Seeing relativity: a virtual journey around (and within) a black hole

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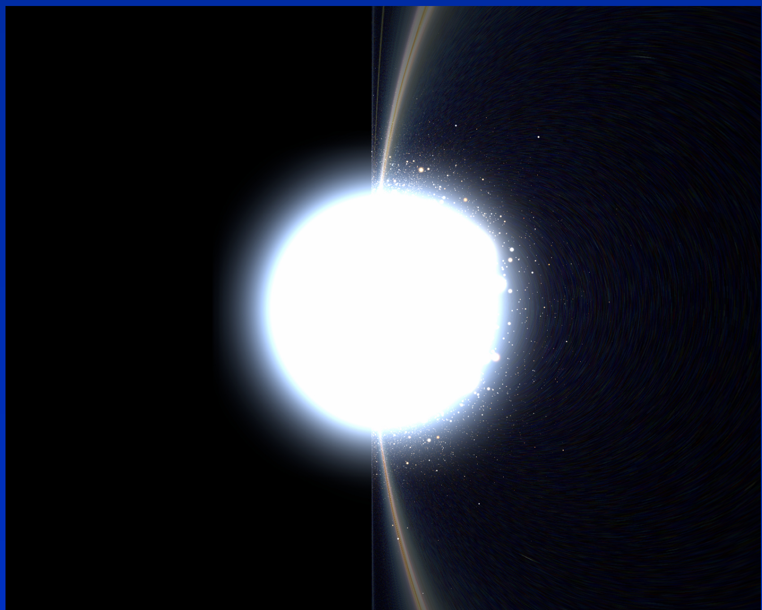


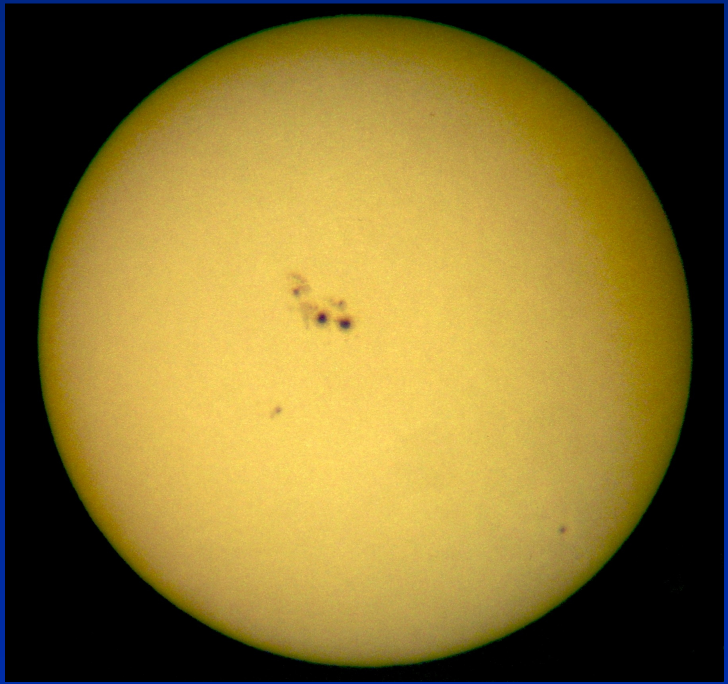
Plan

- Black holes (as considered as non relativistic objects)
- How to form them
- What do they look like

What is a black hole? (I)

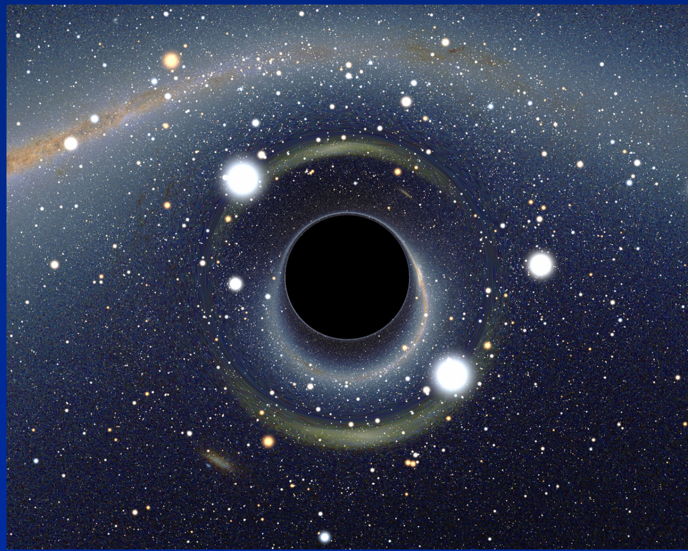
- Historically, black holes arose from an extrapolation of the concet of escape velocity
- $$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}}$$
- For Earth, it is $11.2\,\mathrm{km\,s^{-1}} \ll c$

What is a black hole? (II)



- Can be computed for any celestial body, provided one knows its mass and radius
- Examples
  - Sun : $R_\odot \sim 100\ R_{\rm Earth}$, $M_\odot \sim 300\,000\ M_{\rm Earth}$, $v^{\rm esc}_\odot \sim 600\ {\rm km/s}$
  - Sirius B : $M_{\rm Sir\ B} \sim M_\odot$, $R_{\rm Sir\ B} \sim R_{\rm Earth}$, $v^{\rm esc}_{\rm Sir\ B} \sim 6000\ {\rm km/s}$
  - Crab pulsar : $M_{\rm Crab} \sim 1{,}4\ M_\odot$, $R_{\rm Crab} \sim 10\ {\rm km}$, $v^{\rm esc}_{\rm Crab} \sim 200\,000\ {\rm km/s}$
- The last one is nolonger much small than c...

What is a black hole? (III)



- ... As early as at the end of XVIII$^{\mathrm{th}}$ century, and before knowing of the existence of any compact stellar remnant, it was already considered the possiblity that compact object with escape velocity larger than c could exist, with therefore

$$R \leq \frac{2GM}{c^2}$$

- Should such object exist, light would not escape from them (at least to infinity), and therefore nothing else could. They would be (literally) black holes
- The objects are now known to exist in Nature...
- ... Even though becoming convinced of it was a fairly long process involving people from many different fields and which took several centuries

From concept to physical reality

- Concept originates from independent work (8 years apart) from Laplace and Mitchell at the end of XVIII$^{\mathrm{th}}$ century, but...

- Detailed understanding of the nature + formation pprocess of black holes has necessitated:

  - ♣ To know that stars are Sun-like objects (1838-1872)
  - ♣ To understand stellar structure (∼1920)
  - ♣ To discover their energy source (1938)
  - ♣ To model stellar evolution as a function of their mass (∼1960)...
  - ♣ To deduce that the most massive of them could form black holes
  - ♣ Check this explicitely thanks to observations (1054-1972-????)

Which black hole populations? (I)

- At leasttwo types of black holes are known to exist
  - ♣ Stellar black holes, which are the endpoint of massive star evolution and whose formation process is well understood
    - Number: a few 10^7 per MW-like galaxy (which contains several 10^{11} stars).
    - Mass: Few 3 to a few dozens of Solar masses
  - ♣ Supermassive black holes which lie at the center of many/most/all galaxies. Formation process is not known as very massive black holes seen at very high redshift suggests that SMBH seed is more massive than stellar remnant
    - Number: 1 (or 2) per galaxy
    - Mass: from 10^5's to 10^9's Solar masses

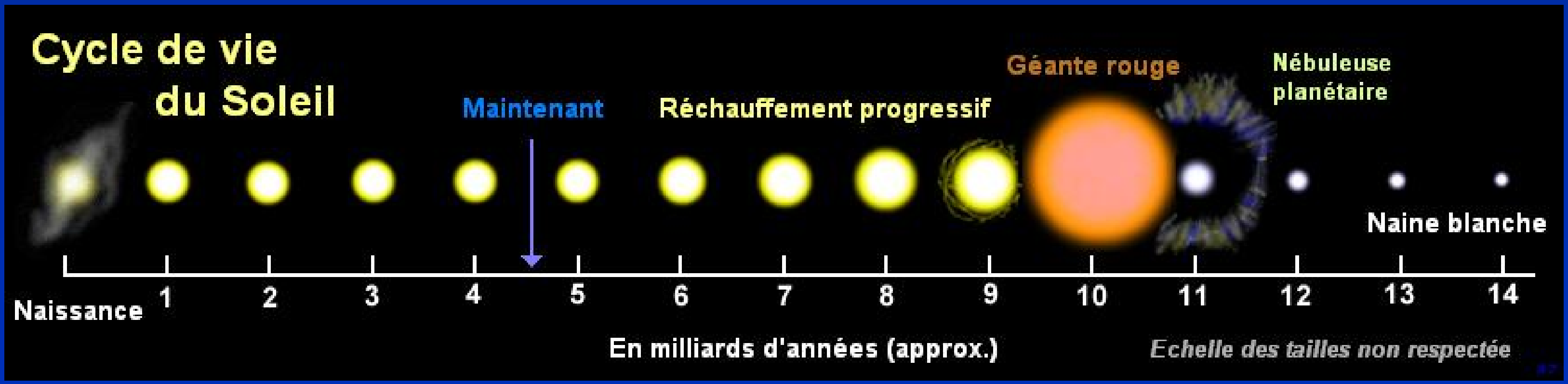
Supermassive black holes are rarer but easier to observe than stellar ones.

Which black hole populations? (II)

- There may be two other types of black holes:
  - Intermediate mass black holes
    - Number: ? (both rare and hard to spot)
    - Mass: from 10^2's to 10^4's of Solar masses (?)
    - Origin: ?
  - Primordial black holes which formed at (very) high density (constant M/R implies $\rho \propto 1/M^2$)
    - Number: ?, only upper limits
    - Mass: ?, but possibly as small as 10^{-12} Earth masses (Hawking limit)
    - Importance: None from an astrophysical point of view, but possible exquisite window of the primordial universe

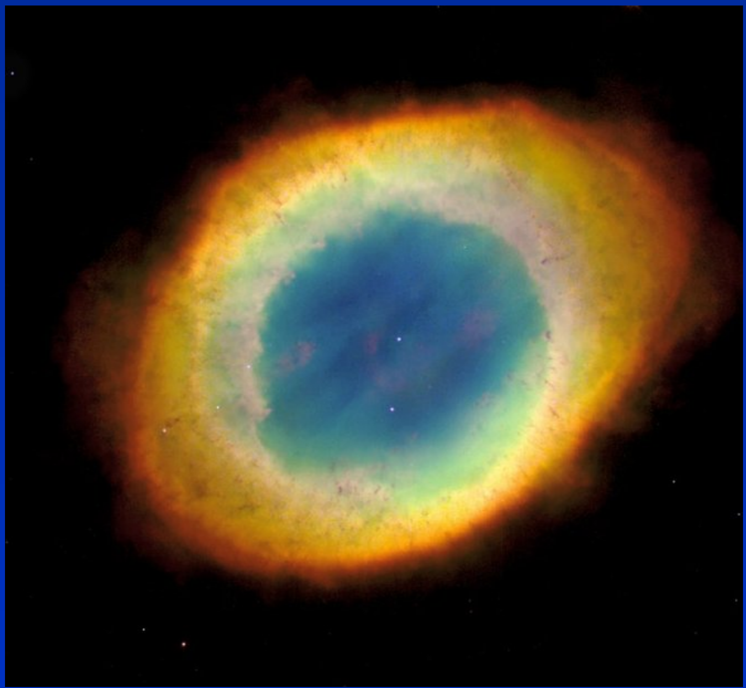
Stars

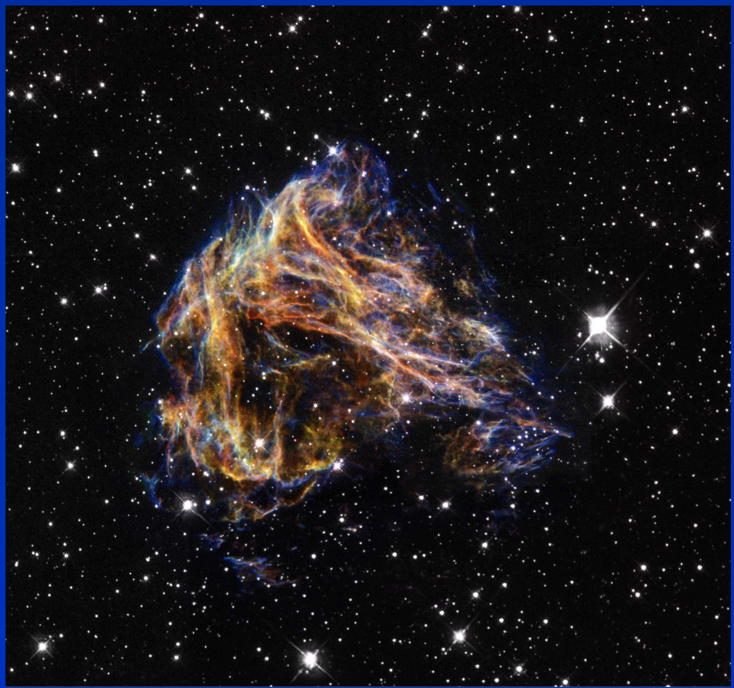
- A star is:
  - A cloud of gas which contracts because of its own gravitational field,
  - eventually fragments into smaller pieces,
  - heats,
  - ignite some nuclear reaction chain,
  - stabilizes,
  - eventually reorganizes itsenf as new reactions are ignited,
  - dies more or less violently leaving behind a compact core



The endpoints of stellar evolution

- Mostly depends on mass (and metallicity)
- Classical physics tells that inert object sustained by electron degeneracy pressure exists only if $M < 1.4 M_\odot$
- Low mass star ($< 0.5 M_\odot$) will remain as they are and shrink to helium white dwarves
- Intermediate mass stars will ignite helium, expel part of (most of) their mass in the ISM (planetary nebula) and their naked core will form a carbon-oxygen(-neon) white dwarf of mass $< 1.4 M_\odot$
- Higher mass stars ($M > 8 M_\odot$) will form an inert iron core which will grow till $1.4 M_\odot$. Then, core collapse $\rightarrow$ Supernova (because of shock wave) and $\rightarrow$ supernova remnant + Neutron star or Black hole (r.)



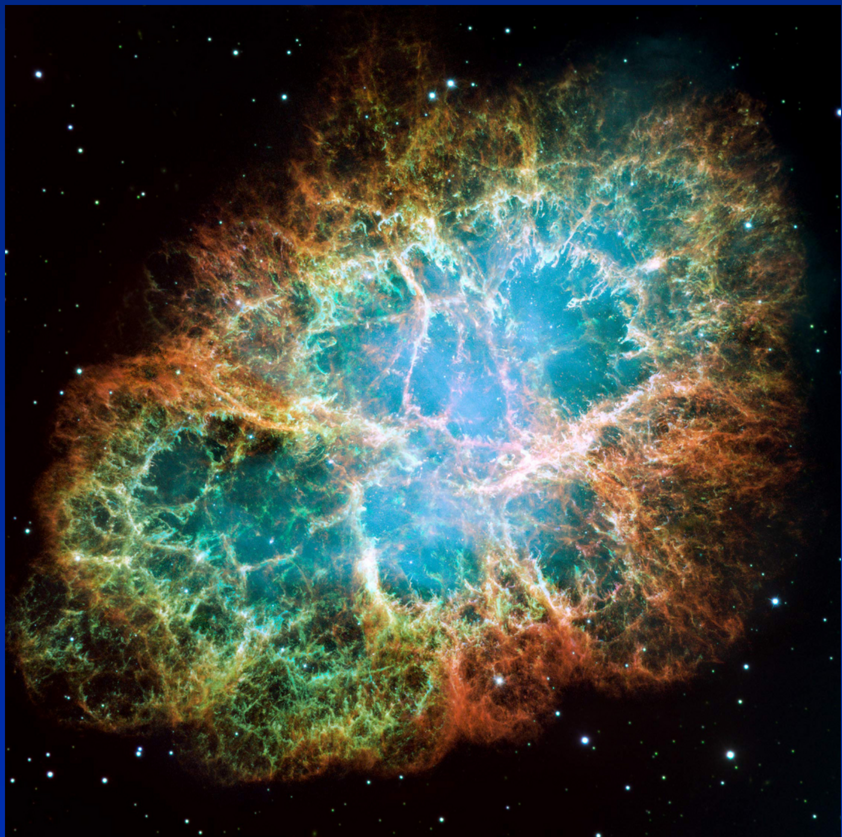
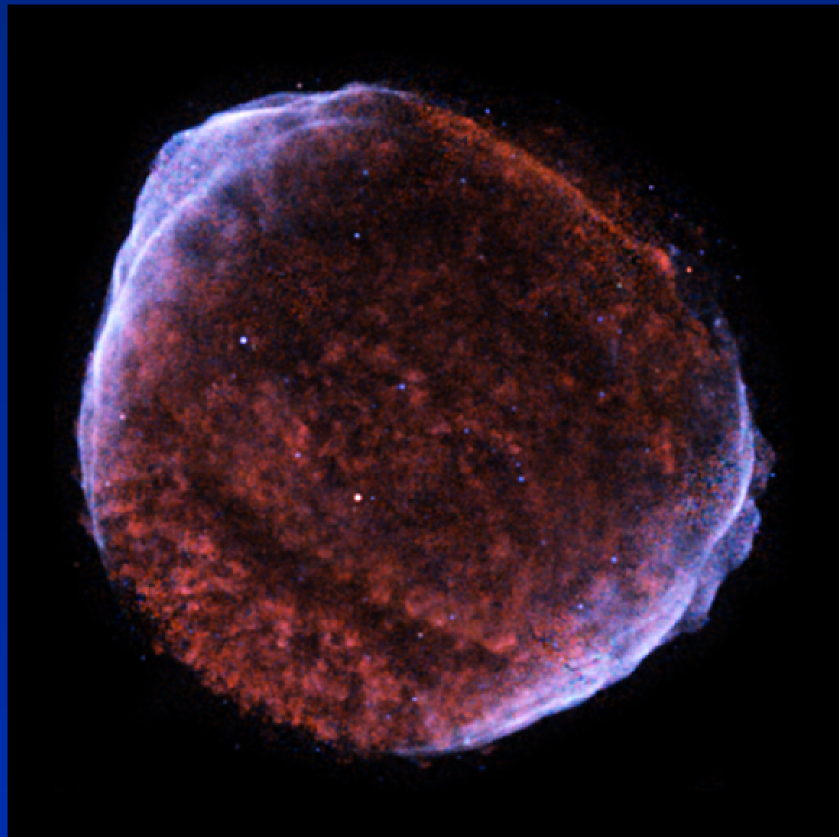


Supernovae (I)

SN 1987A, a few month before (l.) and after (r.) explosion

- This type of supernova (not to be confused with thermonuclear supernova) corresponds to:
  - Implosion of stellar core due to gravitational instability of any inert (i.e., iron) mass sustained by degeneracy pressure above Chandrasekhar mass $\longrightarrow$ Classical physics results
  - A huge ($0.1 M c^2$!) energy release under tho form of neutrinos, a small fraction of it (1% ?) interacts with outer layers $E_{\rm int} \sim 10^{46}\,{\rm J} \sim 10^{-3} M c^2$), which is enough to expel them at fast but non relativistic speed.
  - The disruption of external layers due to the shock wave emitted by the former $\longrightarrow$ Core-collapse supernova

Supernovae (II)

Some famous supernova remnants (l.: SN 1054 ; r.: SN 1006)

- The supernova leaves behind:
  - A supernova remnant that dissipates into the ISM in a few 10^5 years.
  - A compact core (NS or BH) if it is a core collapse SN
  - A lasting memory for some lucky observers (in 185, 386, 393, 1006, 1054, 1181, 1572, 1604, 201?...)
- BUT no observed supernova has (yet, see SN 1979C) consensually given birth to a black hole for sure (study neutrino photometry of fallback matter spectrumto know)

What does a black hole look like?

130 THE SCHWARZSCHILD SPACE-TIME

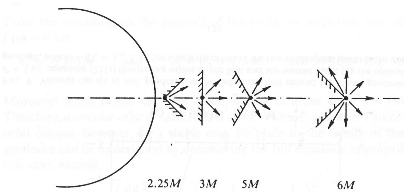


FIG. 10. The cone of avoidance at different distances from the centre.

Therefore, the cone of avoidance is narrow and, at large distances, its angle is that subtended by a disc of radius $(3\sqrt{3})M$. The cone opens out fully at $r = 3M$; and for $r < 3M$, it is directed outward and becomes narrower as $r \to 2M$; and at $r = 2M$, everything is blotted out (see Fig. 10).

(c) *The geodesics of the first kind*

We now consider the case when all the roots of the cubic equation $f(u) = 0$ are real and the two positive roots are distinct. Let the roots be

$$u_1 = \frac{P-2M-Q}{4MP}(<0), \quad u_2 = \frac{1}{P}, \quad \text{and} \quad u_3 = \frac{P-2M+Q}{4MP}, \tag{246}$$

where P denotes the perihelion distance and Q is a constant to be specified presently. The sum of the roots has been arranged to be equal to $1/2M$ as required (cf. equation (226)). Also, it should be noted that the ordering of the roots, $u_1 < u_2 < u_3$, requires that

$$Q + P - 6M > 0. \tag{247}$$

Next, evaluating

$$f(u) = 2M(u-u_1)(u-u_2)(u-u_3) \tag{248}$$

with u_1, u_2, and u_3 as specified in equations (246), and comparing the result with the expression (225), we obtain the relations

$$Q^2 = (P-2M)(P+6M) \tag{249}$$

and

$$\frac{1}{D^2} = \frac{1}{8MP^3}[Q^2 - (P-2M)^2]. \tag{250}$$

THE NULL GEODESICS 131

With the aid of the first of these relations, the second simplifies to give

$$D^2 = \frac{P^3}{P-2M}. \tag{251}$$

Combined with the relation (249), the inequality (247) gives

$$(P-2M)(P+6M) > (P-6M)^2, \tag{252}$$

or, after simplification,

$$P > 3M. \tag{253}$$

From equation (251), we now obtain the further inequality

$$D > (3\sqrt{3})M = D_c \quad \text{(say)}. \tag{254}$$

Therefore, the orbits we are now considering have impact parameters in excess of the critical value leading to the special solutions considered in §(*b*) above. Moreover, they are entirely outside the circle $r = 3M$.

We now make the substitution

$$u - \frac{1}{P} = -\frac{Q-P+6M}{8MP}(1+\cos\chi), \tag{255}$$

or, equivalently,

$$u + \frac{Q-P+2M}{4MP} = \frac{Q-P+6M}{8MP}(1-\cos\chi). \tag{256}$$

By this substitution,

$$\left.\begin{aligned} u = \frac{1}{P} \text{ when } \chi = \pi, \text{ and} \\ u = 0 \text{ and } r \to \infty \quad \text{when } \sin^2\tfrac{1}{2}\chi = \frac{Q-P+2M}{Q-P+6M} = \sin^2\tfrac{1}{2}\chi_\infty \text{ (say).} \end{aligned}\right\} \tag{257}$$

We further find that the substitutions (255) and (256) reduce equation (214) to

$$\left(\frac{d\chi}{d\varphi}\right)^2 = \frac{Q}{P}(1 - k^2\sin^2\tfrac{1}{2}\chi), \tag{258}$$

where

$$k^2 = \frac{1}{2Q}(Q-P+6M). \tag{259}$$

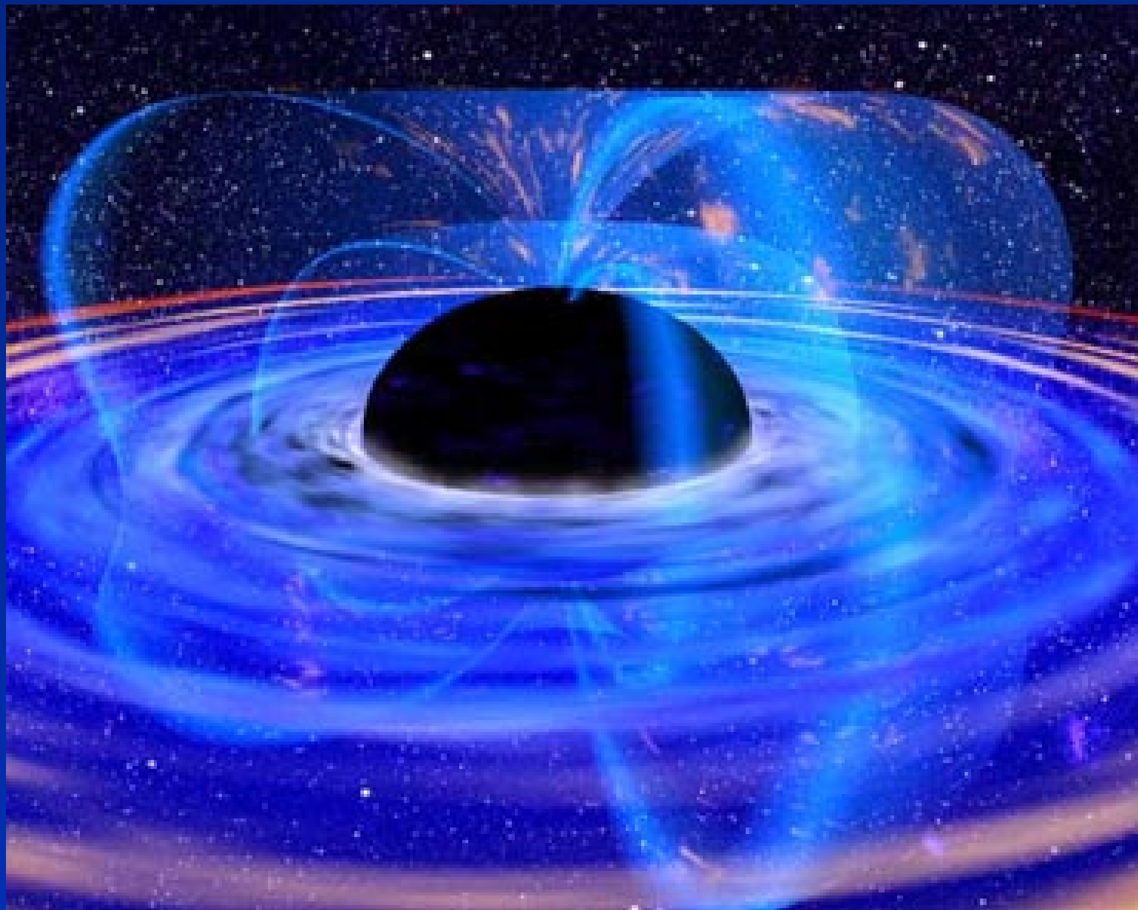
The solution for φ can, therefore, be expressed in terms of the Jacobian elliptic integral in the form

$$\varphi = 2\left(\frac{P}{Q}\right)^{1/2}[K(k) - F(\tfrac{1}{2}\chi, k)], \tag{260}$$

S. Chandresakhar, *The mathematical theory of black holes* (1983), 646 pages

- Few visualization work (till recently)...
  - J.-P. Luminet (1979)
  - J.-A. Marck (1996)
- ... even in special relativity
  - C. M. Savage & A. C. Searle, *Seeing Relativity*, http://www.anu.edu.au/Physics/Searle/ + DVD

What does a black hole *not* look like?



Vue d'artiste NASA

♣ Anything that is represented is incorrect:

- Accretion disk
- Shadow
- Deflection of light

What *will* a black hole look like? (I)

- These are small objects: from a few to a few billion kilometers

$$R_{\mathrm{BH}} = \frac{2GM}{c^2} \sim \frac{M}{M_{\mathrm{Soleil}}} \times 3\ \mathrm{km}$$

- Known black holes are far:
  - Nearby stars = a few light years (= a few 10^{13} km)
  - Black holes are significantly rarer (10^{-4})
  - Only a tiny portion of them, even in our Galaxy, are detected as such ($1/1\,000\,000$)
    $\longrightarrow$ a few dozen only
- Examples :
  - Cyg X-1 : $M \sim 10 M_\odot$, $D \sim 5\ 600$ ly, $\theta \sim 0.6$ nas
    Note : 1 nas = 1 μm on the Moon (or a proton 10 cm away)
  - Sgr A* : $M \sim 4.1 \times 10^6 M_\odot$, $D \sim 26\ 000$ ly, $\theta \sim 50\ \mu$as
    Note : 50 μas = 50 cm at Moon distance
  - M 87 : $M \sim 3.3 \times 10^9 M_\odot$, $D \sim 53\ 000\ 000$ ly, $\theta \sim 20\ \mu$as
- But for the last two, should be achievable within one or two decades

What *will* a black hole look like? (II)

- Problem lies in the diffraction limit when observing at wavelength λ with a telescope of diameter D

$$\text{Resolution} = \frac{\lambda}{D}$$

- 100 m telescope in optical light

$$\text{Resolution} = 1 \text{ mas}$$

- But... extra limitation due to atmosphere $= 0{,}1 - 1$ as

HST resolution is 1000 too rough to resolve Sgr A*...

What *will* a black hole look like? (III)

- This can be overcome with interferometry, i.e. by cleverly combininglight from several telescopes spanning a network of size L

$$\text{Resolution} = \frac{\lambda}{L}$$

- Extremely difficult to perform in optical domain (except short distance, but then resolution gain is poor, i.e., $L \sim 100$ m), but used for decade in radio domain
- Loss of resolution because of larger wavelength factor 2 000 ($0{,}5\,\mu\mathrm{m} \to 1\,\mathrm{mm}$), BUT gain in L is huge, 100 000 as one can span the whole Earth surface ($100\,\mathrm{m} \to 10\,000\,\mathrm{km}$)

The ALMA interferometer

What *will* a black hole look like? (IV)

- Various projects:

  - Long base radio interferometryo (VLBI): $\lambda \sim 3$ mm, $L \sim 10^4$ km, $\theta \sim 100 \to 30$ μas

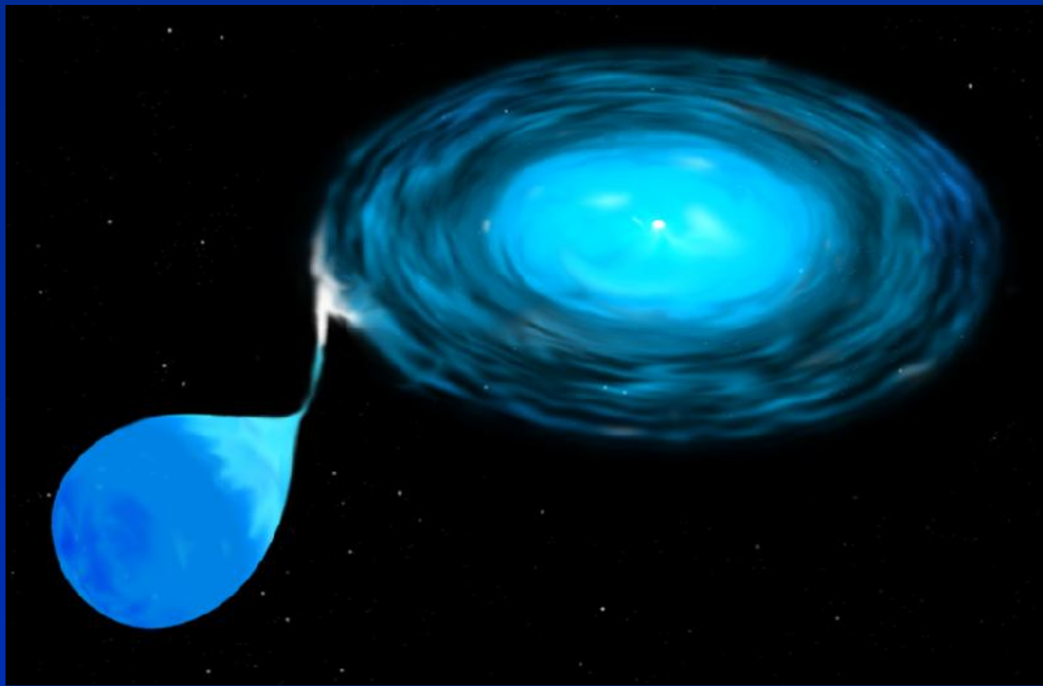
  - GRAVITY: near IR astrometry at the VLT (first light still expected in 2014), $\theta \sim 10 - 15$ μas

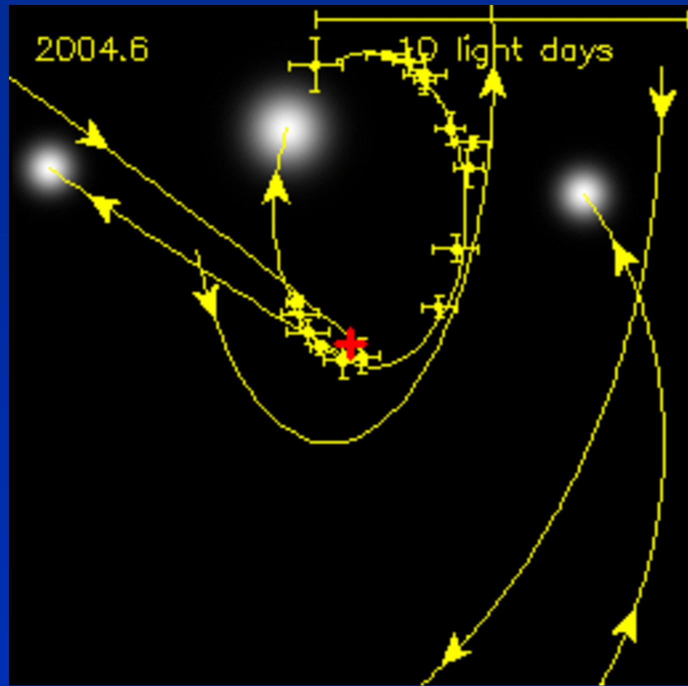
  - MAXIM (*Micro-Arcsecond X-Ray Imaging Mission* : X-ray interferometry (NASA project "Beyond Einstein"), $\theta \sim 1$ μas

    "But no studies exist as to the feasibility and technology requirements to realize an orbiting X-ray interferometer. This is because the technology challenges are severe and this program maybe at least 25-50 years in the future."

What we do today... (I)

- There are indirect but indisputable proofs of BH existence:
  - Study of light emitted by matter before disappearing behind the horizon (l.)...
  - ... or by its gravitationnal interaction with surrounding stars (r.)





- From a theoretical point of view, matter collapse to a BH state is unavoidable provided a very large set of initial conditions, and independently of details of GR or matter equation of state: minimal requirements are that gravity is a metric, causal theory and that dominant energy condition $(\rho + 3P > 0)$ holds.

Warm-up: seeing special relativity

- Orbiting close to a black hole necessitates relativistic speed: approximate 3rd Kepler law translates into

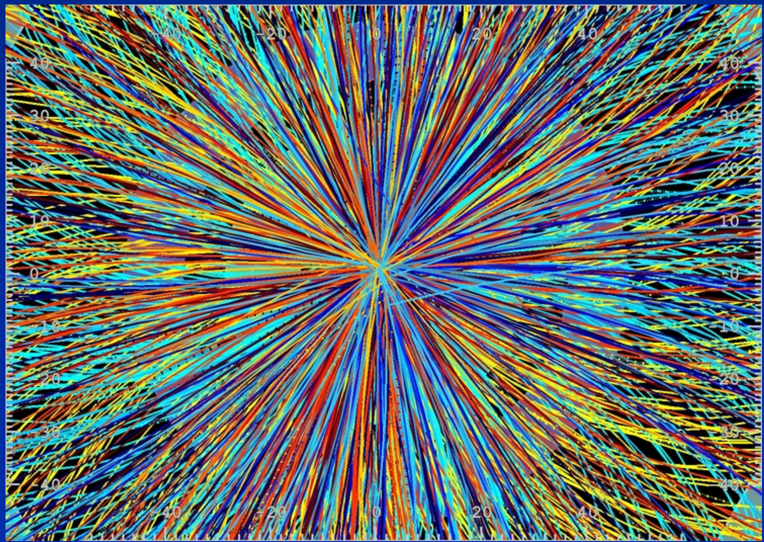
$$v_{\rm orb} \sim c\sqrt{\frac{R_{\rm BH}}{2R_{\rm orb}}}$$

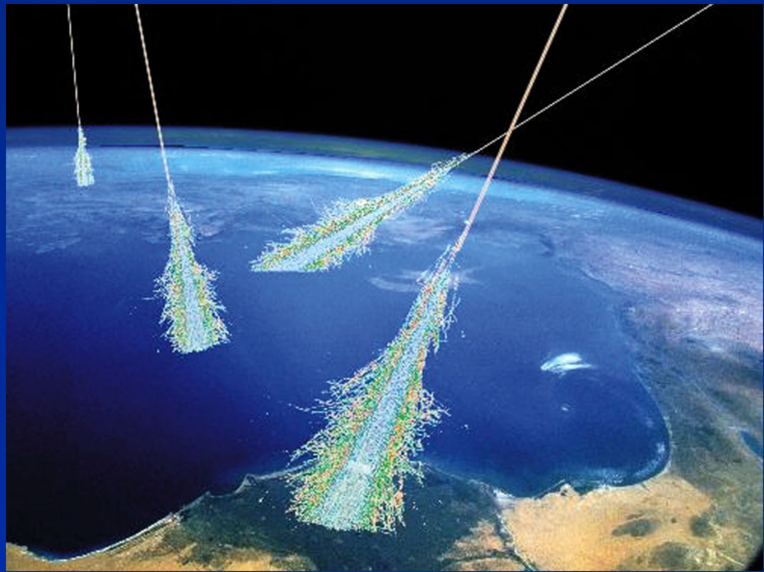
- $\rightarrow$ This has to be taken into account even before considering gravity

- Then, several SR effects arise
  - Aberration
  - Doppler
  - Intensity

Aberration

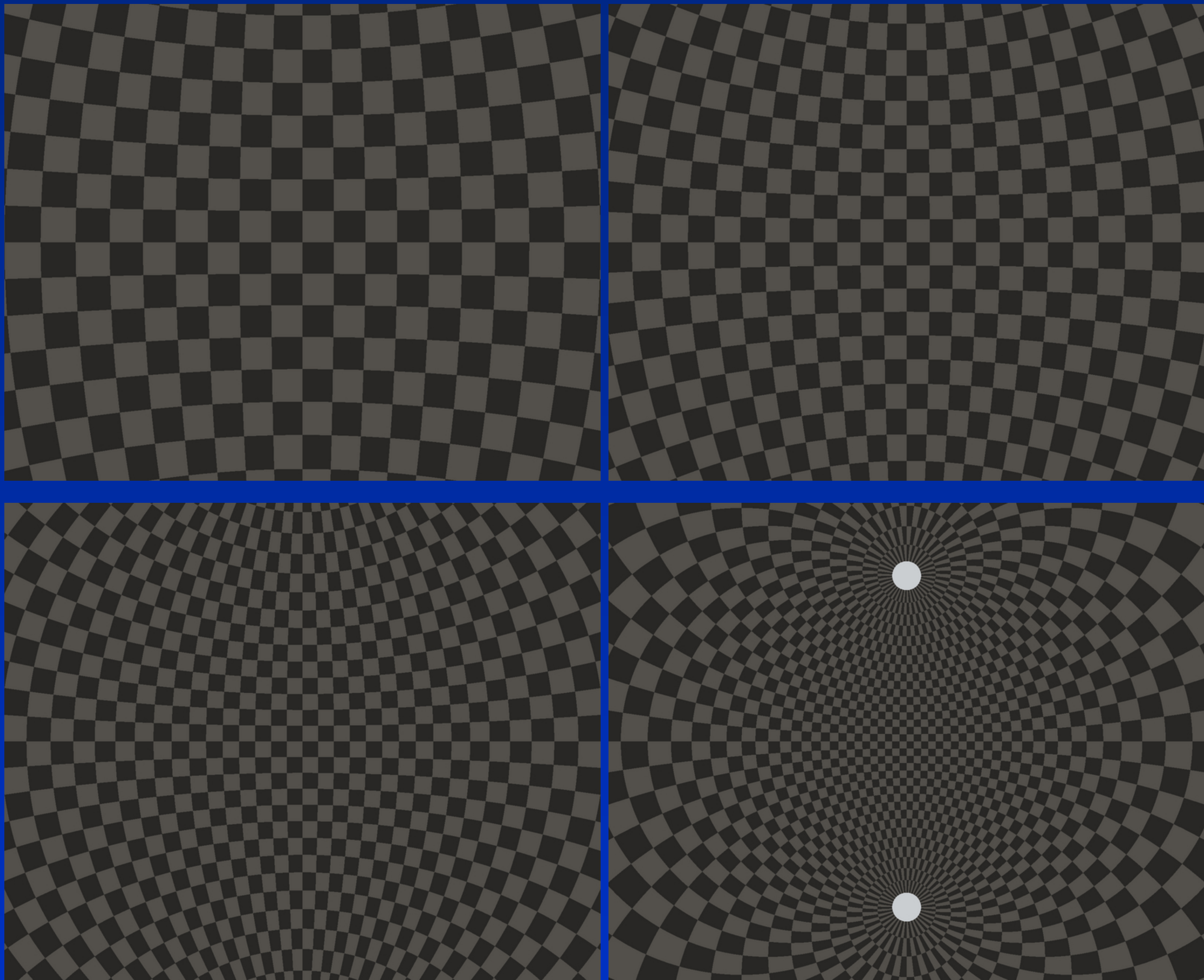
- Aberration = what makes things seen at LHC differ from those at Pierre Auger observatory.



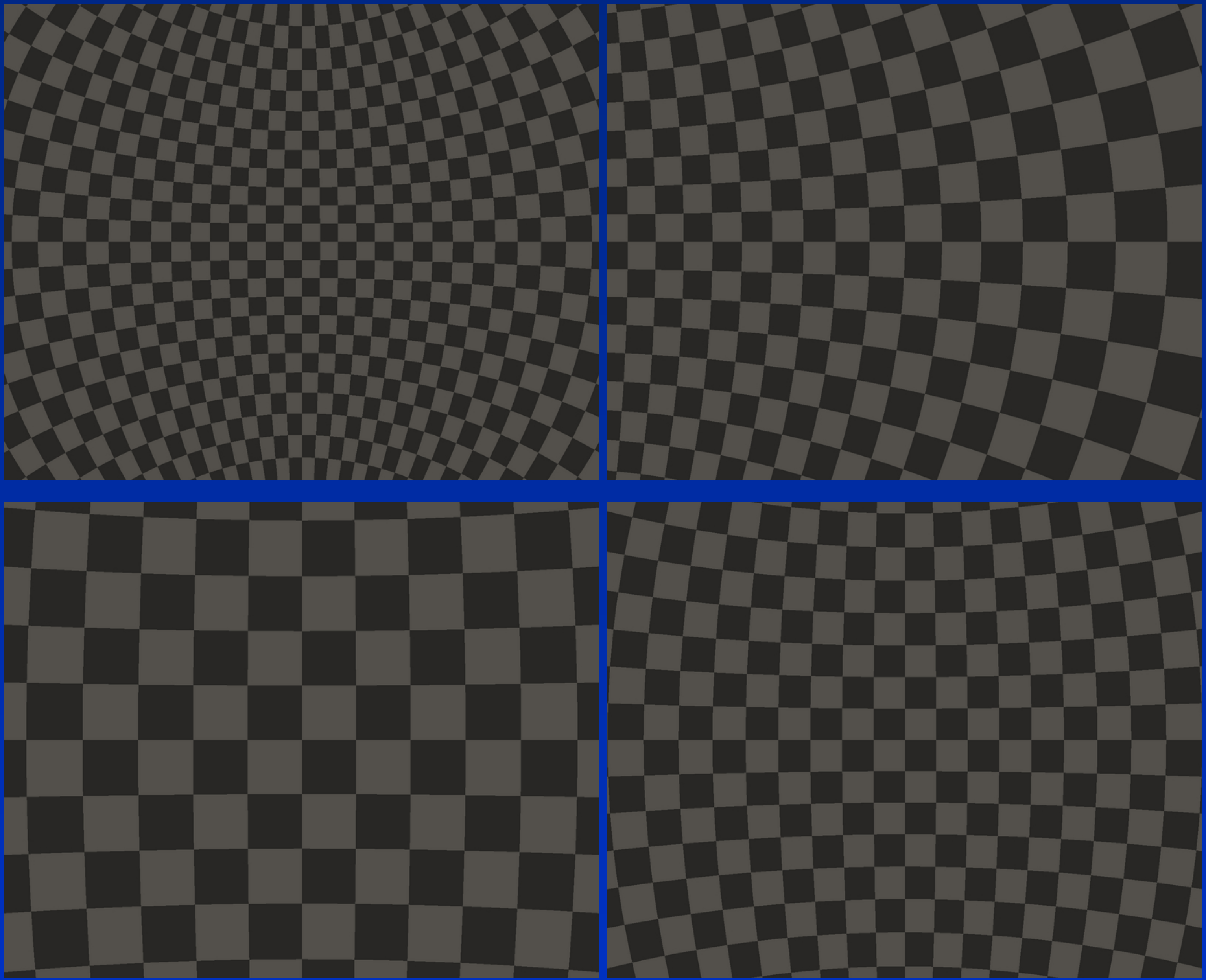


- Or, more down-to-Earth,
  - Rain when driving a car
  - Apparent direction of wind on a sailboat
  - Apparent direction of light (Bradley, 1728-1748)
  - Exists at classical level, even without SR
  - Very counter-intuitive effect, since opposed to parallax

Examples : $v = 0$, $0.3c$, $0.6c$ and $0.9c$



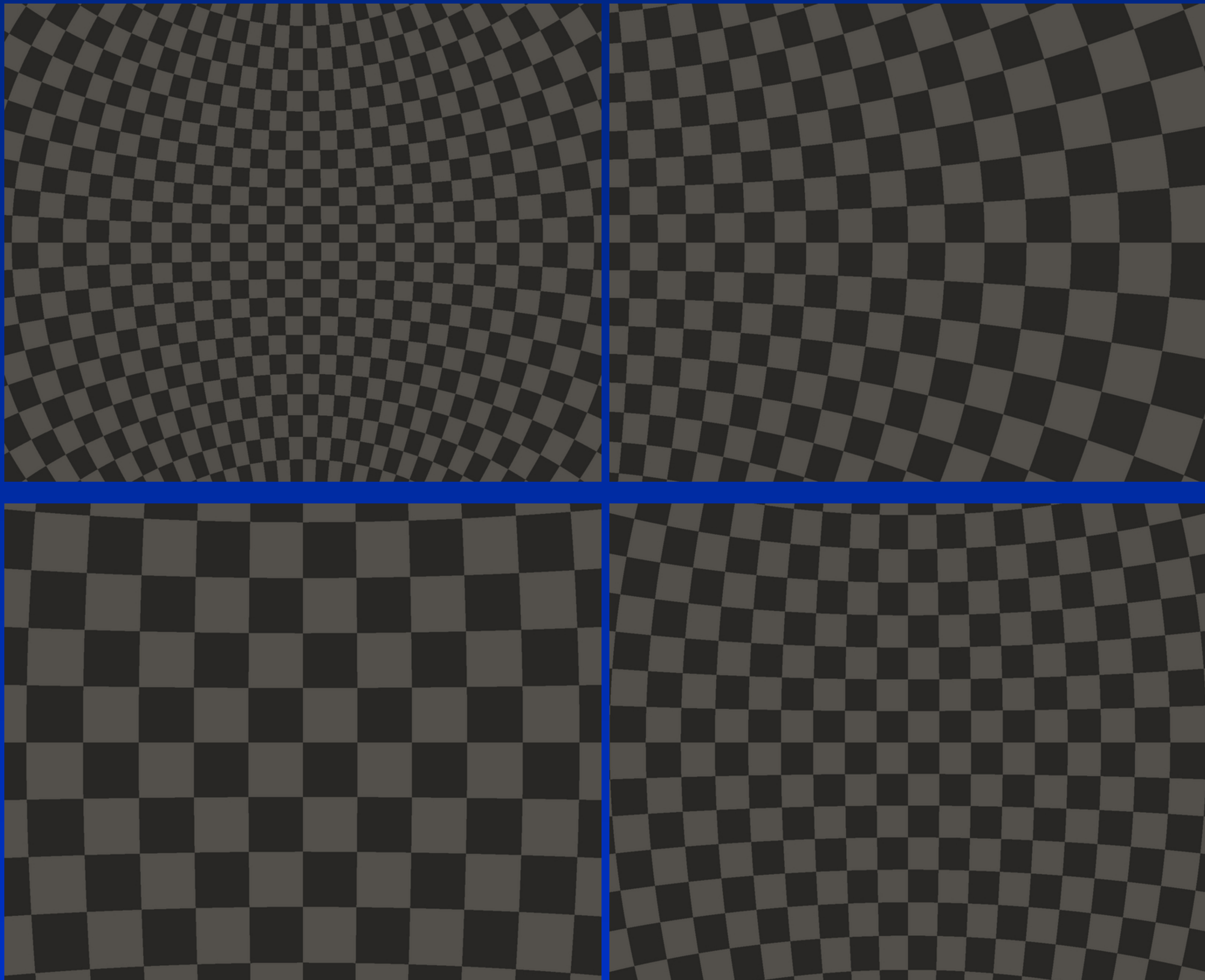
Examples : $v = 0.5c$, front, right and rear views



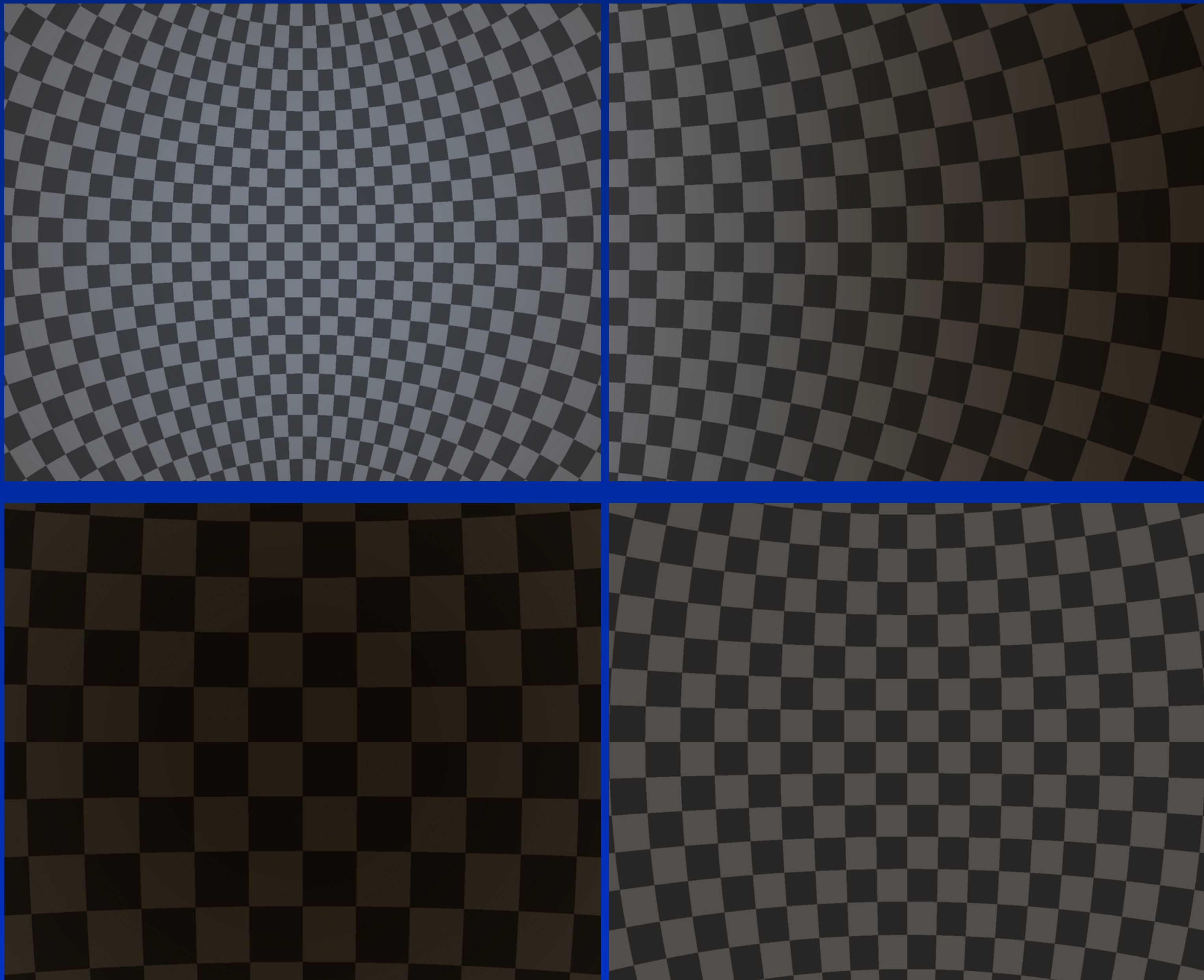
Doppler effect

- Any wave is perceived diffrently depending on the relative velocity of observer wrt wave source
- We are used to Doppler effect with sound waves
- For light:
  - Light source is blueshifted when approching
  - Light source is redshifted when receding
  - Also exists at classical level, but only SR give correct formula at high speed

Examples : $v = 0.5c$, front, right and rear views, aberration only

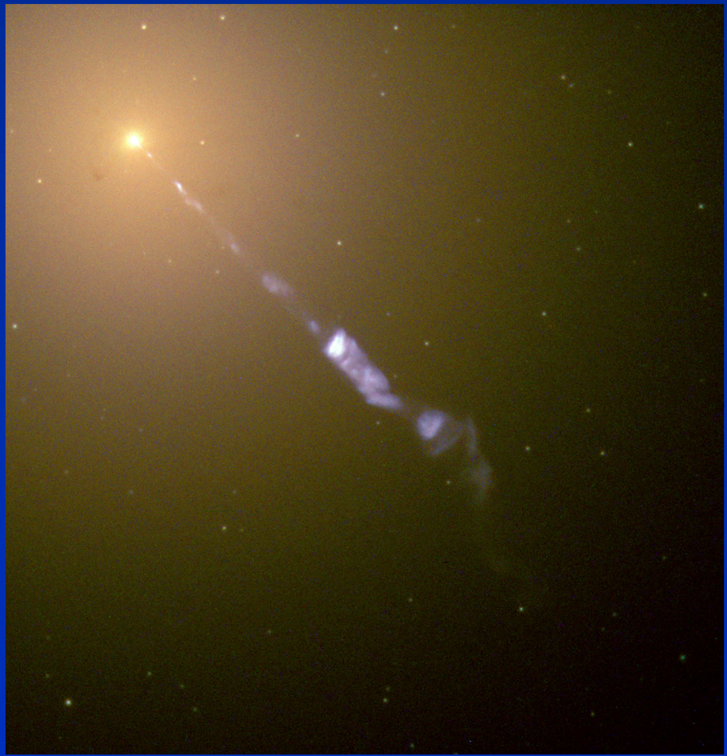


Examples : $v = 0.5c$, front, right and rear views, aberration + Doppler



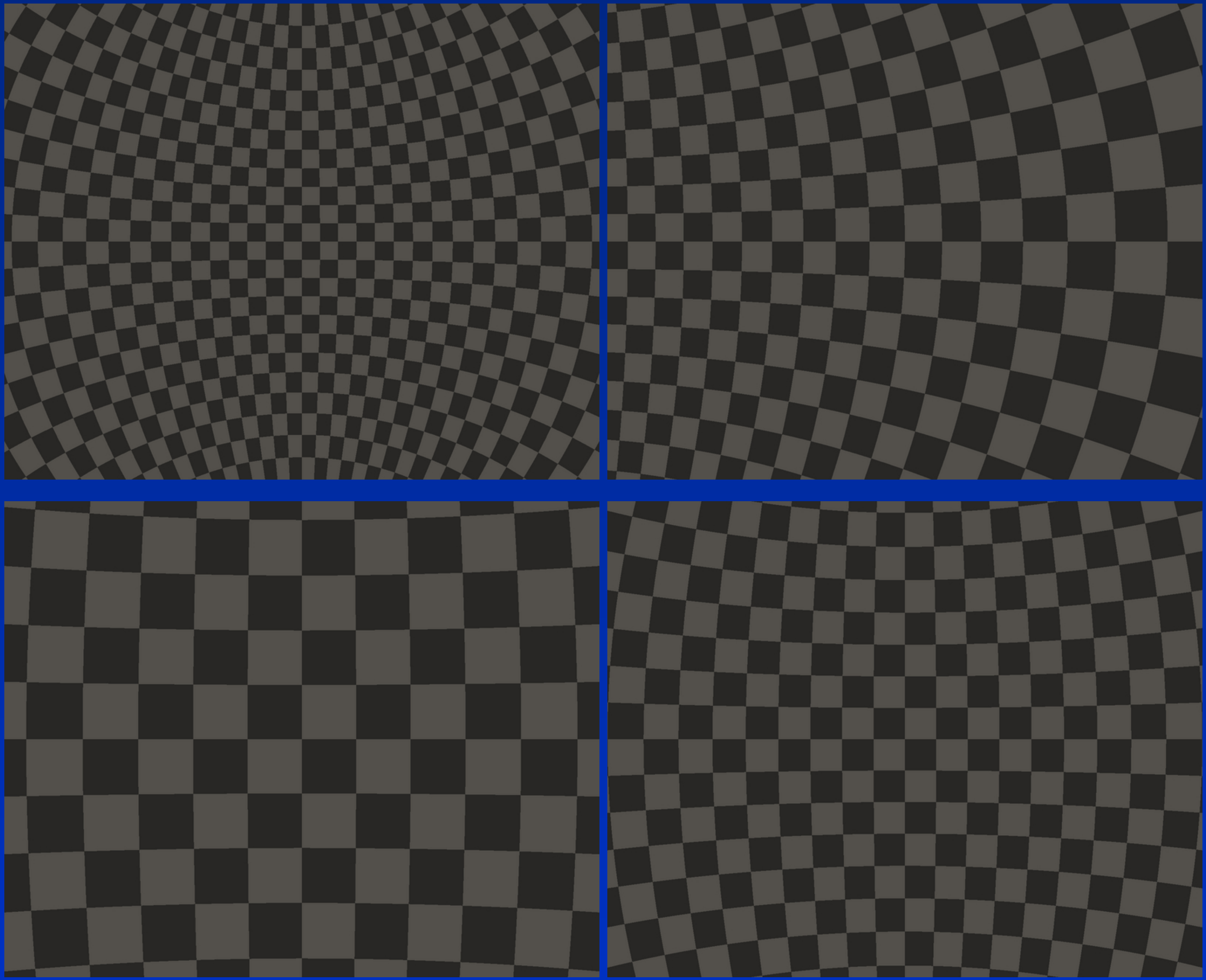
Intensity

- In addition to Doppler effect:
  - Light flux increases when approaching a light source, which therefore will appear brighter
  - Light flux decreases when receding from a light source, which therefore will appear dimmer
- SR formula tells that effect become really large at relativistic speeds, cf the double jet in M87

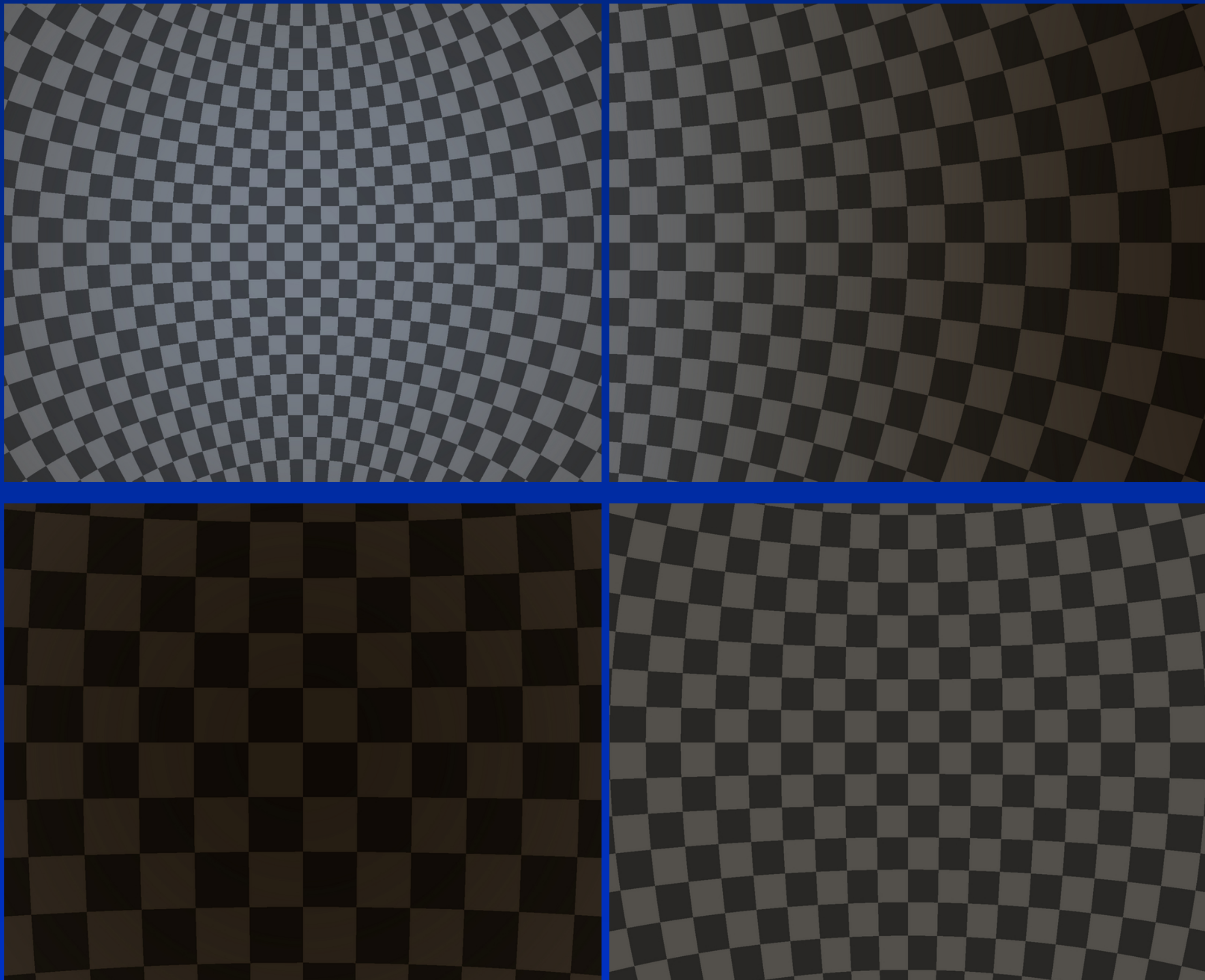


- $\rightarrow$ We will somehow artificially reduce the amplitude of this when necessary

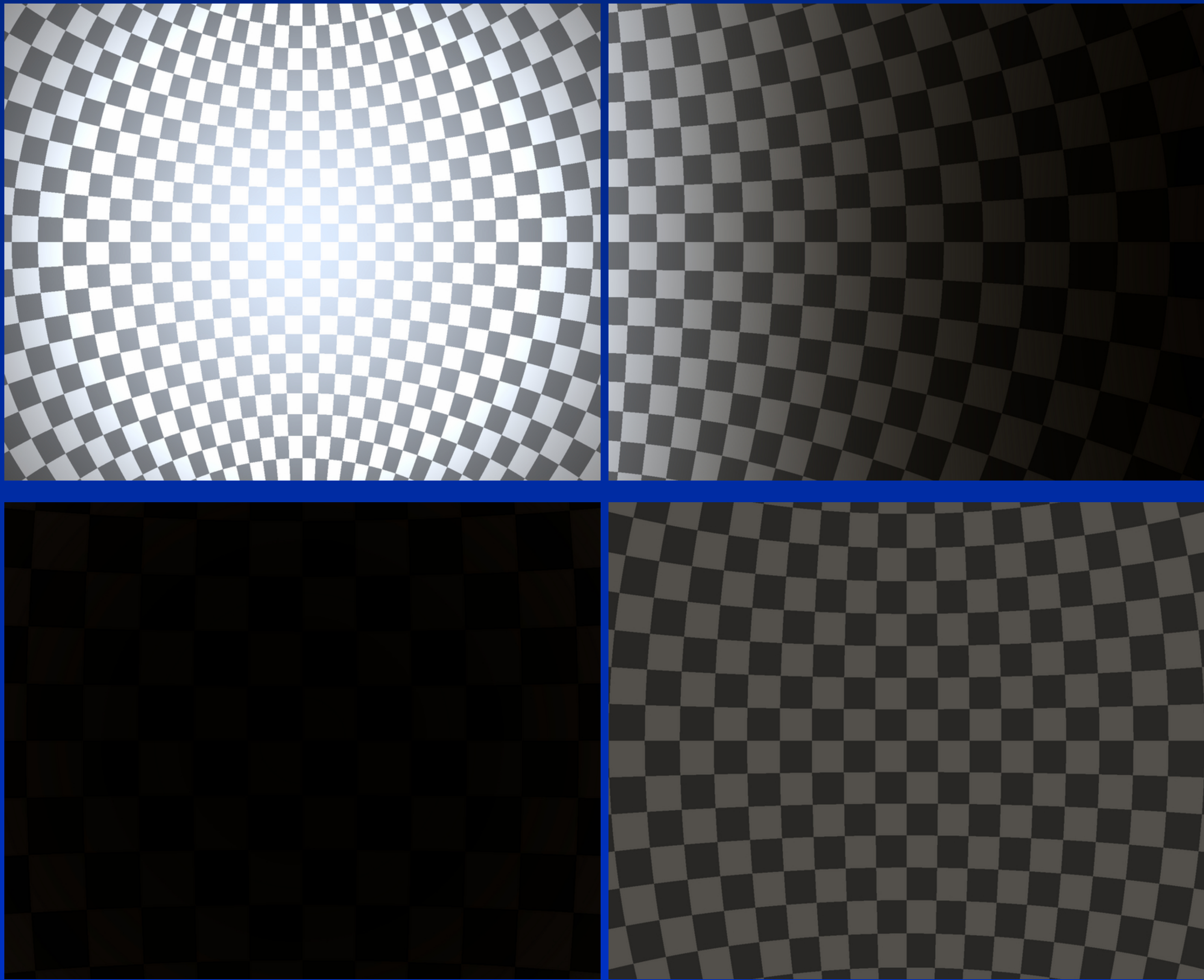
Examples : $v = 0.5c$, front, right and rear views, aberration only



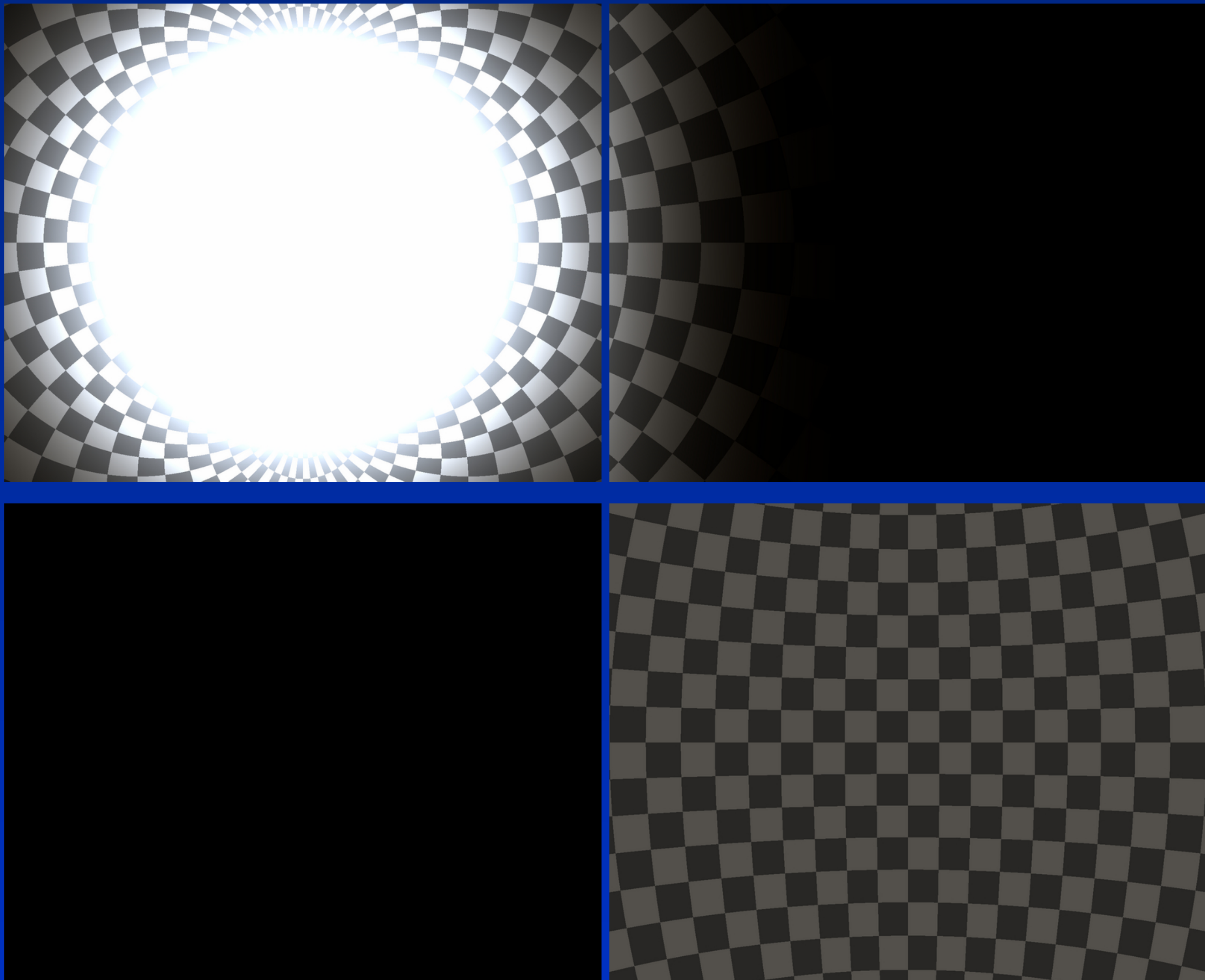
Examples : $v = 0.5c$, front, right and rear views, aberration + Doppler



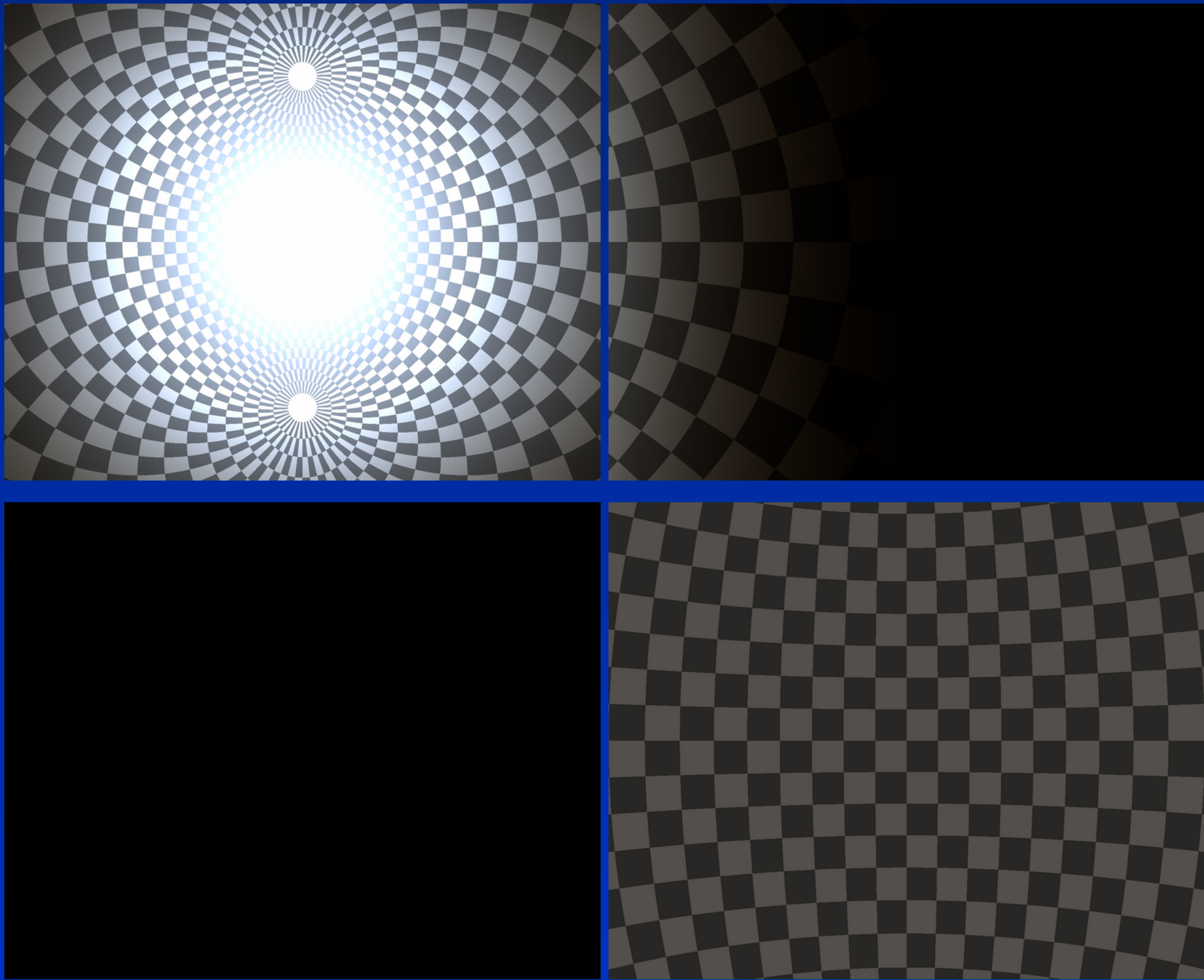
Examples : $v = 0.5c$, front, right and rear views, aberration + Doppler + intensity



Examples : $v = 0.9c$, front, right and rear views, aberration + Doppler + intensity



Examples : $v = 0.9c$, front, right and rear views, aberration + Doppler + corrected intensity



An example of special relativistic acceleration

or: what would we see when trying to visit some relatives in ϵ Eridani ?

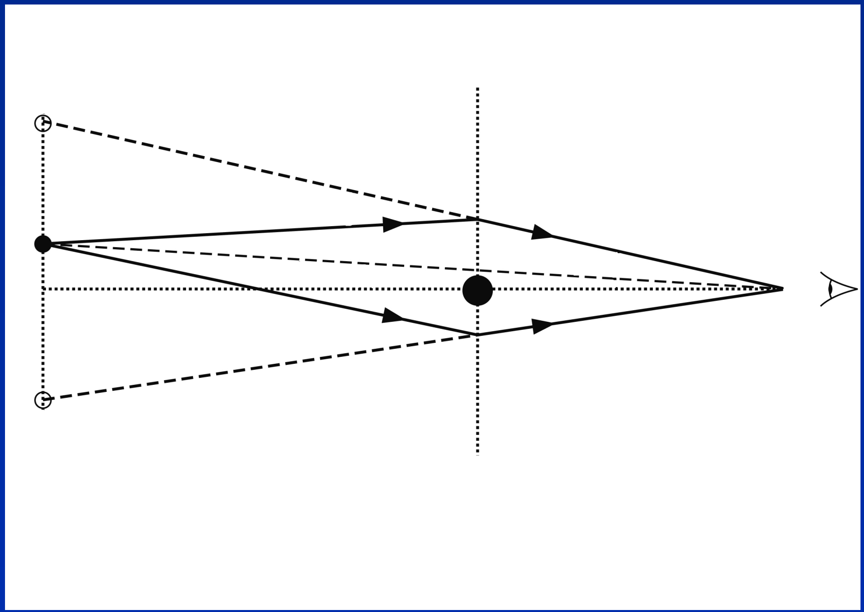
- Going from $v = 0$ to $v = 0.995c$ with constant acceleration a, $v = c\tanh(a\tau/c)$
- Duration $= \tau = 3$ min 20, i.e.,

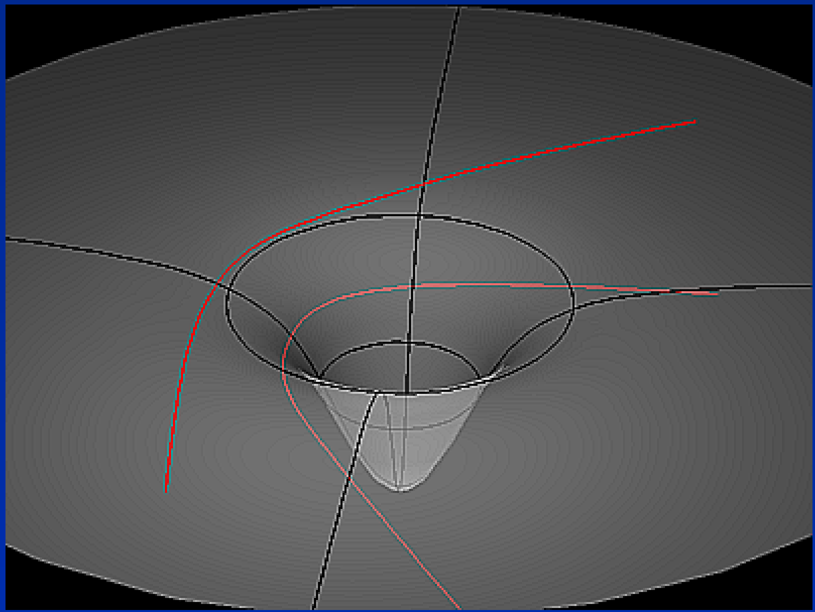
$$a \sim 450\ 000\ g$$

- As a comparison:
  - Trained military aircraft pilot: $a_{\rm max}^{\rm cont} \sim 10\ g$
  - IndyCar/Formula 1 accident: $a_{\rm max}^{\rm choc} \sim 214\ g$, $\Delta l = 2$ m, $\Delta v = 320$ km/h, $\Delta t \sim 0.040$ s (K. Bräck, Texas Motor Speedway 2003)
- Showing a realistic acceleration amounts to switch from 30 fps to 5 frames per day...., i.e. $\tau \to 3$ years...

Black holes (I)

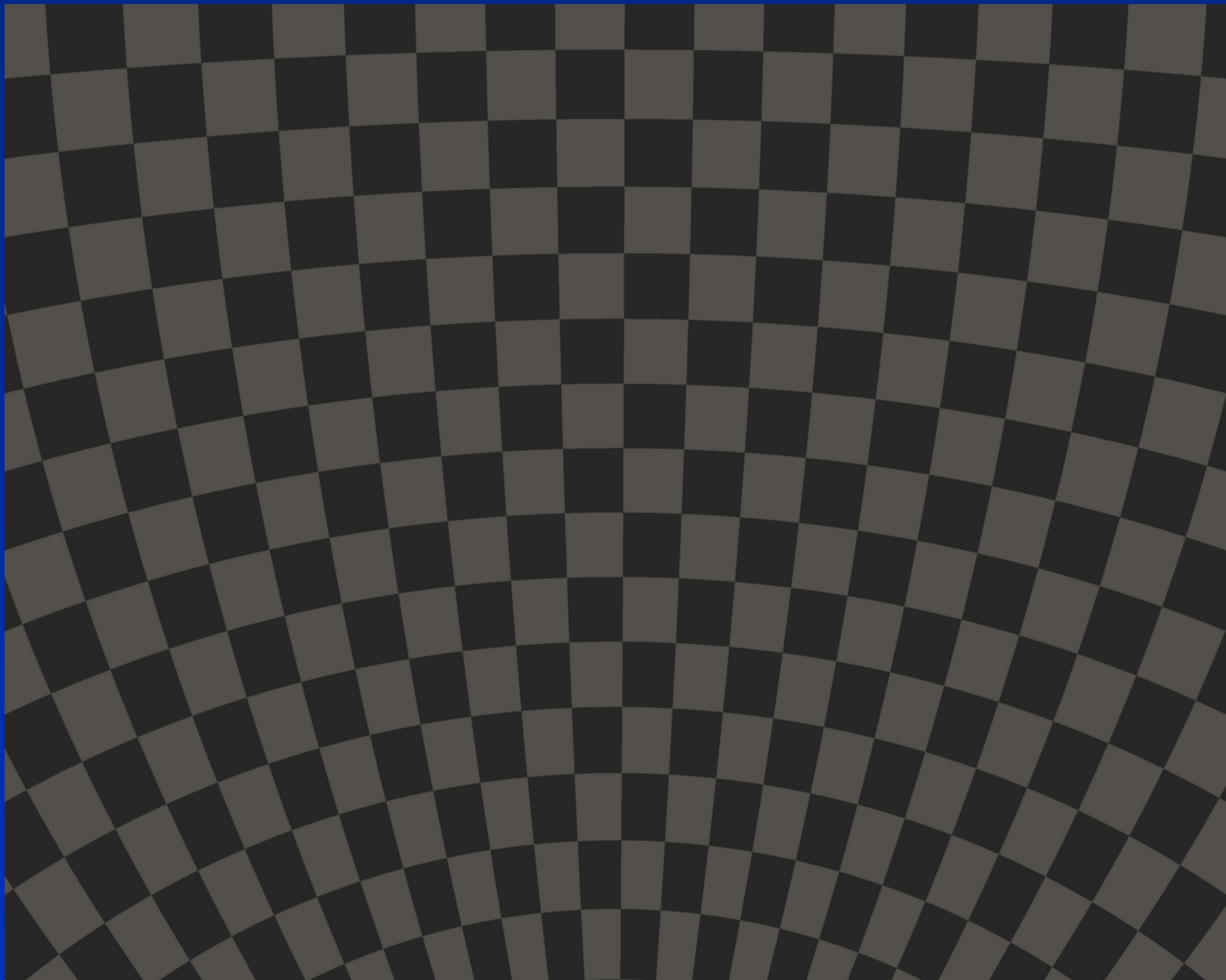
- They affect trajectory and energy of light
- These effects are intuitive but their large amplitude makes them quite confusing





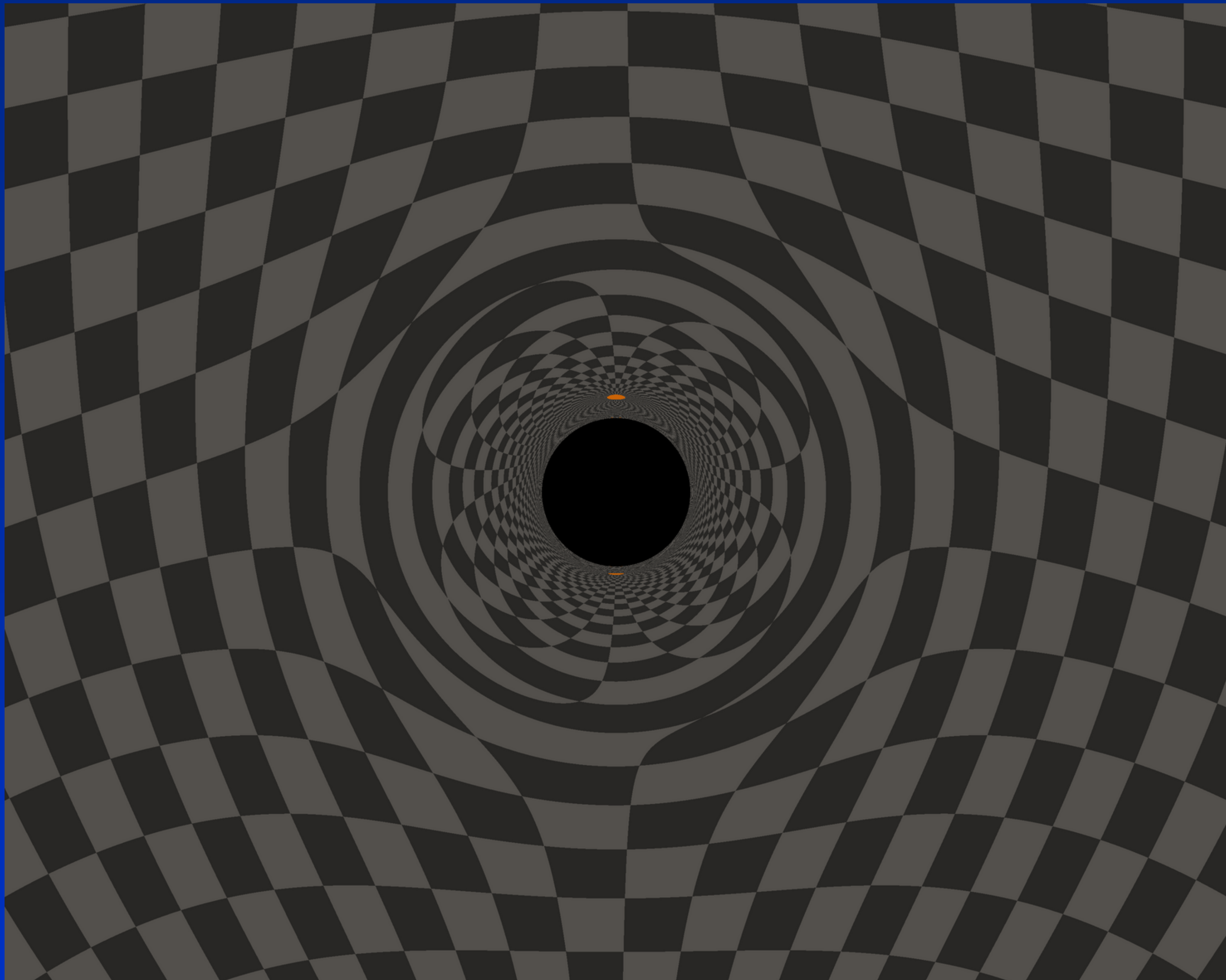
Effect of gravity on light (I)

Without BH



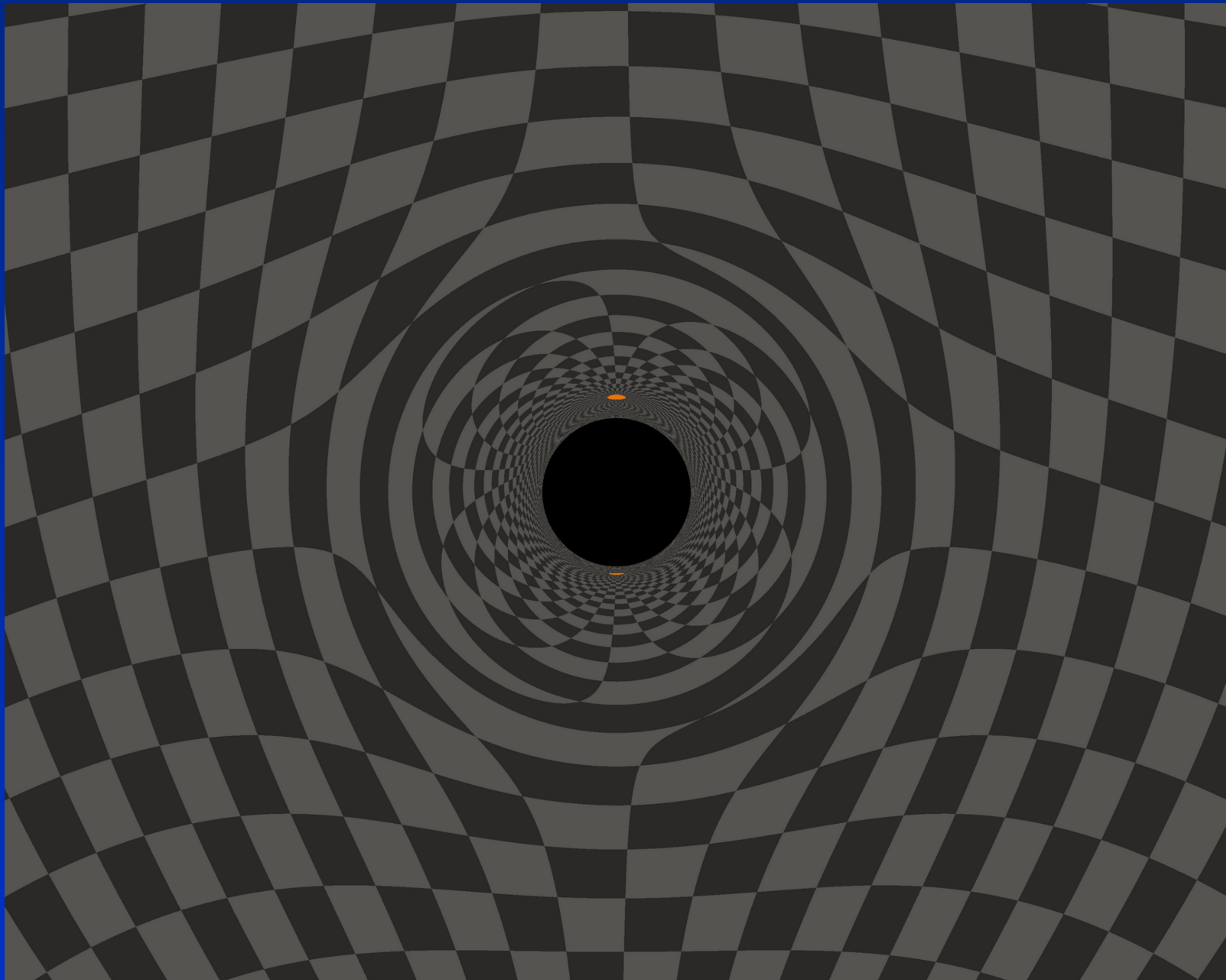
Effect of gravity on light (II)

BH, deflection only



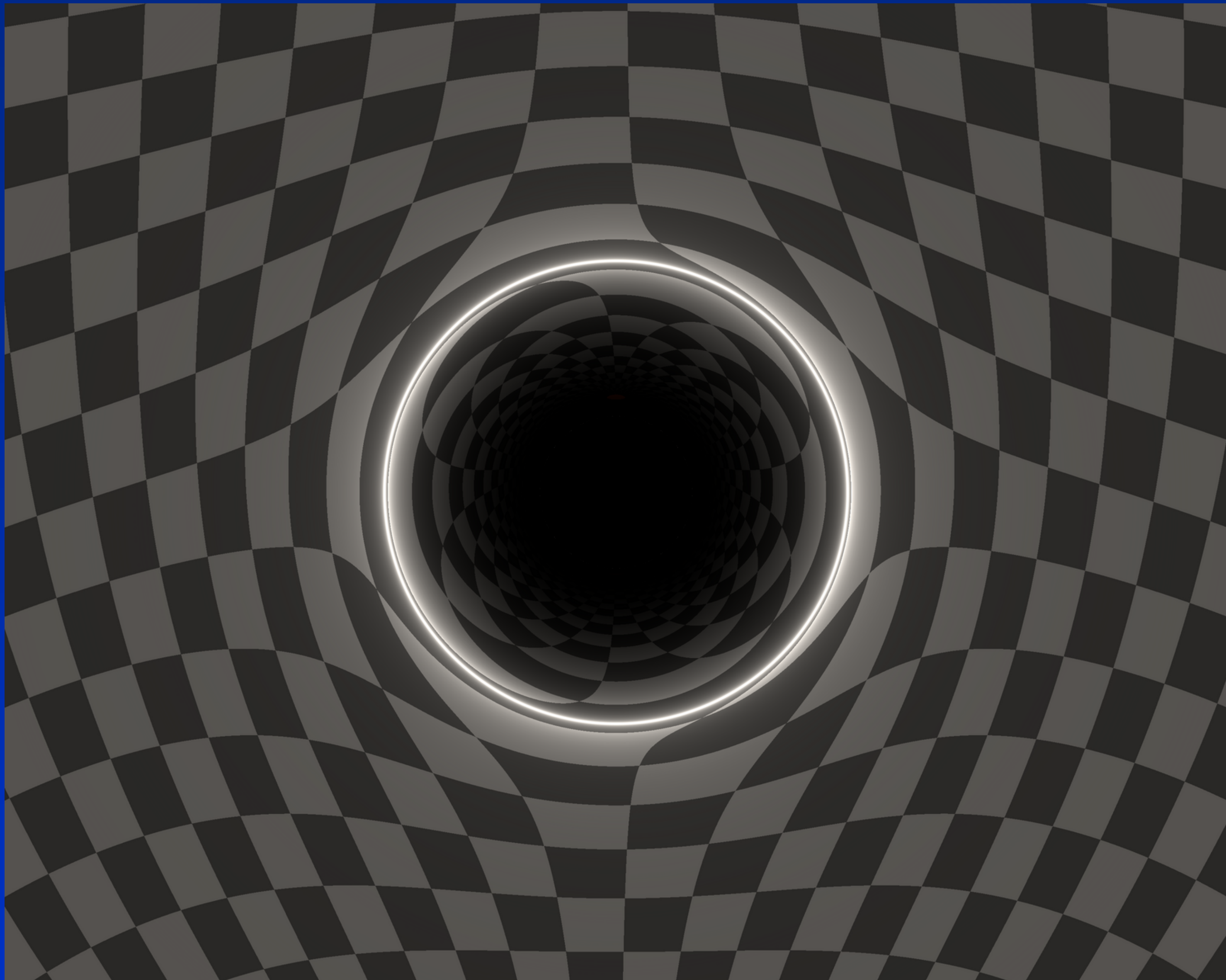
Effect of gravity on light (III)

BH, deflection + gravitational blueshift



Effect of gravity on light (IV)

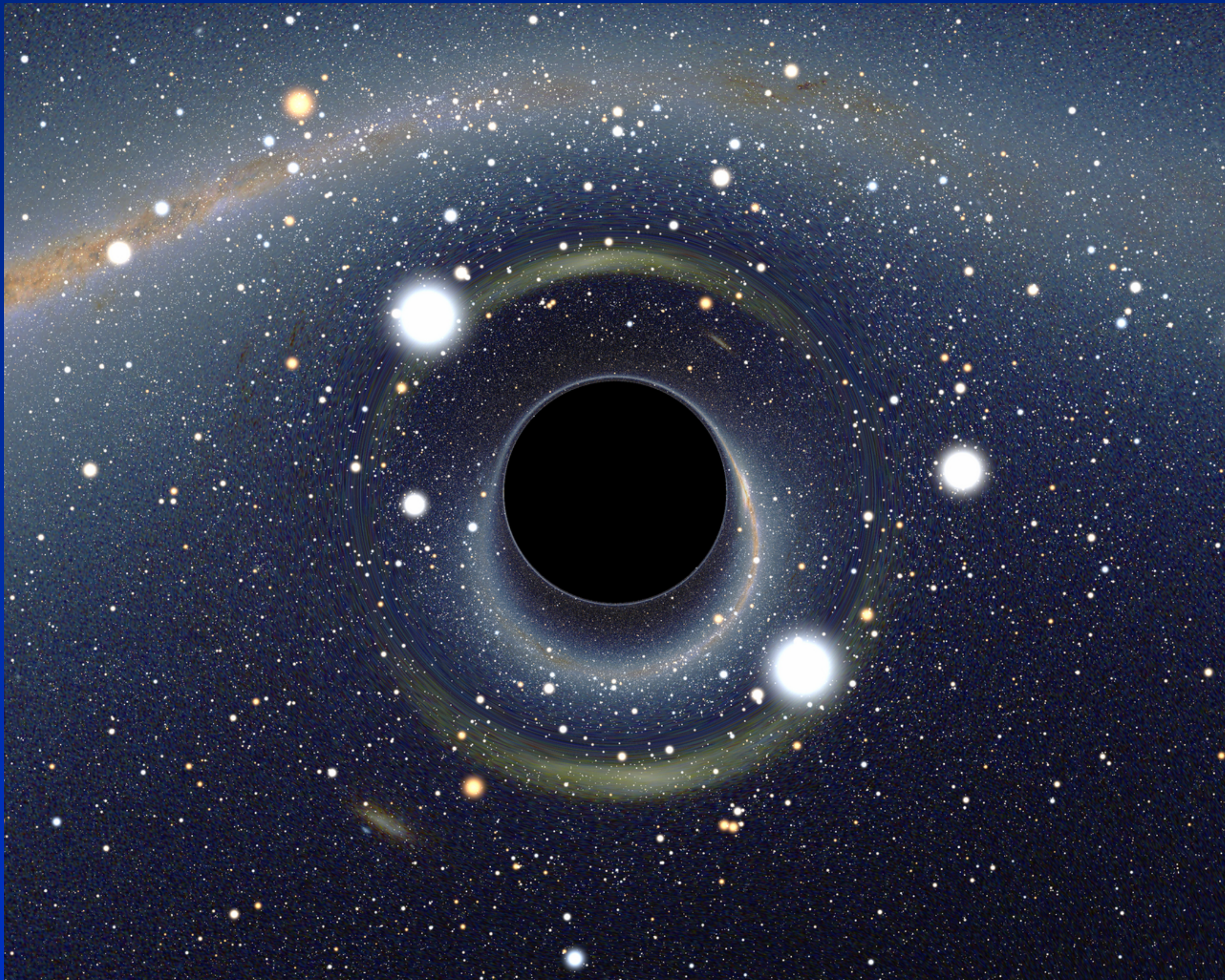
BH, amplification map



Black holes (II)

- They affect trajectories and energy of photons
- What is exactly behind the black hole is not hidden by its shadow, but apppears scattered on a ring surrounding the shadow (Einstein ring)
- Increase of angular size means increase of luminosity (surface brightness is conserved, but surface increases $\rightarrow$ gravitational lens phenomenon
- A whole copy of the celestial sphere lies (heavily distorted) within the Einstein ring, then another, then another, etc de l'anneau d'Einstein et des images multiples de chaque objet
- Some examples...

Taking everything ito account...



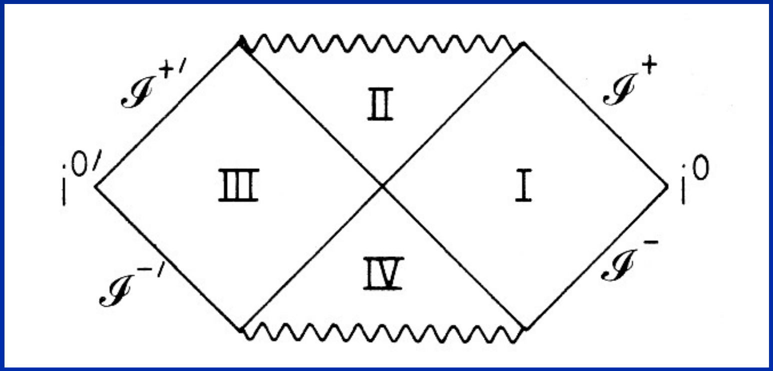
Orbiting around a black hole

- A small sample of trajectories:
  - Circular “far” from the BH
  - Circular “near” the BH
  - “Elliptical”
  - “Parabolic”
  - “Hyperbolic”
  - Very near (therefore non geodesic)

Inside the horizon...

- Un trou noir est un objet qui se forme à un moment donné
  - On peut aller voir (sans espoir de retour) ce qu'il y a dedans...
  - Illustration...

- But there also exists mathematical solutions describing other objects: eternal black holes



  - such objects are not expected to exist, however
  - Forgetting this detail, we have in fact two different universes which are connected through the black hole
  - On can enter in the BH from any of these two universe, but escape is still impossible
  - However, the neighbour universe can be seen (altough it is unatteinble) once in the black hole
  - A nice movie showing this...

D’autres types de trous noirs

- Some other types of eternal black hole exist, either possessing electric charge or angular momentum (or both)

- They still do not correspond to physically realistic configuration

- ... but possess even more fascinatig properties:

  - On ne peut être piégé une fois à l’intérieur, mais au contraire, on en est forcément expulsé
  - They possess two entrances and two exits

  - Science-fiction made them popular, considering that they could exist and help to perform fast and safe (???) interstellar travel

- Still another movie...

Conclusion

- It works!
- And it is very different from what we could naively expect
- To see more (in French): http://tinyurl.com/riazuel3