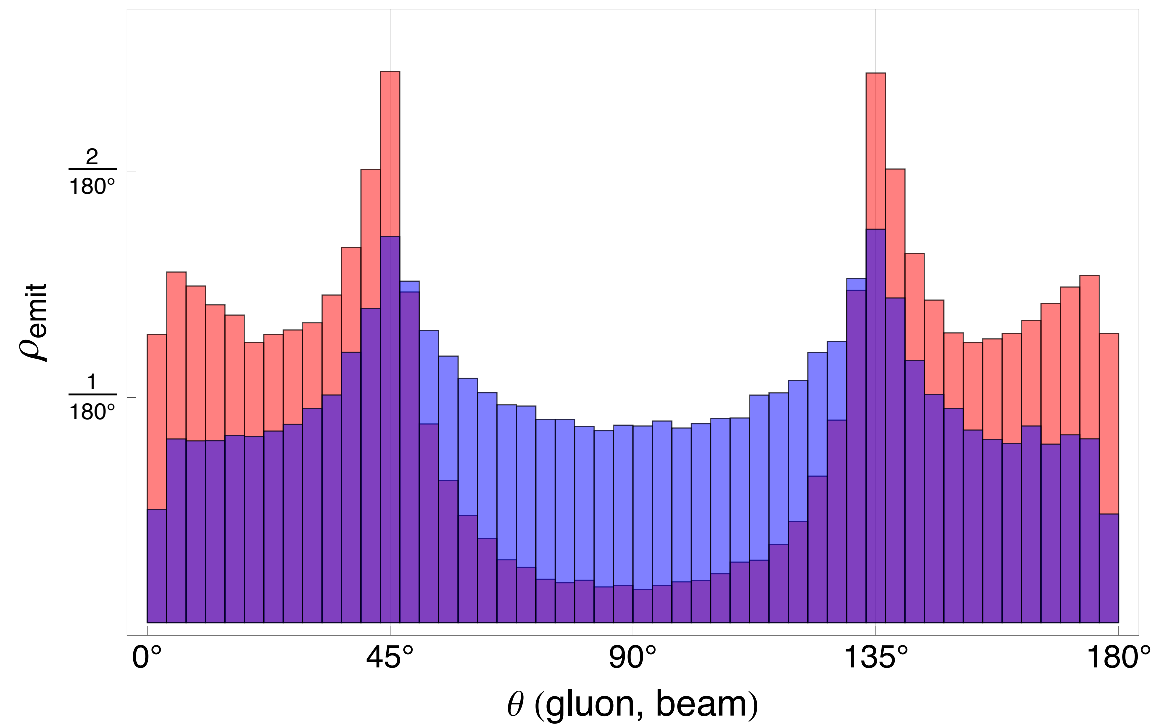


Figure 4: Angular distribution of the first gluon emission in $qq \to qq$ scattering at 45°, for the two different color flows. The light (red) histogram shows the emission density for the forward flow, and the dark (blue) histogram shows the emission density for the backward flow.

Another good recent example is the SM contribution to the Tevatron top-quark forward-backward asymmetry from coherent showers, see: PS, Webber, Winter, JHEP 1207 (2012) 151

Antennae

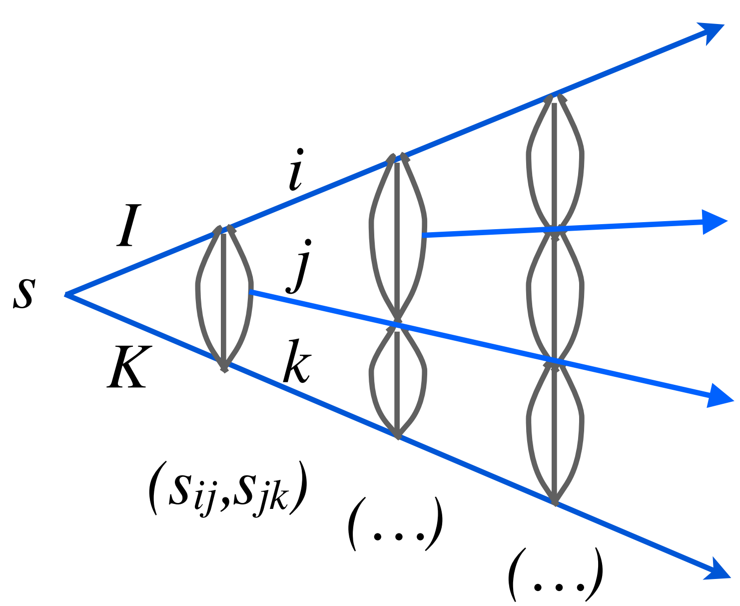
Observation: the evolution kernel is responsible for generating real radiation.

→ Choose it to be the ratio of the real-emission matrix element to the Born-level matrix element

→ AP in coll limit, but also includes the Eikonal for soft radiation.

Dipole-Antennae
(E.g., ARIADNE, VINCIA)

$$\mathrm{d}\mathcal{P}_{IK\to ijk} = \frac{\mathrm{d}s_{ij}\mathrm{d}s_{jk}}{16\pi^2 s}\; a(s_{ij}, s_{jk})$$



2→3 instead of 1→2
(→ all partons on shell)

$$a_{q\bar{q}\to qg\bar{q}} = \frac{2C_F}{s_{ij}s_{jk}}\left(2s_{ik}s + s_{ij}^2 + s_{jk}^2\right)$$

$$a_{qg\to qgg} = \frac{C_A}{s_{ij}s_{jk}}\left(2s_{ik}s + s_{ij}^2 + s_{jk}^2 - s_{ij}^3\right)$$

$$a_{gg\to ggg} = \frac{C_A}{s_{ij}s_{jk}}\left(2s_{ik}s + s_{ij}^2 + s_{jk}^2 - s_{ij}^3 - s_{jk}^3\right)$$

$$a_{qg\to q\bar{q}'q'} = \frac{T_R}{s_{jk}}\left(s - 2s_{ij} + 2s_{ij}^2\right)$$

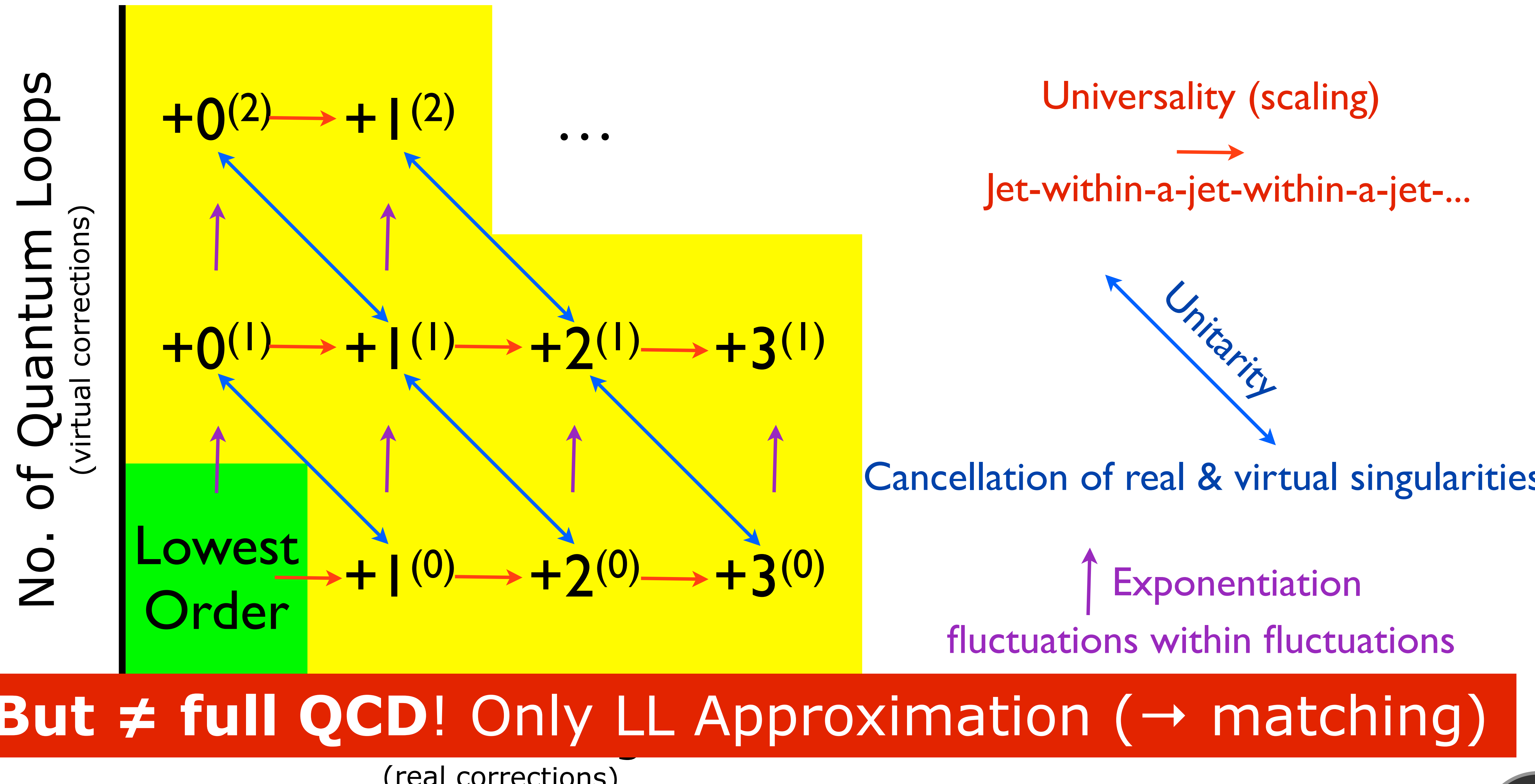
$$a_{gg\to g\bar{q}'q'} = a_{qg\to q\bar{q}'q'}$$

… + non-singular terms

Bootstrapped Perturbation Theory

Start from an **arbitrary lowest-order** process (green = QFT amplitude squared)

Parton showers generate the bremsstrahlung terms of the rest of the perturbative series (approximate infinite-order resummation)



But ≠ full QCD! Only LL Approximation (→ matching)

The Shower Operator

Born $\left.\frac{d\sigma_H}{d\mathcal{O}}\right|_{\mathrm{Born}} = \int d\Phi_H \; |M_H^{(0)}|^2 \; \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$

H = Hard process

{p} : partons

But instead of evaluating O directly on the Born final state, first insert a showering operator

Born + shower $\left.\frac{d\sigma_H}{d\mathcal{O}}\right|_{\mathcal{S}} = \int d\Phi_H \; |M_H^{(0)}|^2 \; \mathcal{S}(\{p\}_H, \mathcal{O})$

{p} : partons

S : showering operator

Unitarity: to first order, S does nothing

$$\mathcal{S}(\{p\}_H, \mathcal{O}) = \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_H)\right) \; + \; \mathcal{O}(\alpha_s)$$

The Shower Operator

To ALL Orders

(Markov Chain)

$$S(\{p\}_X, \mathcal{O}) = \Delta(t_{\mathrm{start}}, t_{\mathrm{had}})\delta(\mathcal{O}-\mathcal{O}(\{p\}_X))$$

"Nothing Happens" → "Evaluate Observable"

$$-\int_{t_{\mathrm{start}}}^{t_{\mathrm{had}}} \mathrm{d}t \frac{\mathrm{d}\Delta(t_{\mathrm{start}}, t)}{\mathrm{d}t} S(\{p\}_{X+1}, \mathcal{O})$$

"Something Happens" → "Continue Shower"

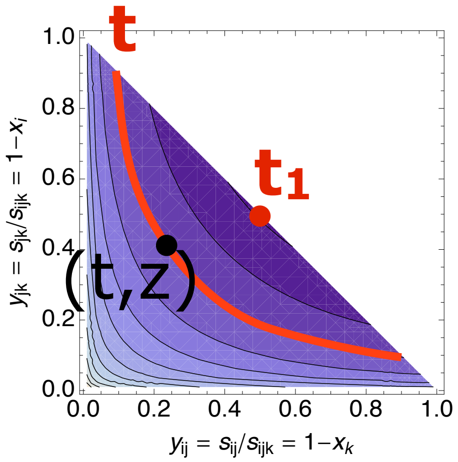
All-orders Probability that nothing happens

$$\Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \mathrm{d}t \, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$$

(Exponentiation)
Analogous to nuclear decay
N(t) ≈ N(0) exp(-ct)

A Shower Algorithm

Note: on this slide, I use results from the theory of Random numbers, interesting in itself but would need more time to give details



1. Generate Random Number, R ∈ [0,1]

Solve equation $R = \Delta(t_1, t)$ for t (with starting scale t_1)

Analytically for simple splitting kernels,
else numerically (or by trial+veto)
→ t scale for next branching

2. Generate another Random Number, R_z ∈ [0,1]

To find second (linearly independent) phase-space invariant

Solve equation $R_z = \frac{I_z(z,t)}{I_z(z_{\max}(t),t)}$ for z (at scale t)

With the "primitive function" $I_z(z,t) = \int_{z_{\min}(t)}^{z} \mathrm{d}z \left.\frac{\mathrm{d}\Delta(t')}{\mathrm{d}t'}\right|_{t'=t}$

3. Generate a third Random Number, R_φ ∈ [0,1]

Solve equation $R_\varphi = \varphi/2\pi$ for φ → Can now do 3D branching

Perturbative Ambiguities

The final states generated by a shower algorithm will depend on

1. The choice of perturbative evolution variable(s) $t^{[i]}$.

   ← Ordering & Evolution-scale choices

2. The choice of phase-space mapping $\mathrm{d}\Phi^{[i]}_{n+1}/\mathrm{d}\Phi_n$.

   ← Recoils, kinematics

3. The choice of radiation functions a_i, as a function of the phase-space variables.

   ← Non-singular terms, Reparametrizations, Subleading Colour

4. The choice of renormalization scale function μ_R.

5. Choices of starting and ending scales.

   ← Phase-space limits / suppressions for hard radiation and choice of hadronization scale

→ gives us additional handles for uncertainty estimates, beyond just μ_R
(+ ambiguities can be reduced by including more pQCD → matching! Tomorrow)

The Tyranny of Carlo

J. D. Bjorken

"Another change that I find disturbing is the rising tyranny of Carlo. No, I don't mean that fellow who runs CERN, but the other one, with first name Monte.

The simultaneous increase in detector complexity and in computation power has made simulation techniques an essential feature of contemporary experimentation. The Monte Carlo simulation has become the major means of visualization of not only detector performance but also of physics phenomena. So far so good.

But it often happens that the physics simulations provided by the the MC generators carry the authority of data itself. They look like data and feel like data, and if one is not careful they are accepted as if they were data. All Monte Carlo codes come with a GIGO (garbage in, garbage out) warning label. But the GIGO warning label is just as easy for a physicist to ignore as that little message on a packet of cigarettes is for a chain smoker to ignore. I see nowadays experimental papers that claim agreement with QCD (translation: someone's simulation labeled QCD) and/or disagreement with an alternative piece of physics (translation: an unrealistic simulation), without much evidence of the inputs into those simulations."

Account for parameters + pertinent cross-checks and validations
Do serious effort to estimate uncertainties, by salient MC variations

Uncertainty Estimates

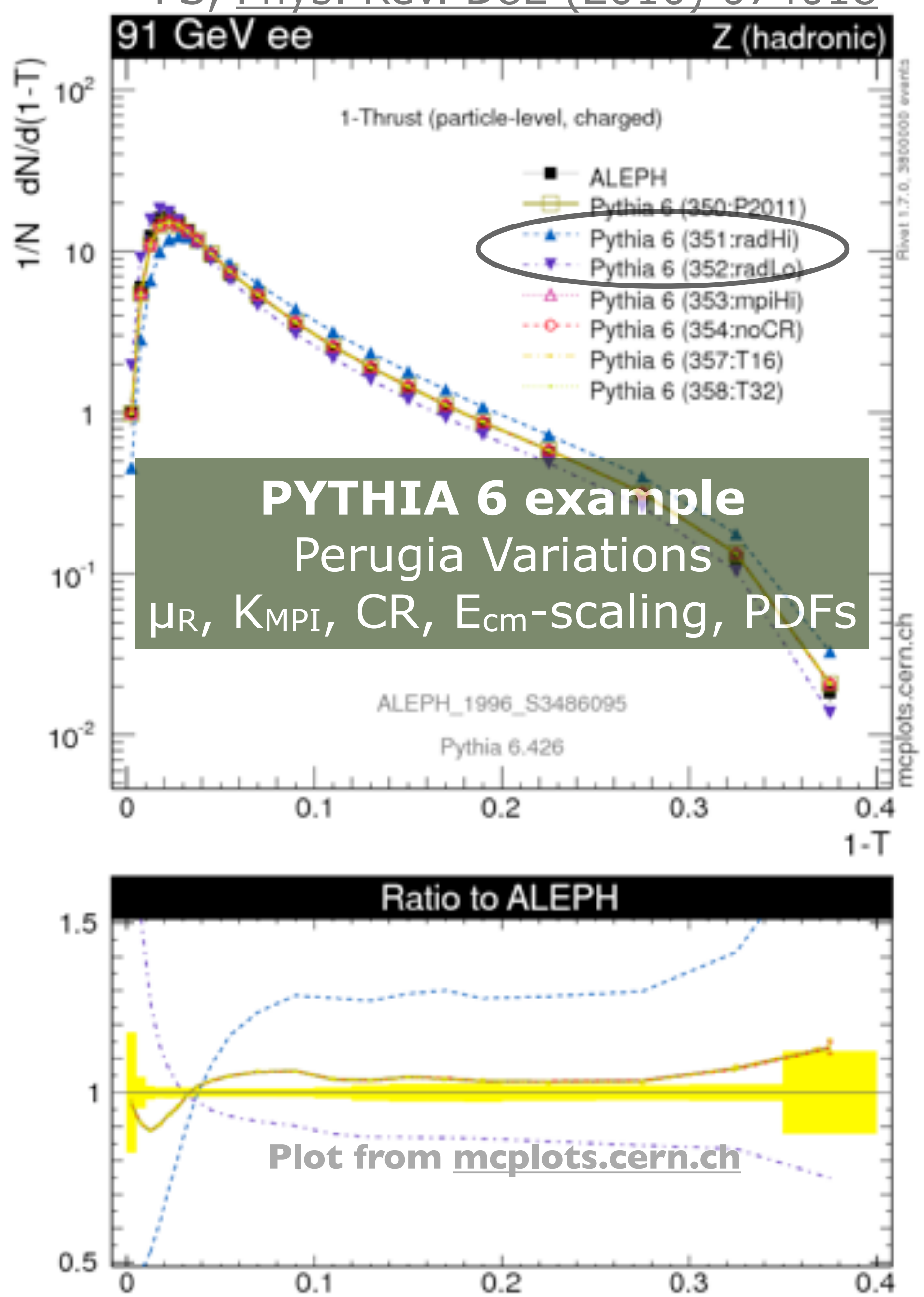
a) Authors provide specific "tune variations"

Run once for each variation→ envelope

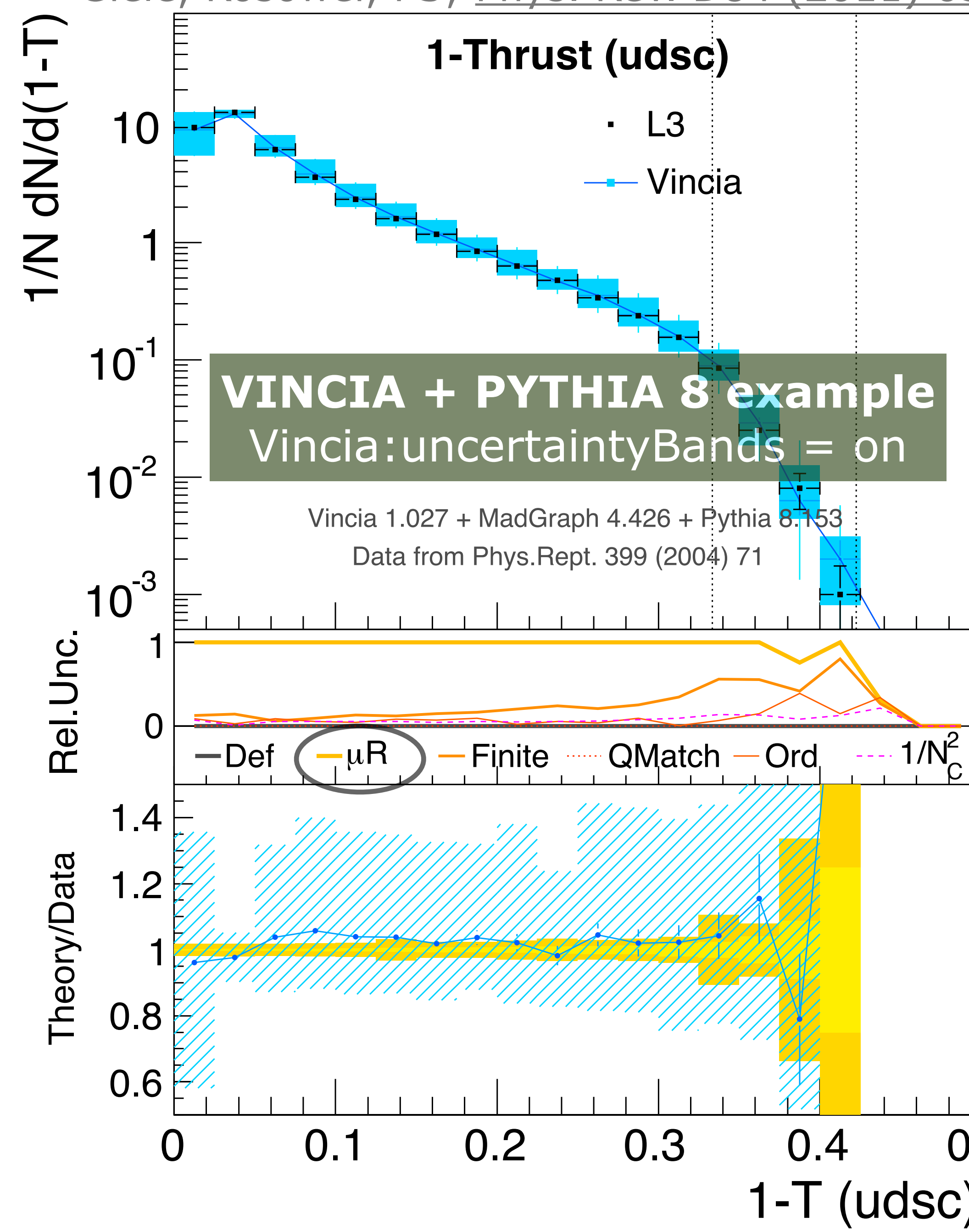
b) **One** shower run

+ unitarity-based uncertainties → envelope

PS, Phys. Rev. D82 (2010) 074018



Giele, Kosower, PS; Phys. Rev. D84 (2011) 054003

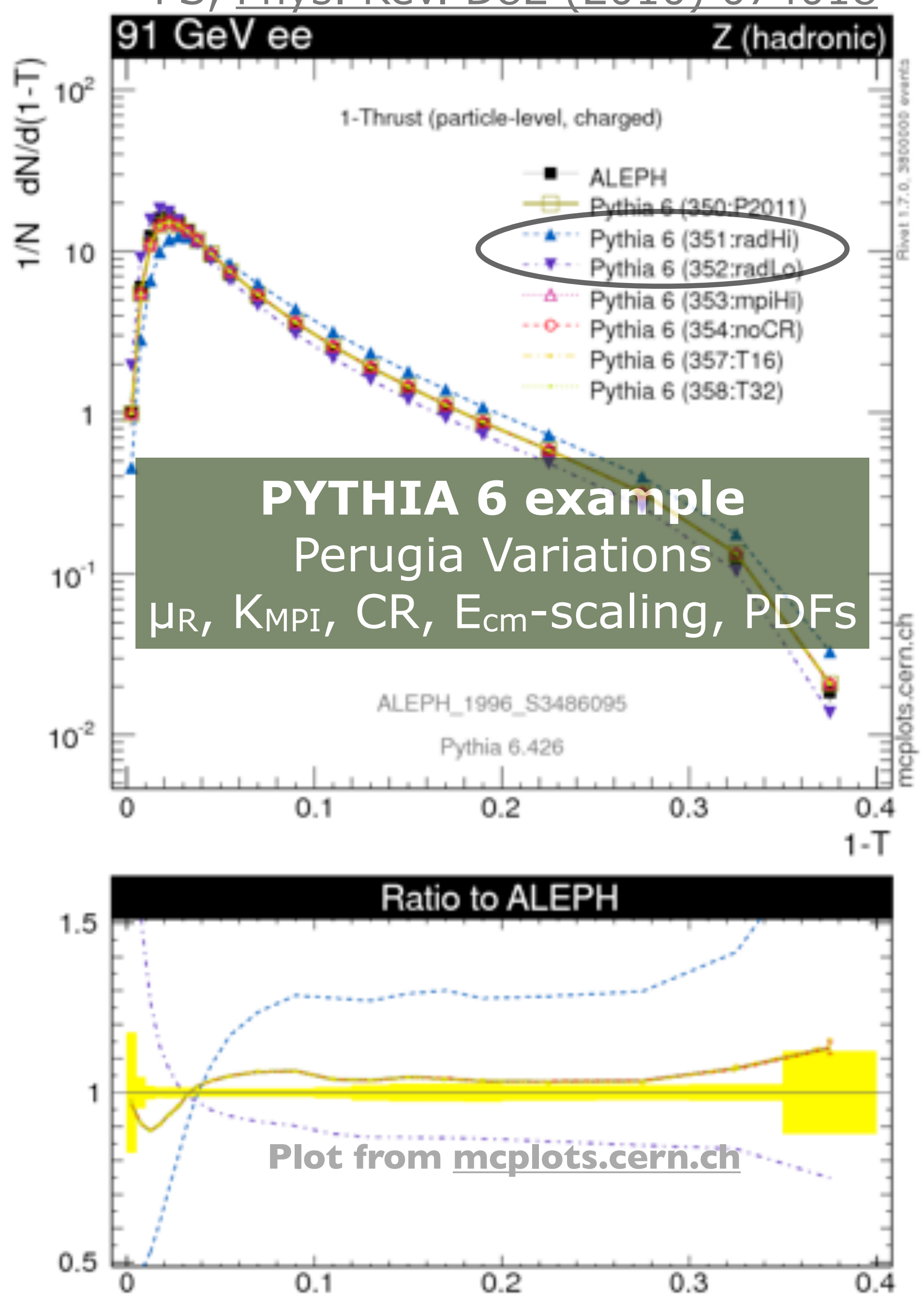


Uncertainty Estimates

a) Authors provide specific “tune variations”

Run once for each variation→ envelope

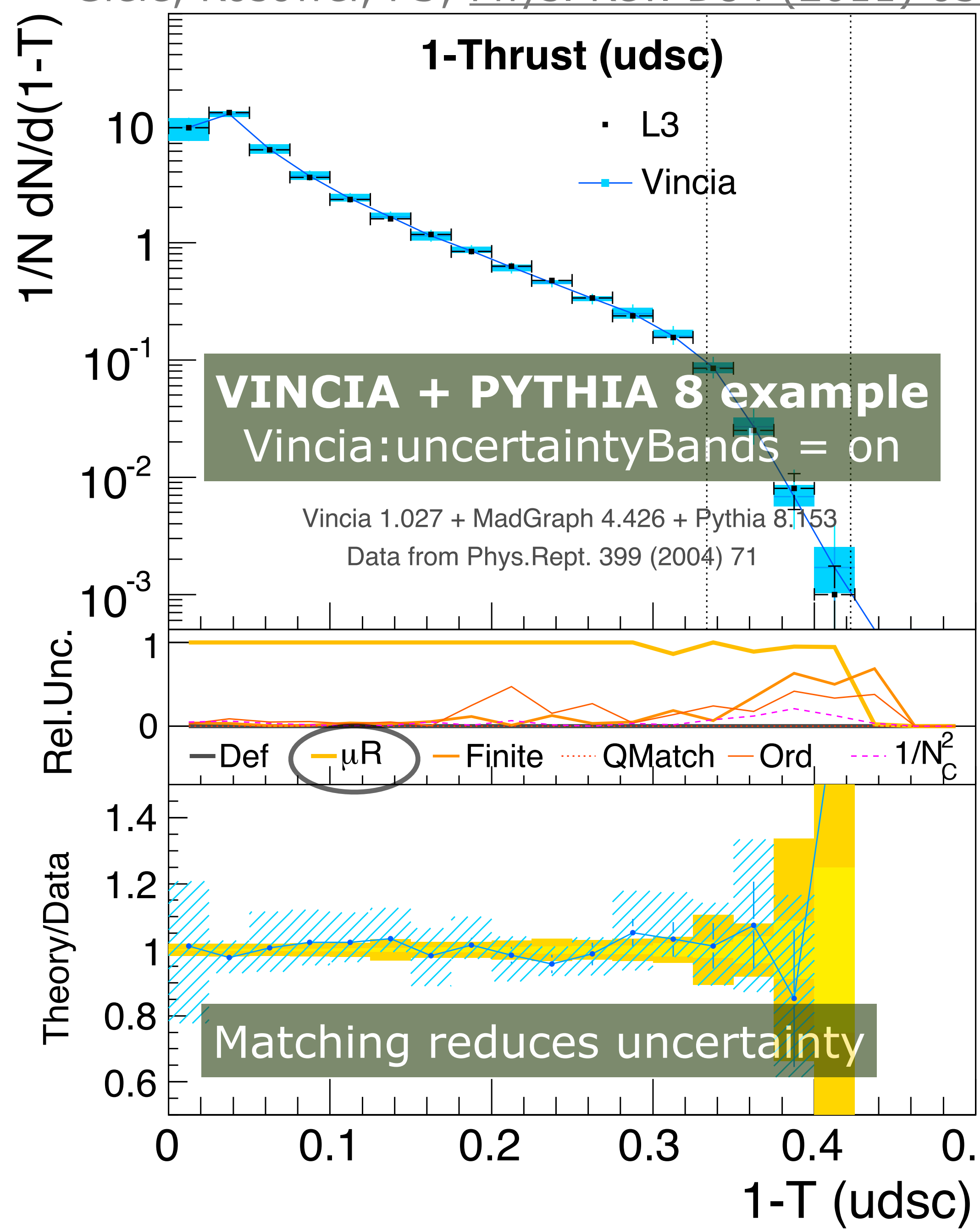
PS, Phys. Rev. D82 (2010) 074018



b) **One** shower run

+ unitarity-based uncertainties → envelope

Giele, Kosower, PS; Phys. Rev. D84 (2011) 054003



Automatic Uncertainty Estimates

One shower run (VINCIA + PYTHIA)

+ unitarity-based uncertainties → envelope

Giele, Kosower, PS; Phys. Rev. D84 (2011) 054003

```
*-------  PYTHIA Event and Cross Section Statistics  -------------------------------------------------------------*
|                                                                                                                 |
| Subprocess                                        Code |            Number of events       |      sigma +- delta    |
|                                                        |       Tried   Selected   Accepted |     (estimated) (mb)   |
|                                                        |                                   |                        |
|-----------------------------------------------------------------------------------------------------------------|
|                                                        |                                   |                        |
| f fbar -> gamma*/Z0                                221 |       10511      10000      10000 |   4.143e-05  0.000e+00 |
|                                                        |                                   |                        |
| sum                                                    |       10511      10000      10000 |   4.143e-05  0.000e+00 |
|                                                                                                                 |
*-------  End PYTHIA Event and Cross Section Statistics ----------------------------------------------------------*

*-------  VINCIA Statistics  -------------------------------------------------------------------------------------*
|                                                                                                                 |
|                                                                                                                 |
| Number of nonunity-weight events                                 =      none                                    |
| Number of negative-weight events                                 =      none                                    |
|                                                                                                                 |
|                                         weight(i)         Avg Wt   Avg Dev  rms(dev)     kUnwt     Expected effUnw |
| This run                                    i =    IsUnw     <w>    <w-1>                 1/<w>    Max Wt  <w>/MaxWt |
|    User settings                                 0    yes    1.000     0.000         -     1.000        -          - |
|    Var : VINCIA defaults                         1    yes    1.000     0.000         -     1.000     1.000      1.000 |
|    Var : AlphaS-Hi                               2     no    0.996 -3.89e-03         -     1.004    22.414   4.44e-02 |
|    Var : AlphaS-Lo                               3     no    1.020  1.99e-02         -     0.981    43.099   2.37e-02 |
|    Var : Antennae-Hi                             4     no    1.000  2.61e-04         -     1.000     5.417      0.185 |
|    Var : Antennae-Lo                             5     no    0.996 -4.33e-03         -     1.004    10.753   9.26e-02 |
|    Var : NLO-Hi                                  6    yes    1.000     0.000         -     1.000     1.000      1.000 |
|    Var : NLO-Lo                                  7    yes    1.000     0.000         -     1.000     1.000      1.000 |
|    Var : Ordering-Stronger                       8     no    1.004  4.48e-03         -     0.996    14.225   7.06e-02 |
|    Var : Ordering-mDaughter                      9     no    1.033  3.25e-02         -     0.968    55.954   1.85e-02 |
|    Var : Subleading-Color-Hi                    10     no    1.001  7.37e-04         -     0.999     1.505      0.665 |
|    Var : Subleading-Color-Lo                    11     no    1.006  6.44e-03         -     0.994     5.283      0.191 |
|                                                                                                                 |
*-------  End VINCIA Statistics ----------------------------------------------------------------------------------*
```

Summary: Parton Showers

Aim: generate events in as much detail as mother nature

→ Make stochastic choices ~ as in Nature (Q.M.) → Random numbers

Factor complete event probability into separate universal pieces, treated independently and/or sequentially (Markov-Chain MC)

Improve lowest-order perturbation theory by including 'most significant' corrections

Resonance decays (e.g., $t\to bW^+$, $W\to qq'$, $H^0\to\gamma^0\gamma^0$, $Z^0\to\mu^+\mu^-$, …)

Bremsstrahlung (FSR and ISR, exact in collinear and soft* limits)

Hard radiation (matching, discussed tomorrow)

Hadronization (strings/clusters, discussed tomorrow)

Additional Soft Physics: multiple parton-parton interactions, Bose-Einstein correlations, colour reconnections, hadron decays, …

Coherence*

Soft radiation → Angular ordering or Coherent Dipoles/Antennae

See also: MCnet Review (long): Phys.Rept. 504 (2011) 145-233 and/or PDG Review on Monte Carlo Event Generators, and/or PS, TASI Lectures (short): arXiv:1207.2389