Introduction to Lattice QCD
and
some applications to Nuclear and Hadronic physics.

Jaume Carbonell

Ecole Joliot-Curie, Fréjus, 29 sep – 8 oct 2013

Second Lecture: the state of the Art

Hadron spectrum

Baryon ground states

Related topic (sigma terms)

Excites states: in trouble !

More complex systems: from baryons to nuclei

Scattering states

Hadron-Hadron Potential

Baryon Structure observables

Still problems in the simplest cases

Some thoughts about the (nuclear) Yukawa model on the lattice (non QCD!)

HADRON SPECTRUM

I. Ground states

ETMC

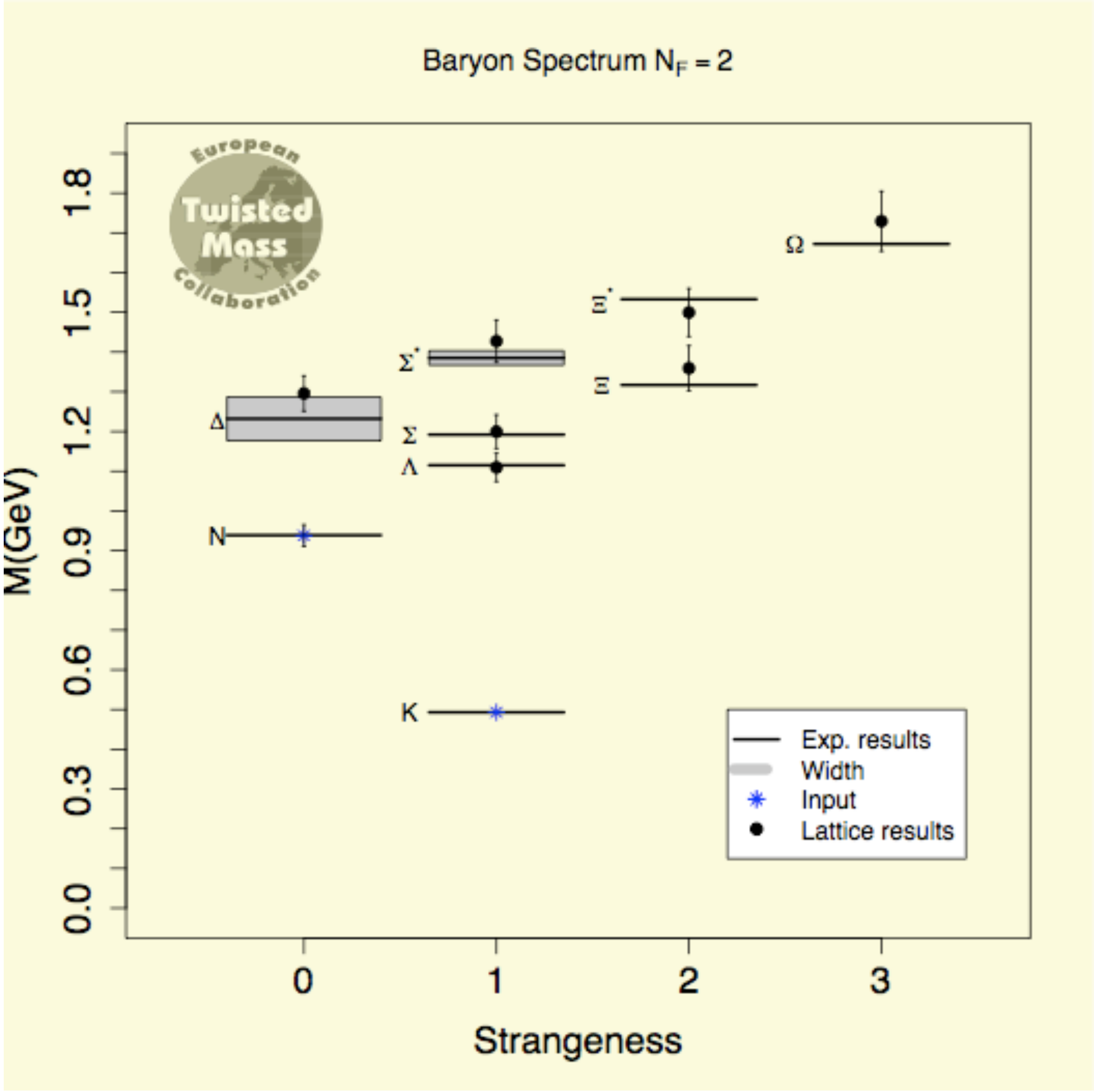
Alexandrou et al PRD80 (2009) 114503

Twisted mass + thee level Symanzik G action
N_f=2
a=0.07-0.09 fm
L=2-2.7 fm
m_π=280-290 MeV $m_\pi L$>3.3

« a » fixed from N

m_s determined from K (ETMC)

Continuum limit+Chiral fits with
- Cubic expansion on m_π+SU3 (N,Δ)
- NLO SU(2) HBχPT (S>0)



$$M_N^{p^3}(m_\pi) = M_N^{(0)} - 4c_N^{(1)}\, m_\pi^2 - \frac{3g_A^2}{16\pi f_\pi^2}\, m_\pi^3$$

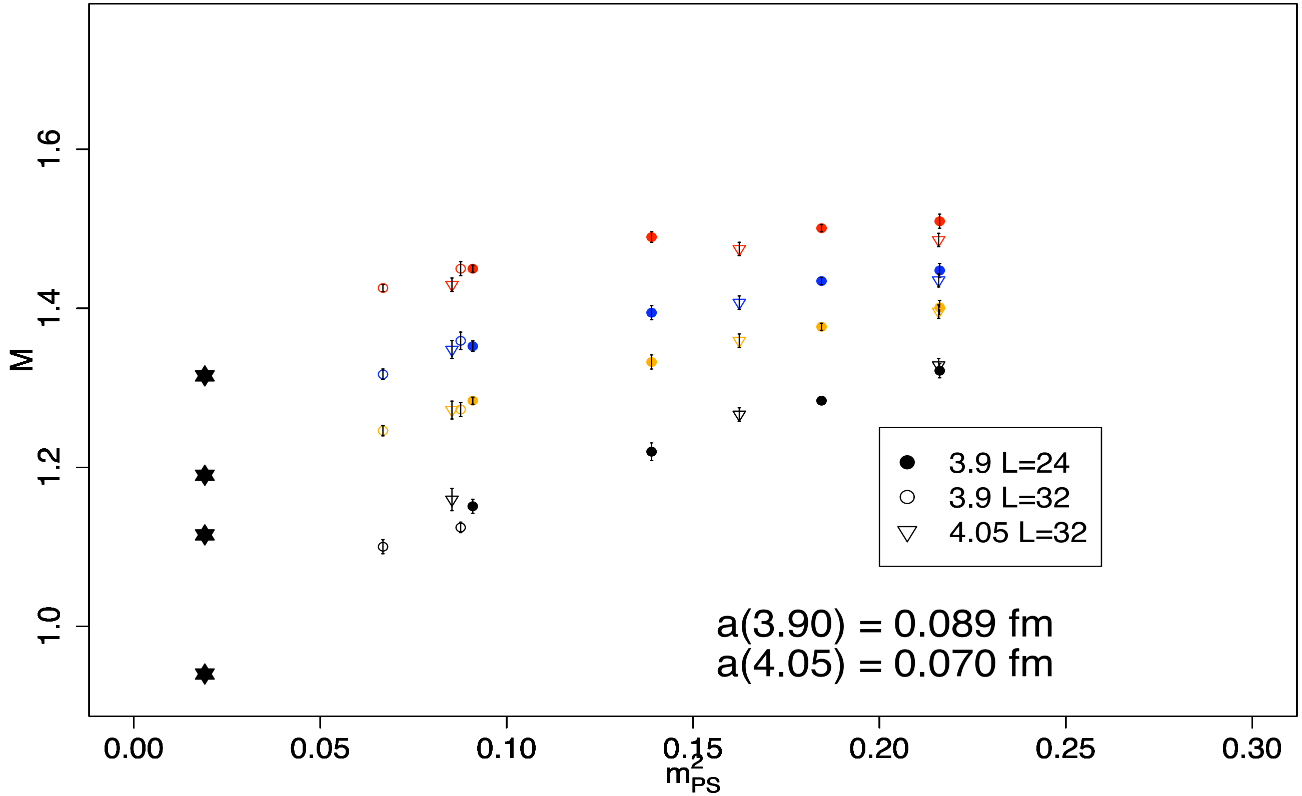
$$M_\Lambda^{p^3}(m_\pi) = M_\Lambda^{(0)} - 4c_\Lambda^{(1)}\, m_\pi^2 - \frac{g_{\Lambda\Sigma}^2}{16\pi f_\pi^2}\, m_\pi^3$$

$$M_\Sigma^{p^3}(m_\pi) = M_\Sigma^{(0)} - 4c_\Sigma^{(1)}\, m_\pi^2 - \frac{2g_{\Sigma\Sigma}^2 + \frac{g_{\Lambda\Sigma}^2}{3}}{16\pi f_\pi^2} m_\pi^3$$

$$M_\Xi^{p^3}(m_\pi) = M_\Xi^{(0)} - 4c_\Xi^{(1)}\, m_\pi^2 - \frac{3g_{\Xi\Xi}^2}{16\pi f_\pi^2} m_\pi^3$$

The main source of errors comes from the fact that simulations are not done with physical q

Results must be "extrapolated to physical point"

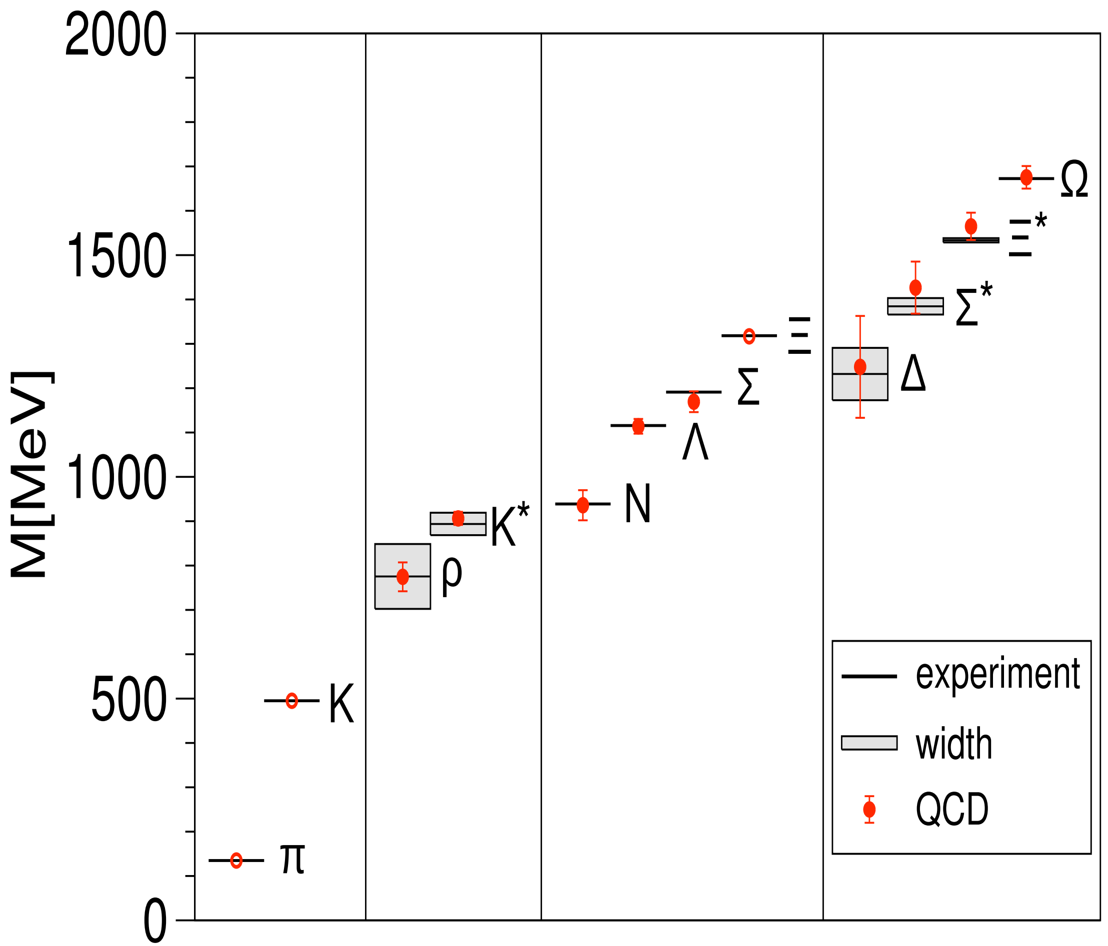


Is the Higgs particle the origin of the mass ?

BMW

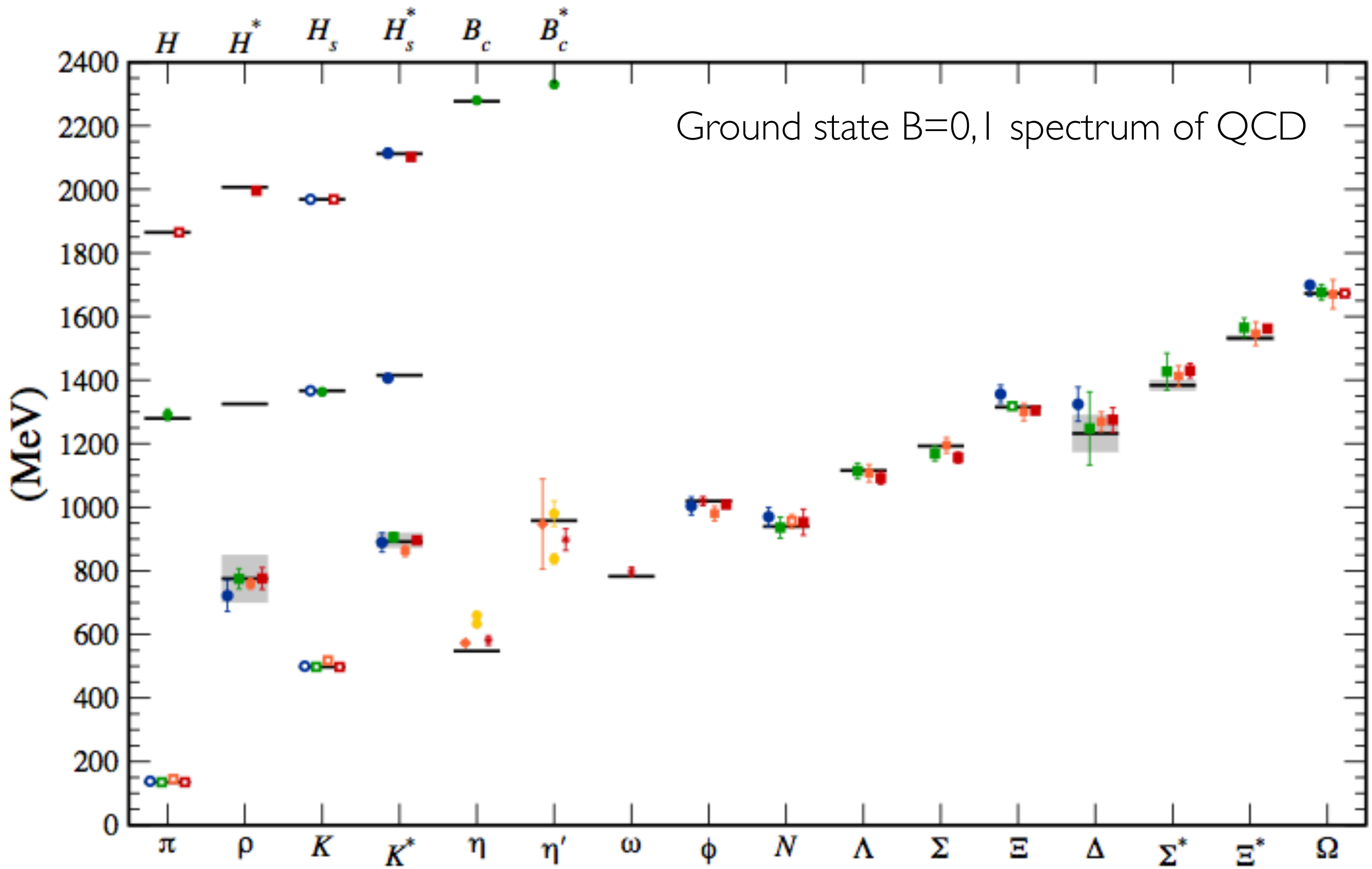
S.Durr et al. Science 322 (2008) 1224

Wilson-Clover + tree level Symanzik G action
N_f=2+1
a=0.06-0.12 fm
m_π=190MeV
$m_\pi L$>4



Since, they succeed simulations with m_π=120 MeV !

Summary of different collaborations



[A Kronfeld, 1209.3468]

Groud state masses are entering now a precision era…

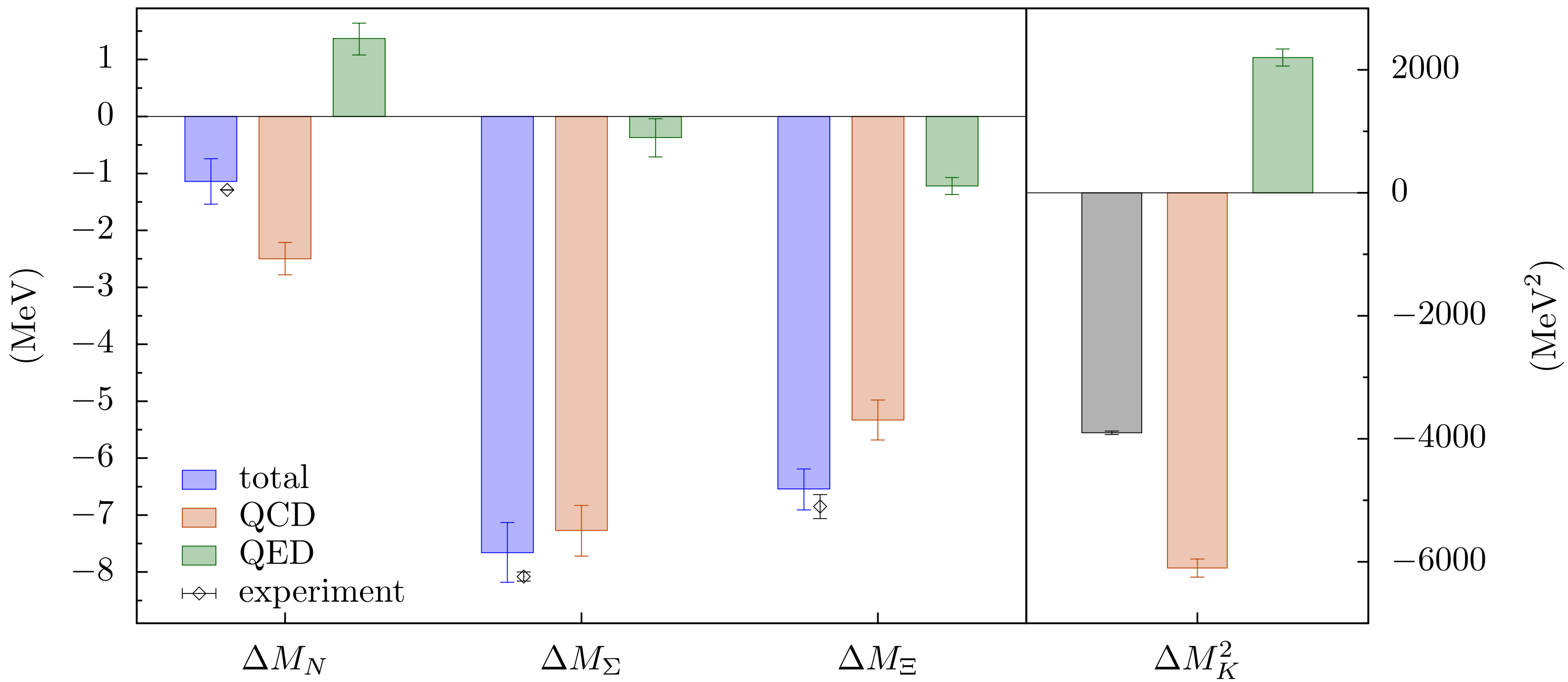
Isospin symmetry is broken in the more recent calculations

- m_u # m_d
- Incorporate electromagnetic effects between quarks (QCD+QED "quenched")

The mass difference (1°/₀₀) in isospin multiplets (N,Σ,Ξ,K) has been calculated

Of particular interest is M_n-M_p which governs the weak decay and stability of nuclear chart

It results from a cancellation of opposite tendencies (if m_u=m_d , M_p>M_n … still H atomes ?)



$\Delta M_N = M_p - M_n$

PhD A. Portelli, CPT Marseille 2012 (BMW)

Related topics: sigma terms

Baryon scalar form factor at zero momentum transfert
Of special interest is the Nucleon case: key obervable in the direct detection of dark matter

- Nucleon πN sigma term

$$\sigma_{\pi N} = m_l \langle N \mid \bar{u}u + \bar{d}d \mid N \rangle \qquad m_l = \frac{m_u + m_d}{2}$$

- Nucleon strange sigma term

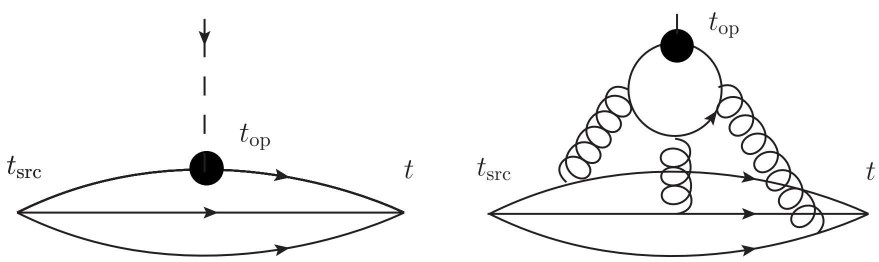
$$\sigma_s = m_s \langle N \mid \bar{s}s \mid N \rangle$$

- Strange content of N

$$y = \frac{2\langle N \mid \bar{s}s \mid N \rangle}{\langle N \mid \bar{u}u + \bar{d}d \mid N \rangle}$$

They can be computed :

- As matrix elements ("direct") ...
  but disconnected contributions



(a) (b)

- As derivatives of the baryon masses (Feynman-Hellman Th;)

$$\sigma_{\pi N} = m_\pi^2 \frac{\partial M}{\partial m_\pi^2} = m_l \frac{\partial M}{\partial m_l} \qquad \sigma_s = m_s \frac{\partial M}{\partial m_s}$$

Significant differences remain in the "sigma term" $\sigma_{\pi N}$ (coef $c^{(1)}_X$ in M_X)

ETMC	$\sigma_{\pi N}$=66.7 +/- 1.3 MeV	Alexandrou et al. PRD78 (2008) 014509
BMW	$\sigma_{\pi N}$=39(4) (+18)(-7) MeV	Durr et al. PRD85 (2012)
PACS-CS	$\sigma_{\pi N}$=45 +/- 6 MeV	an. by Shanahan et al, PRD87 (2013) 0745034

Furthermore, when ETMC results are analyzed by Adelaide method one finds

$\sigma_{\pi N}$=46.5 +/- 1.2 MeV (Th. Thomas, P. Shanahan, R. Young, Private Communication)

But experimentally is not much better, say equally bad…. 45+/-8 and…. 64+/-7 MeV (!!!???)

Concerning the N strange content y_N

BMW	FH method	y_N=0.20(7) (+13)(-17)	compatible with 0 !!!
PACS-CS		y_N=0.04 +/- 0.01	P.E. Shanahan et al, PRD87 (2013) 0745034

MODELS

Gasser Leutwiller χpT LO + y=0.2 + m_s/m=25 $\quad \sigma_{\pi N} = 33 \text{ MeV}$

Higer order corrections + y=0.2 $\quad \sigma_{\pi N} = 45 \text{ MeV}$

From πN scatt data + new χpT method (Δ) $\quad \sigma_{\pi N} = 59(7) \text{ MeV}$

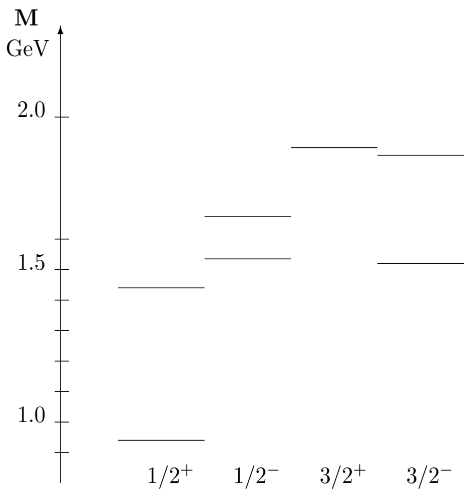
Alarcon et al, PRD85 (2012) 051503 $\quad y_N = 0.02(13)(10)$

I. Excited states

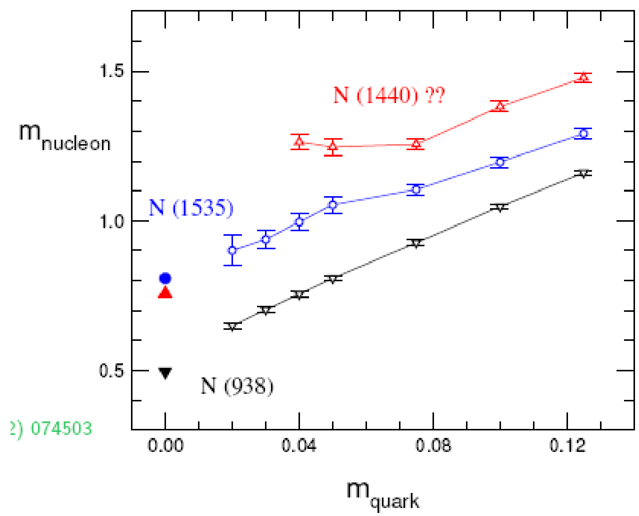
Excited states

The situation is much less clear…
The main problem is the first radial excitation of the $1/2^+$ baryon octet (N,Λ,Σ,Ξ)
Experimentally it lies below the first negative parity excitation ($1/2^-$, $3/2^-$)

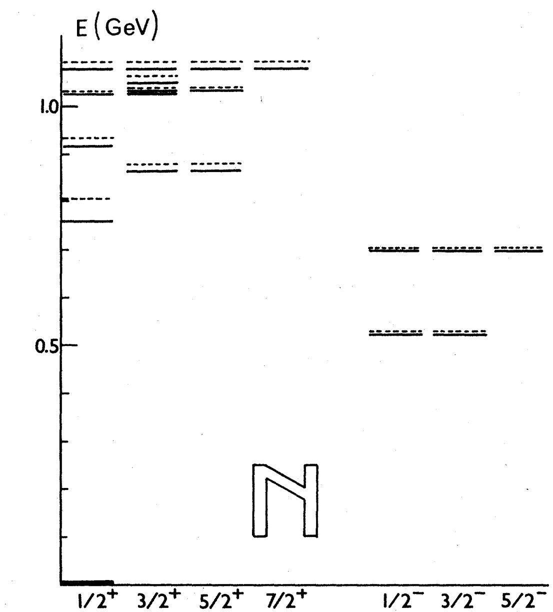
Experimental



Lattice (quenched)



Quark Models (NR)

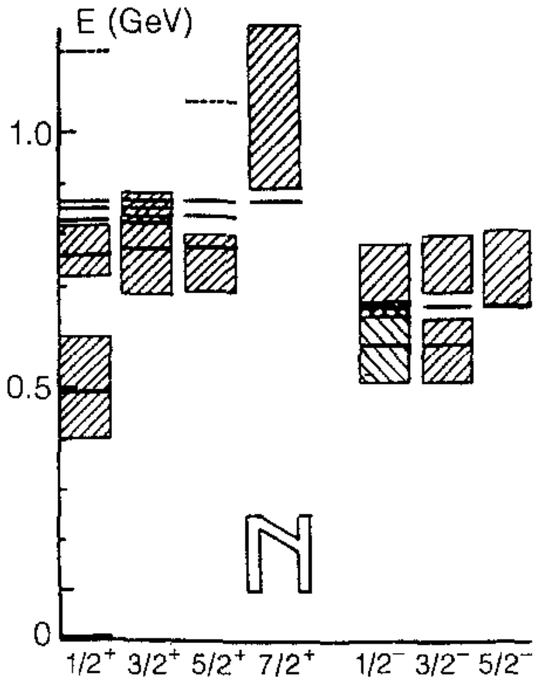


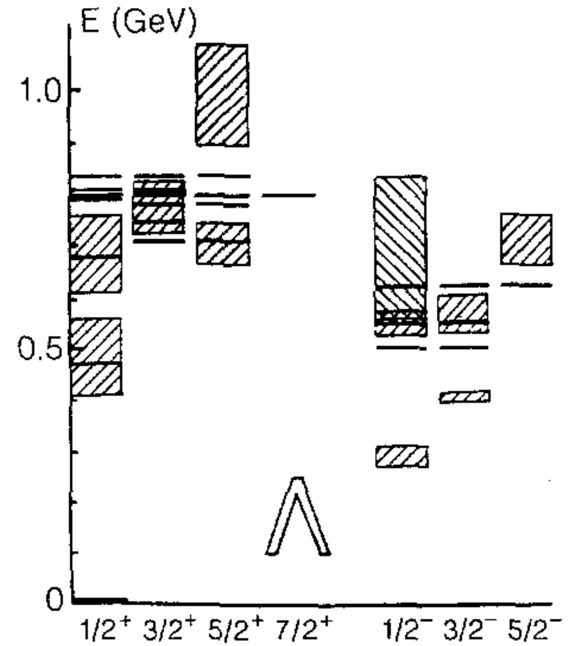
Silvestre-Brac et al Phys Rev D32 (1985)

Some progress in the QM side

I. Introducing 3-q forces in the NRCQM

B. Desplanques et al, Z. Phys. A343 (1992)





Good Ropers for N, Λ Σ but miss Λ negative parities

I'. Roper as N « breathing-mode »

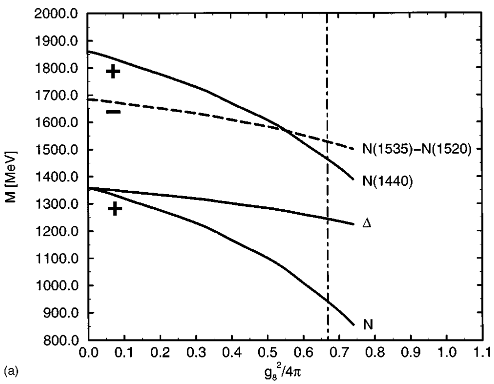
P. Guichon PLB 164 (1985) 361

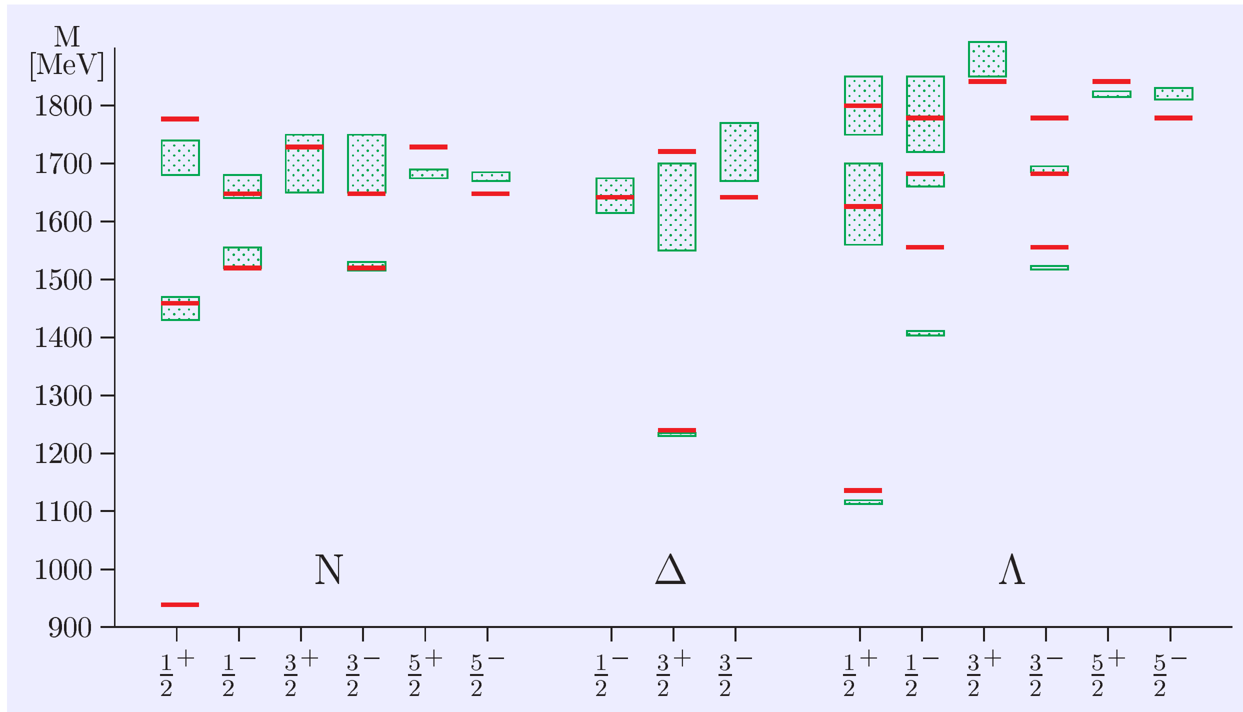
Non static MIT bag model

Ad-hoc, tautological,… but still !

I. Deeply modifying the qq dynamics

Graz Relativistic Constituent Quark Model (psGBE 98, EGBE 05)
Key: Introduce 0^- exchange between quarks to account for broken chiral dynamics





L. Glozmann, W. Plessas, Varga, Wagenbraunn, PRD58 (1998)
L. Glozmann, Papp, W. Plessas, Varga, WagenbraunnPRC57 (1998)
K. Glantschnig, R. Kainhofer, W. Plessasa, B. Sengl, and R.F. Wagenbrunn, Eur. Phys. JA23 (2005) 507

Some disagreement in the Λ and Σ first negative parity excitations

Achieved a consistent description of Baryon in RCQM

The model provides an acceptable agreement in a wide set of observables
…although at the price of increasing the number of « parameters while LQCD remains 2-3

Table 1. Predetermined parameters of the extended GBE CQM (for both cases, without and with spin-orbit forces). For additional explanations see the text.

$m_u = 340$ MeV	$m_d = 340$ MeV	$m_s = 507$ MeV
$\mu_\pi = 139$ MeV	$\mu_K = 494$ MeV	$\mu_\eta = 547$ MeV
$\mu_{\eta'} = 958$ MeV	$\mu_\rho = 770$ MeV	$\mu_{K^*} = 892$ MeV
$\mu_{\omega_8} = 947$ MeV	$\mu_{\omega_0} = 869$ MeV	$\mu_\sigma = 680$ MeV
$\mu_{a_0} = 980$ MeV	$\mu_\kappa = 980$ MeV	$\mu_{f_0} = 980$ MeV
$g^2_{\mathrm{ps},8}/4\pi = 0.67$	$(g^{\mathrm{V}}_{\mathrm{v},8})^2/4\pi = 0.55$	$(g^{\mathrm{V}}_{\mathrm{v},0})^2/4\pi = 1.107$
$(g_{\mathrm{ps},0}/g_{\mathrm{ps},8})^2 = 1$	$(g^{\mathrm{T}}_{\mathrm{v},8})^2/4\pi = 0.16$	$(g^{\mathrm{T}}_{\mathrm{v},0})^2/4\pi = 0.0058$
$g^2_{\mathrm{s}}/4\pi = 0.67$		

Table 3. Free parameters of the extended GBE CQM with spin-orbit forces.

$C = 1.935\ \mathrm{fm}^{-2}$	$\mathrm{V}_0 = -336$ MeV	
$\Lambda_\pi = 834$ MeV	$\Lambda_\rho = 1145$ MeV	$\Lambda_\sigma = 1513$ MeV
$\Lambda_K = 1420$ MeV	$\Lambda_{\eta'} = 1400$ MeV	
$(g^{\mathrm{LS}})^2/4\pi = 0.8$		

III. Including thresholds effects

P. Gonzalez et al, PRC77 (2008) 065213

The main point is that coupling a resonance state to a scattering meson-baryon chanel can significantly decrease its energy.

Most of the disagreements in CQM can be explained by threshold effects
N* (1/2+,1440) as σN
Λ (1/2-, 1405) as Kbar N

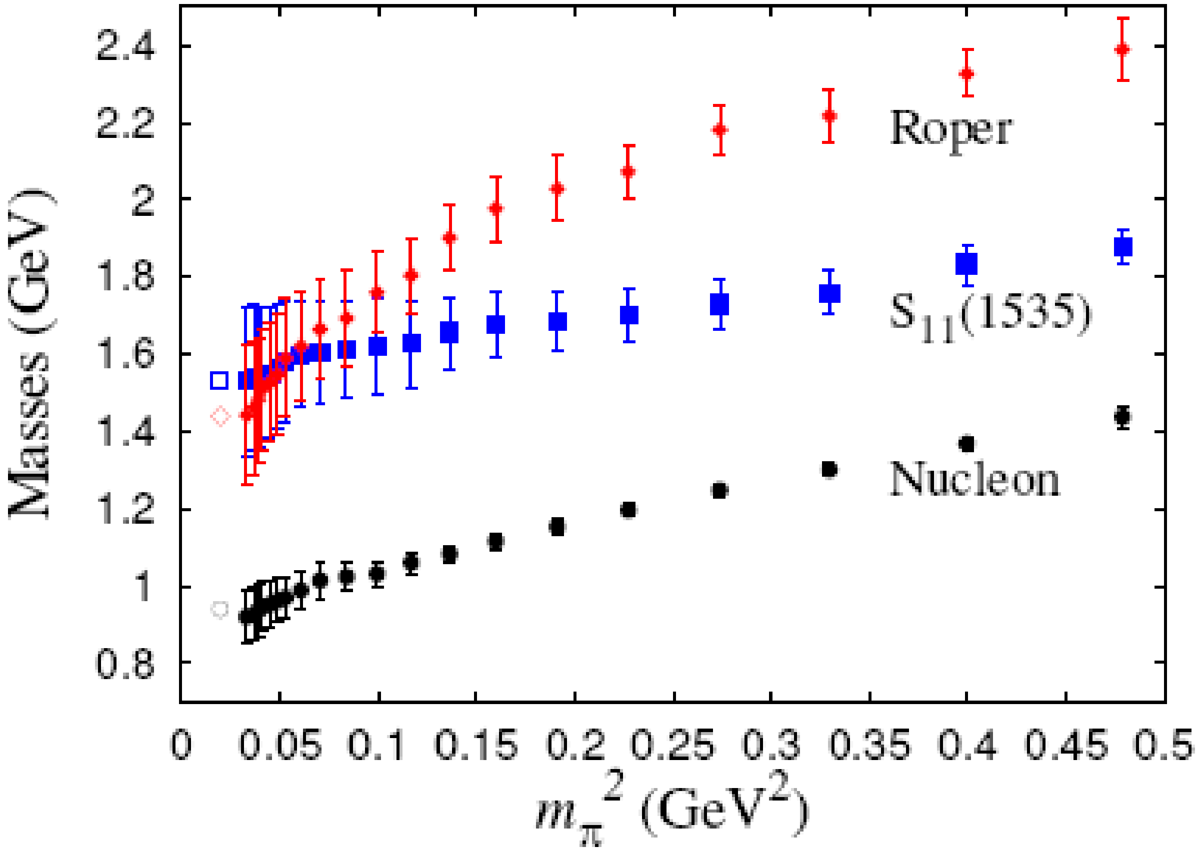
« si non è vero è bene trovato »

… and give us a good hint of what happens in the Lattice results

In LQCD the situation is quite confuse….

Some hope (m_π=180 MeV, quenched) … very much in the spirit of Graz model

Mathur, Chen,Dong, et al, Phys Lett B 605 (2005) 137



Unfortunately not confimed ...

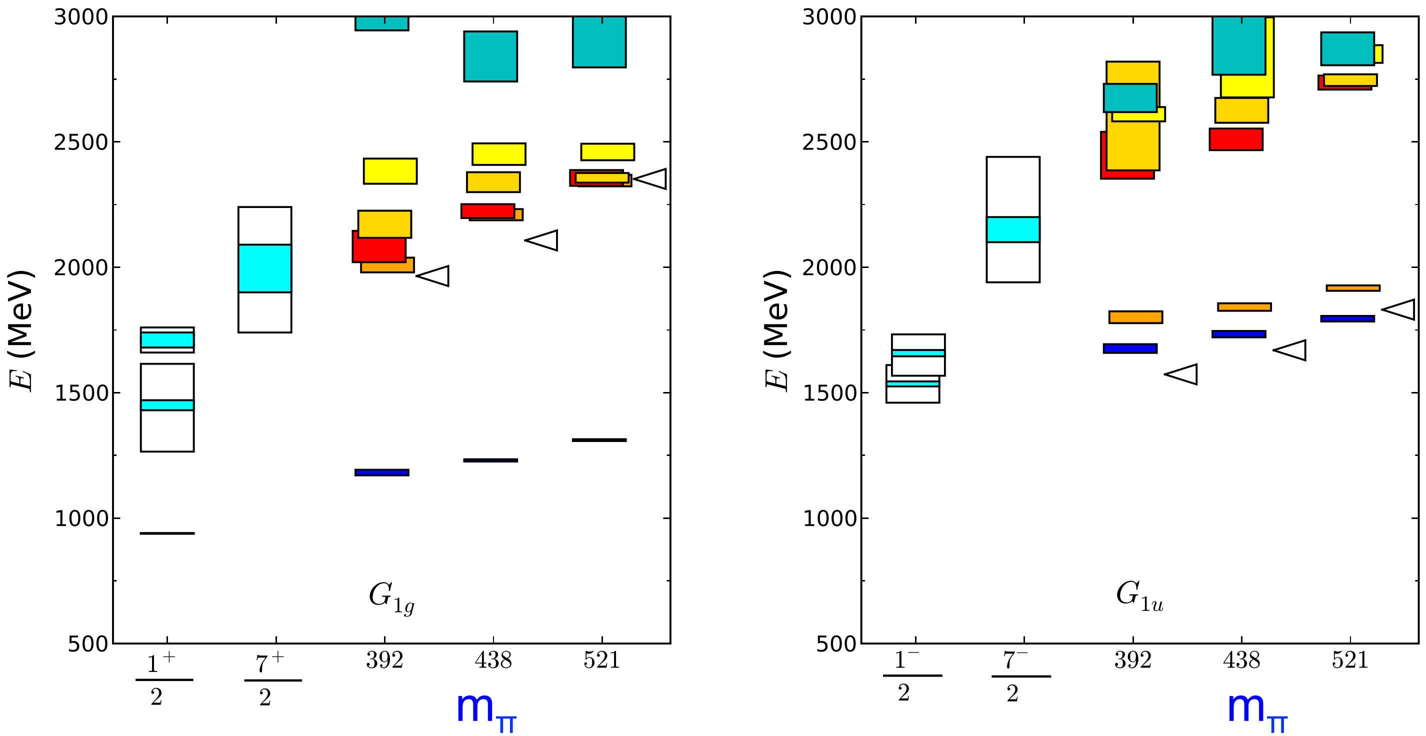
Bern-Graz-Regensburg Collaboration,
Lang, Erice Lectures 2007 , T. Burch et al., Phys. Rev. D 74 (2006) 014504

There is no clear evidence from LQCD that Roper is below the negative parity states

Calculations are very difficult since:
(i) one must disantagle ground and excited contributions in a sum of exponentials (instable fit !)
(ii) states dominated by decay (N*➔Nπ, Nππ,..) "physical π" are essential to dont inhibit scattering states !

(i) sems to be solved by recently developed methods (matrix correlator functions and distillation)
M. Peardon et al., QCD, PRD 80, 054506 (2009)
(ii) remains a serious drawback

Proper coupling to decay channels is mandatory

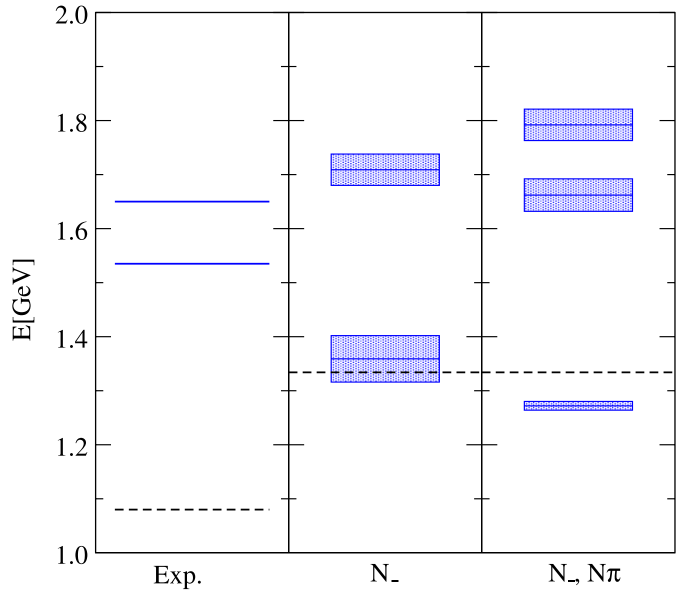


J. Bulava, Edwards, Morningstar et al, PRD 82, 014507 (2010) n_f=2+1 but m_π>390 MeV

PHYSICAL REVIEW D **87,** 054502 (2013)

Scattering in the πN negative parity channel in lattice QCD

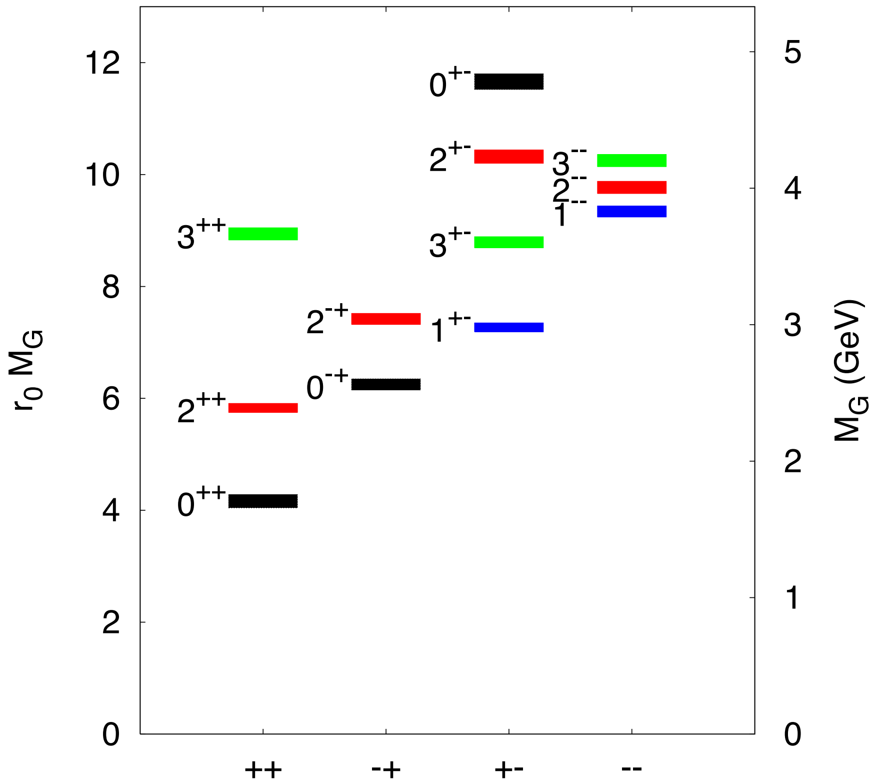
C. B. Lang* and V. Verduci†



Glueballs: a QCD crucial prediction…. with nothing behind !

The more serious calculations in LQCD are pure glue (quenched).

Y. Chen et al.,
PRD73, 014516 (2006)



No one has been clearly experimentally confirmed (hard!)...

Are quenched LQCD calculations reliable ?

First « unquenched » results: n_f=2+1, m_π=360 MeV

Gregory et al, JHEP 120 (2012) 170

Only lowest 0^{++} consistent with the quenched values

Others…. at the price of large error bars !

Others are missing …

+ some new states …

J^{PC}	Mass MeV			
	Unquenched	Quenched		
	This work	M&P	Ky	Meyer
0^{-+}		2590(40)(130)	2560(35)(120)	2250(60)(100)
2^{-+}	3460(320)	3100(30)(150)	3040(40)(150)	2780(50)(130)
0^{-+}	4490(590)	3640(60)(180)		3370(150)(150)
2^{-+}				3480(140)(160)
5^{-+}				3942(160)(180)
0^{--} (exotic)	5166(1000)			
1^{--}		3850(50)(190)	3830(40)(190)	3240(330)(150)
2^{--}	4590(740)	3930(40)(190)	4010(45)(200)	3660(130)(170)
2^{--}				3.740(200)(170)
3^{--}		4130(90)(200)	4200(45)(200)	4330(260)(200)
1^{+-}	3270(340)	2940(30)(140)	2980(30)(140)	2670(65)(120)
3^{+-}	3850(350)	3550(40)(170)	3600(40)(170)	3270(90)(150)
3^{+-}				3630(140)(160)
2^{+-} (exotic)		4140(50)(200)	4230(50)(200)	
0^{+-} (exotic)	5450(830)	4740(70)(230)	4780(60)(230)	
5^{+-}				4110(170)(190)
0^{++}	1795(60)	1730(50)(80)	1710(50)(80)	1475(30)(65)
2^{++}	2620(50)	2400(25)(120)	2390(30)(120)	2150(30)(100)
0^{++}	3760(240)	2670(180)(130)		2755(30)(120)
3^{++}		3690(40)(180)	3670(50)(180)	3385(90)(150)
0^{++}				3370(100)(150)
0^{++}				3990(210)(180)
2^{++}				2880(100)(130)
4^{++}				3640(90)(160)
6^{++}				4360(260)(200)

Models:

Vincent Mathieu,* Fabien Buisseret,+ and Claude Semay‡

PHYSICAL REVIEW D 77, 114022 (2008)

J^{PC}	Lattice	Lattice [36]	CGQCD [8]	Model A		Model B	
0^{++}	$1.710 \pm 0.050 \pm 0.080$ [3]	$1.475 \pm 0.030 \pm 0.065$	1.980	1.655	$\|^1S_0\rangle$	1.724	$\|S_+;0^+\rangle$
	$2.670 \pm 0.180 \pm 0.130$ [2]	$2.755 \pm 0.070 \pm 0.120$	3.260	2.696	$\|^1S_0\rangle$	2.543	$\|S_+;0^+\rangle$
		$3.370 \pm 0.100 \pm 0.150$		3.101	$\|^5D_0\rangle$	3.234	$\|S_+;0^+\rangle$
		$3.990 \pm 0.210 \pm 0.180$		3.496	$\|^1S_0\rangle$	3.839	$\|S_+;0^+\rangle$
0^{-+}	$2.560 \pm 0.035 \pm 0.120$ [3]	$2.250 \pm 0.060 \pm 0.100$	2.220	2.500	$\|^3P_0\rangle$	2.624	$\|S_-;0^-\rangle$
	$3.640 \pm 0.060 \pm 0.180$ [2]	$3.370 \pm 0.150 \pm 0.150$	3.430	3.305	$\|^3P_0\rangle$	3.443	$\|S_-;0^-\rangle$
1^{-+}				2.500	$\|^3P_1\rangle$	Forbidden	
1^{++}				3.101	$\|^5D_1\rangle$	Forbidden	
2^{++}	$2.390 \pm 0.030 \pm 0.120$ [3]	$2.150 \pm 0.030 \pm 0.100$	2.420	1.655	$\|^5S_2\rangle$	2.588	$\|D_+;2^+\rangle$
		$2.880 \pm 0.100 \pm 0.130$	3.110	2.696	$\|^5S_2\rangle$	3.077	$\|S_+;2^+\rangle$
				3.101	$\|^{1,5}D_2\rangle$	3.325	$\|D_+;2^+\rangle$
2^{-+}	$3.040 \pm 0.040 \pm 0.150$ [3]	$2.780 \pm 0.050 \pm 0.130$	3.090	2.500	$\|^3P_2\rangle$	3.077	$\|S_-;2^-\rangle$
	$3.890 \pm 0.040 \pm 0.190$ [3]	$3.480 \pm 0.140 \pm 0.160$	4.130	3.304	$\|^3P_2\rangle$	3.732	$\|S_-;2^-\rangle$
3^{++}	$3.670 \pm 0.050 \pm 0.180$ [3]	$3.385 \pm 0.090 \pm 0.150$	3.330	3.101	$\|^5D_3\rangle$	3.254	$\|D_-;3^+\rangle$
			4.290	3.783	$\|^5D_3\rangle$	3.882	$\|D_-;3^+\rangle$
3^{-+}				3.601	$\|^3F_3\rangle$	Forbidden	
4^{++}	$3.650 \pm 0.060 \pm 0.180$ [37]	$3.640 \pm 0.090 \pm 0.160$	3.990	3.101	$\|^5D_4\rangle$	3.768	$\|D_+;4^+\rangle$
			4.280	3.784	$\|^5D_4\rangle$	3.961	$\|S_+;4^+\rangle$
				4.038	$\|^{1,5}G_4\rangle$	4.328	$\|D_+;4^+\rangle$
4^{-+}			4.270	3.601	$\|^3F_4\rangle$	3.961	$\|S_-;4^-\rangle$
			4.980	4.204	$\|^3F_4\rangle$	4.499	$\|S_-;4^-\rangle$
5^{++}				4.038	$\|^5G_5\rangle$	4.207	$\|D_-;5^+\rangle$
5^{-+}				4.432	$\|^3H_5\rangle$	Forbidden	
6^{++}		$4.360 \pm 0.260 \pm 0.200$		4.038	$\|^5G_6\rangle$	4.598	$\|D_+;6^+\rangle$
				4.585	$\|^5G_6\rangle$	4.708	$\|S_+;6^+\rangle$
				4.793	$\|^{1,5}I_6\rangle$	5.073	$\|D_+;6^+\rangle$

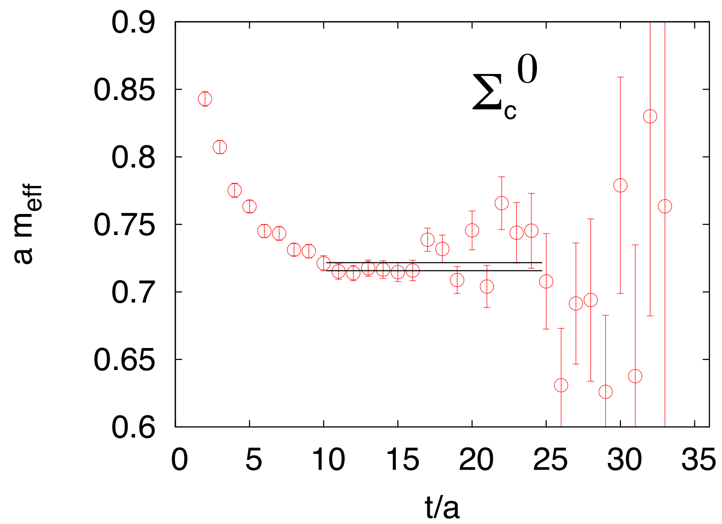
Parameters were adjusted to reproduce (unquenched) LQCD results

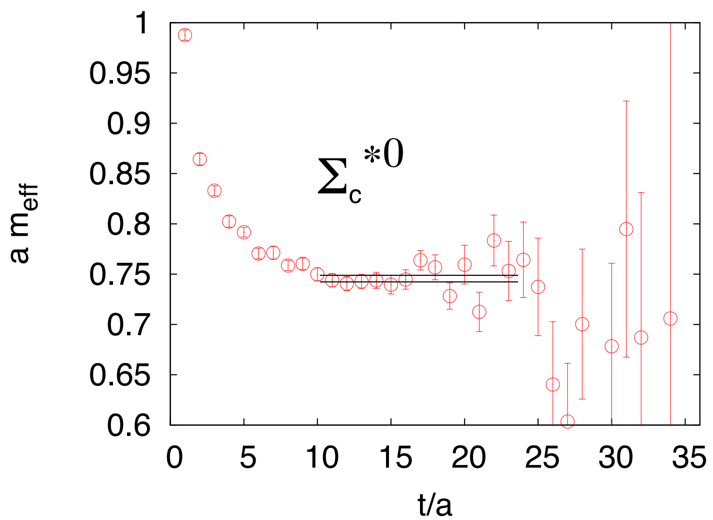
… go to 1

Some predictions for charmed-strange baryons ….

Without any parmater: unambuguous prediction for « exotic » baryons
For instance: baryons J=1/2,3/2 stranges and charmed with C=1,2,3

C. Alexandrou et al (ETMC), Phys.Rev. D86 (2012) 114501

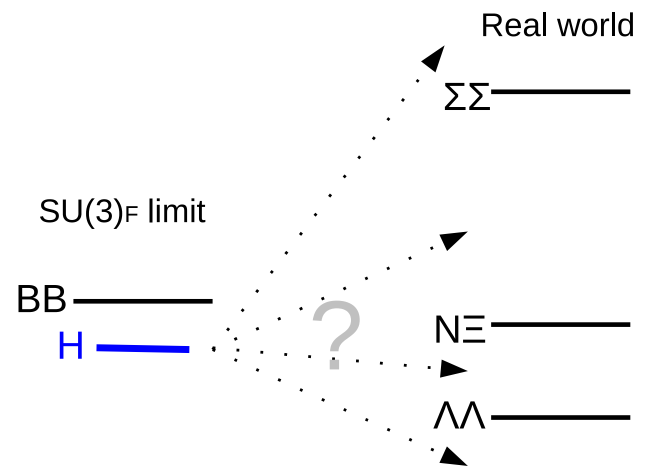




Particle(PDG)	m_B^0 (GeV)	$-4c_B$ (GeV^{-1})	c (GeV^{-2})	χ^2/d.o.f.	m (GeV)
$\Sigma_{c,av}(2.454)$	2.437(25)	1.92(54)	-2.09(91)	1.1	2.468(17)(23)
Ξ_{cc}^+	3.476(35)	2.39(83)	-3.39(1.5)	2.7	3.513(23)(14)
$\Lambda_c^+(2286)$	2.198(40)	2.99(96)	-3.6(1.7)	0.10	2.246(27)(15)
$\Sigma_{c,av}^*(2.520)$	2.520(25)	2.37(51)	-2.96(86)	1.3	2.556(18)(51)
$\Xi_{cc,av}^*$	3.571(25)	2.02(57)	-2.62(99)	1.0	3.603(17)(21)
Ω_{ccc}	4.6706(53)	0.327(35)	0.	2.5	4.6769(46)(30)

Two Baryons on a Lattice

Let us consider H, the most famous one
Jaffe 77 MIT bag and $SU3_F$ limit B=100 MeV
Experimentally: nothing
From $He_{\Lambda\Lambda}$: if bound at all, B_H<7 MeV



Several quark models found it unbound
…as soon as $SU3_F$ is broken

NPLQCD 2011

Evidence for a bound H-dibaryon from LQCD (n_f=2+1) !!!!

S. Beane et al Phys. Rev. Lett. 106 (2011) 162001

B= 16.6 +/- 2.1 +/- 4.6 MeV

In fact m_π=400 MeV , a=0.12 fm , a*L=3.9 fm
« QCD » changes fast when approaching the physical point

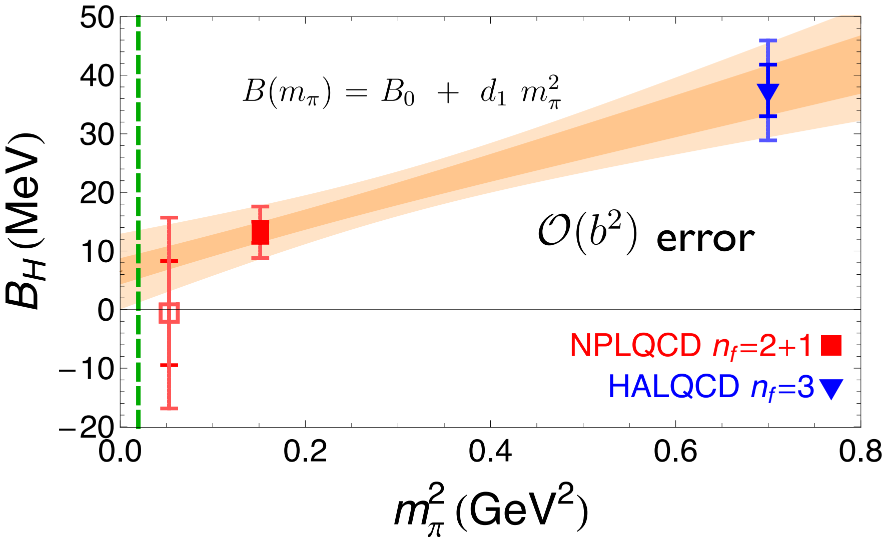
So no real evidence from « real » QCD ….

Two Baryons on a Lattice

NPLQCD 2012

The H-dibaryon and ΞΞ systems are bound at unphysical quark masses.

Naive chiral extrapolation of the existing lattice data indicate that at 2-sigma level H can be unbound or independent of the quark masses



$$B_H^{\text{quadratic}} = 7.4 \pm 2.1 \pm 5.8 \text{ MeV}$$

Two Baryons on a Lattice

HAL QCD (Aoki, Doi, Hatsuda, Ishi,..)

T. Inoue, T. Doi in Lattice 2012, Inoue et al NPA881(2012)28

Wilson-Clover, N_f=3 a=0.12 fm m_π=0.47-1.2 GeV

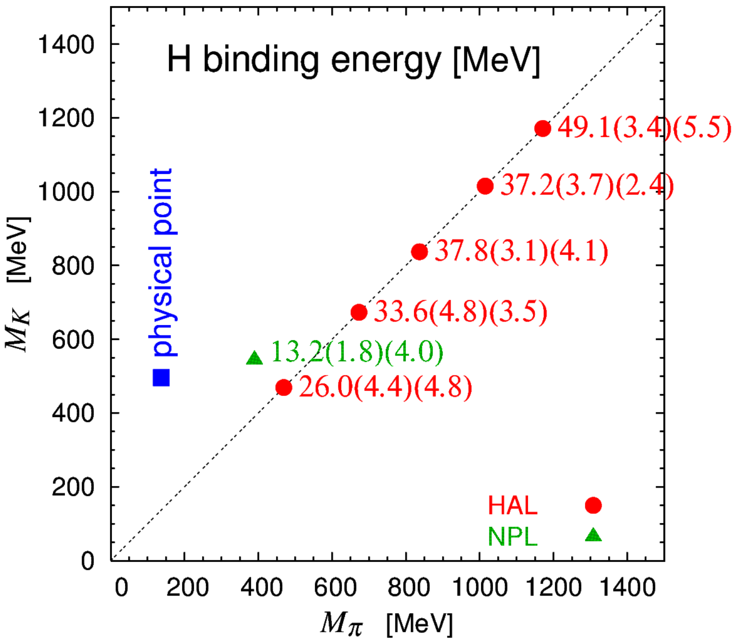
T. Doi Lattice 2012

With m_π=470 MeV=M_K, H bound with B= 26-49 MeV

T. Inoue Lattice 2012

When SU3 is broken, H goes through ΛΛ:

it is unlikely that H is bound



Difficult to follow…. but LQCD seems to confirm model predictions:

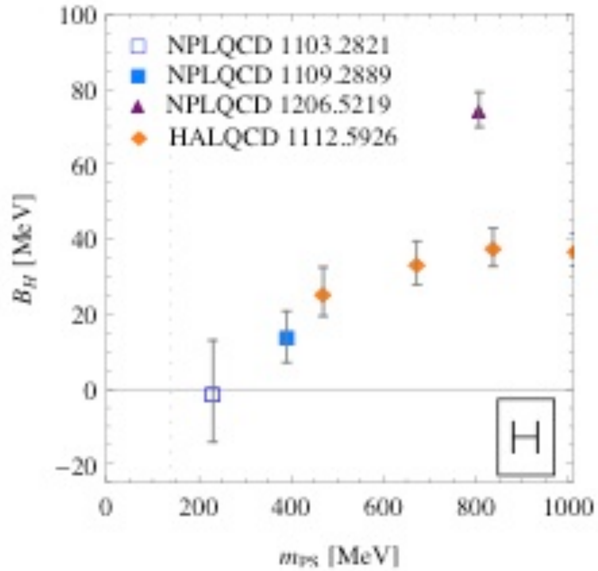
H bound in SU3 (recovered by large pion mass)

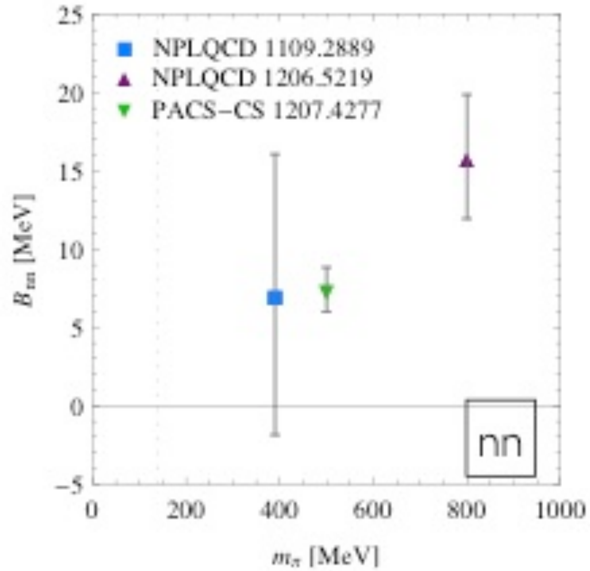
B_H drecreases when going to physical point

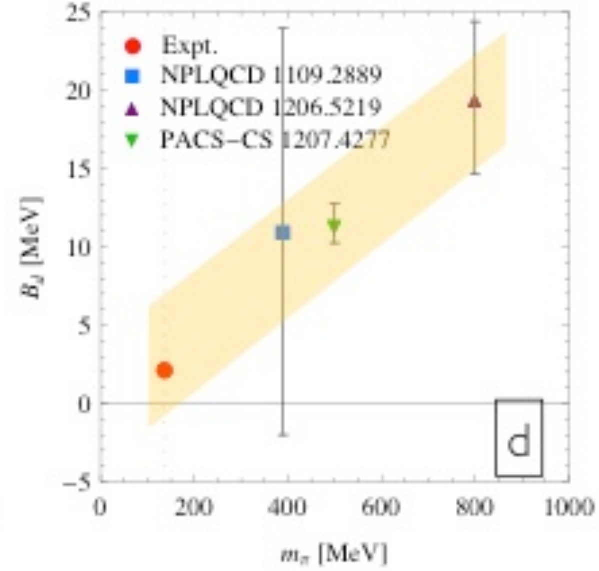
Progress are great but there is no firm conclusion.

Only « impressions » : NOT BOUND !

The very last results (two weeks ago)







Three and Four Nucleons on a Lattice (I)

With a lot of courage (+ PhD grants + Postdocs) one can compute 6A-point euclidean correlators to obtain A=3,4 bound states… not much more !

Some japanees and american groups (PACS CS, HAL QCD) had all that …

It is extremly complex: number of "Wick contractions" increases factorially

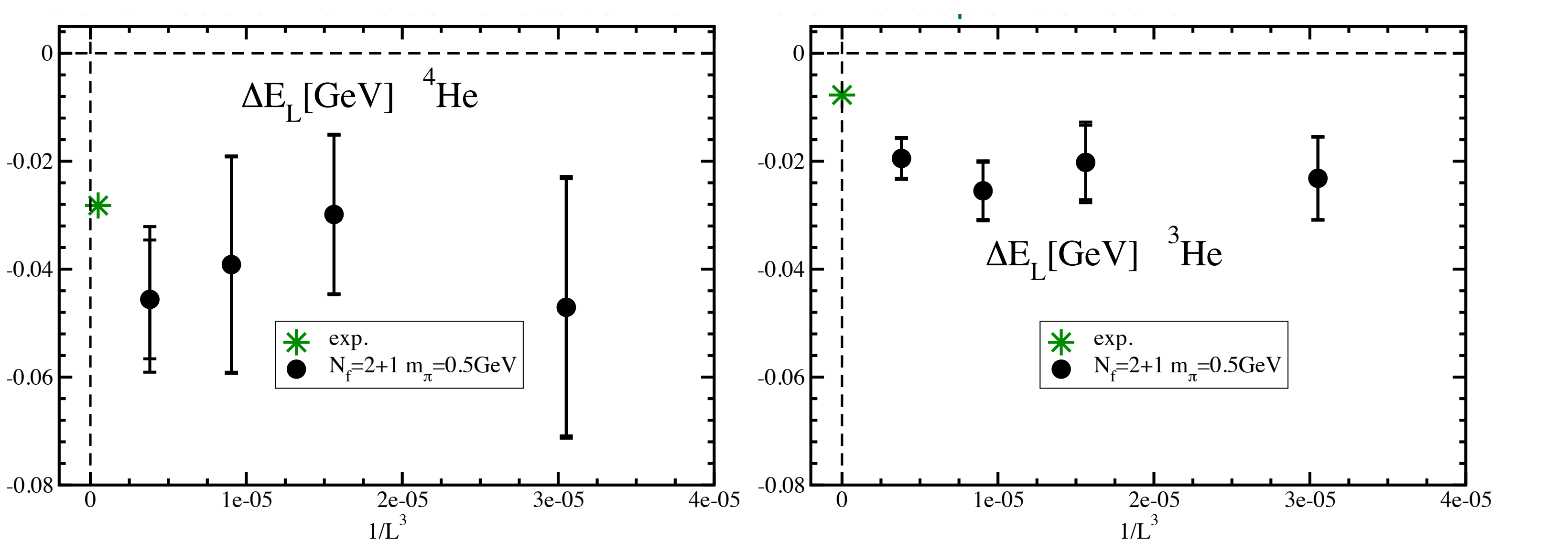
For ^{4}He about 500 000 contraction ….

Signal/Noise is very small and the extraction of effective masses very difficult

Physically is easyest to get A=3,4 than deuteron, quite a fragile and extended object

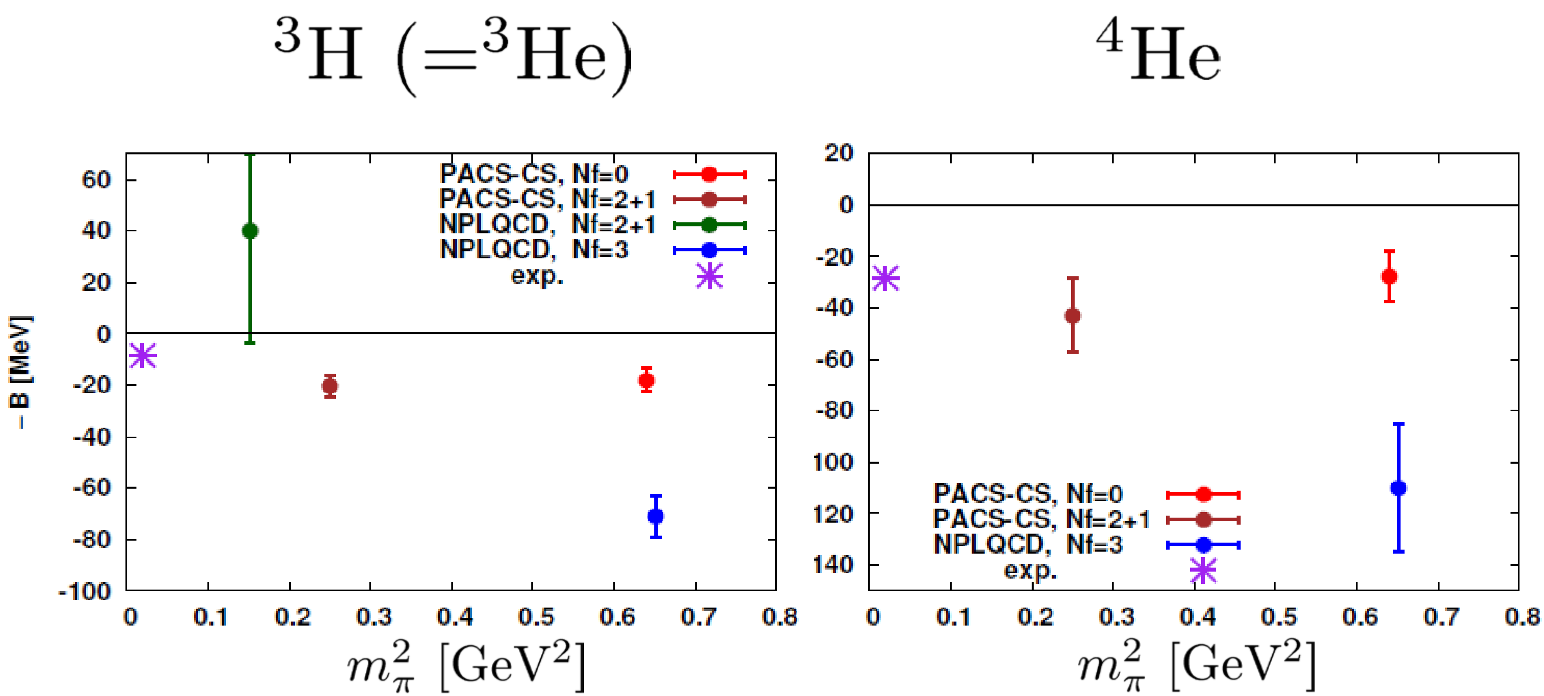
The r^2 (^{4}He) is smaller than deuteron and B/A=7…. 7 times bigger

Yamazaki (PACS CS) at Lattice 2012



Three and Four Nucleons on a Lattice (II)

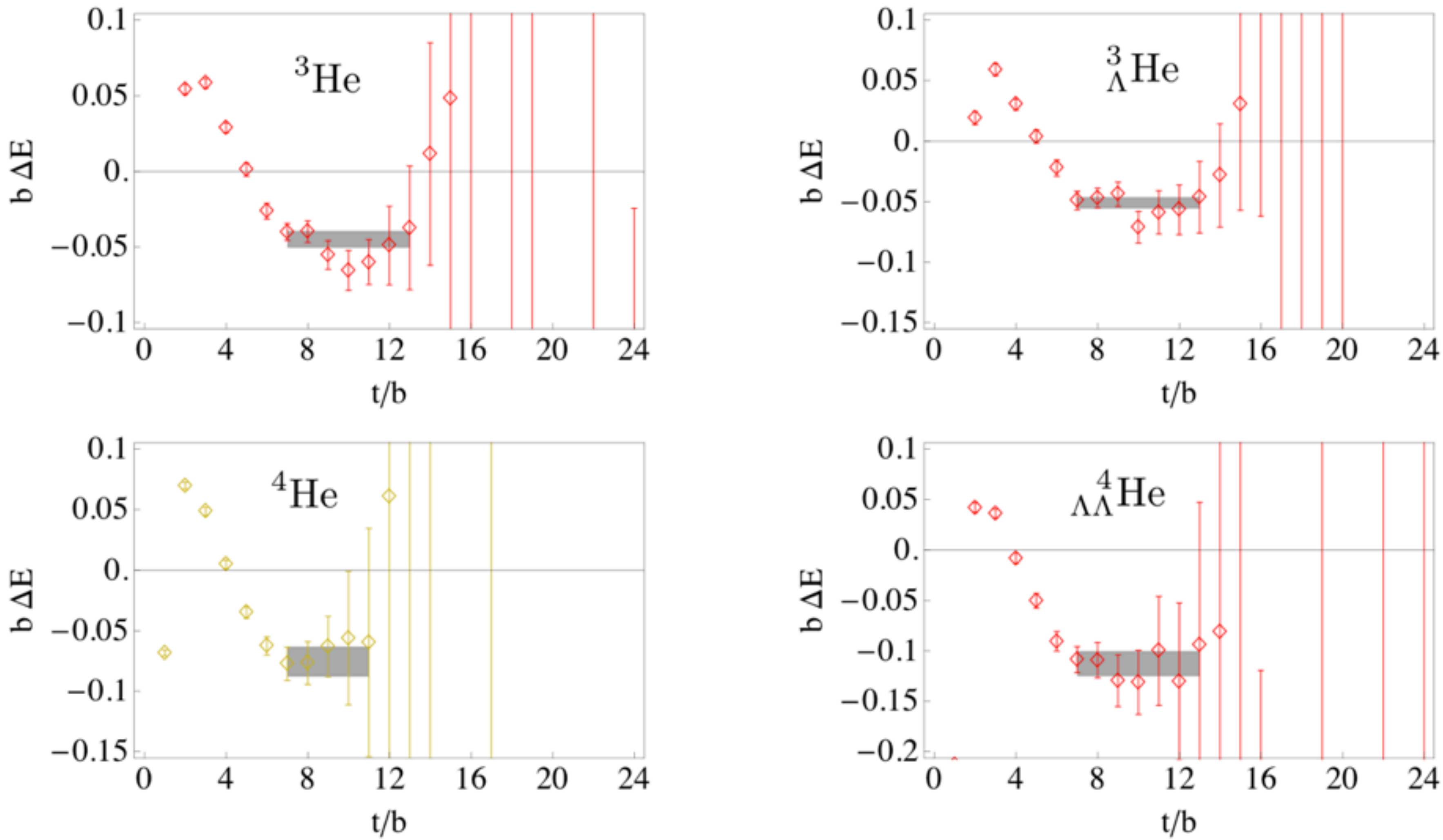
T. Doi, Plenary talk at Lattice 2012



For a nuclear physicist it is very impressive to « get out a nuclei » from (almost) nothing !
V_{NN} « models » (conventional meson-exchange or QCD-inspired EFT) have 20-40 parameters

However this result will not very useful to the Nuclear Physics community….
Models will remain mandatories for usual nuclear physics beyond A=4

Nuclei (A=3,4)



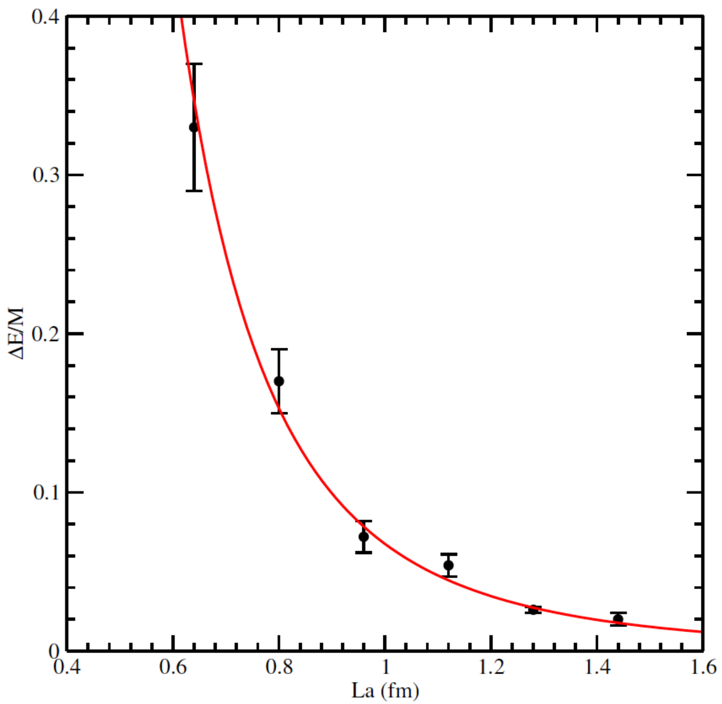
NPLQCD arXiv:1206.5219

SCATTERING STATES

At the begining (Maiani-Testa): « no scattering » in Euclidean
In addition: 2 particles are always confined on a lattice with peridic boundary condition….

Luscher 87: The energy levels $\varepsilon_n(L)$ of 2-particle states in a box (L) provide the phase-shifts

In the simplest case: ground state $\varepsilon_0(L)$ provides the scattering length A_0



$$\epsilon_0(L) = \frac{4\pi A_0}{(aL)^3}\left\{1 + c_1\left(\frac{A_0}{aL}\right) + c_2\left(\frac{A_0}{aL}\right)^2 + \ldots\right\}$$

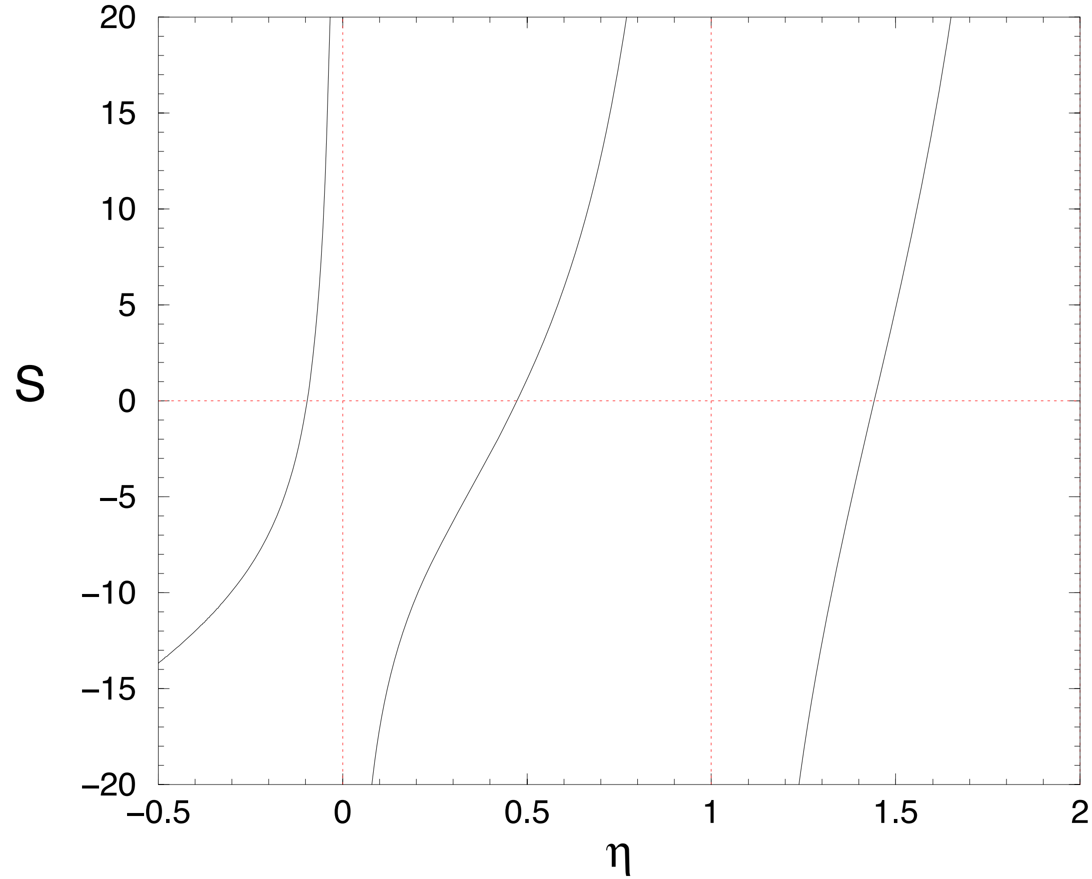
c_1 and c_2 are universal known coefficients

The phase-shifts are obtained in a similar way

$$\eta = \frac{(aL)^2 \epsilon_0(L)}{4\pi^2}$$

$$k \cot \delta_0(k) = \frac{1}{\pi a L}\, S(\eta)$$

S being some universal function (depending on L)



Many interesting results have been obtained (most of them with m_p>>140 MeV) on Meson-Meson, Meson-Baryon, Baryon-Baryon

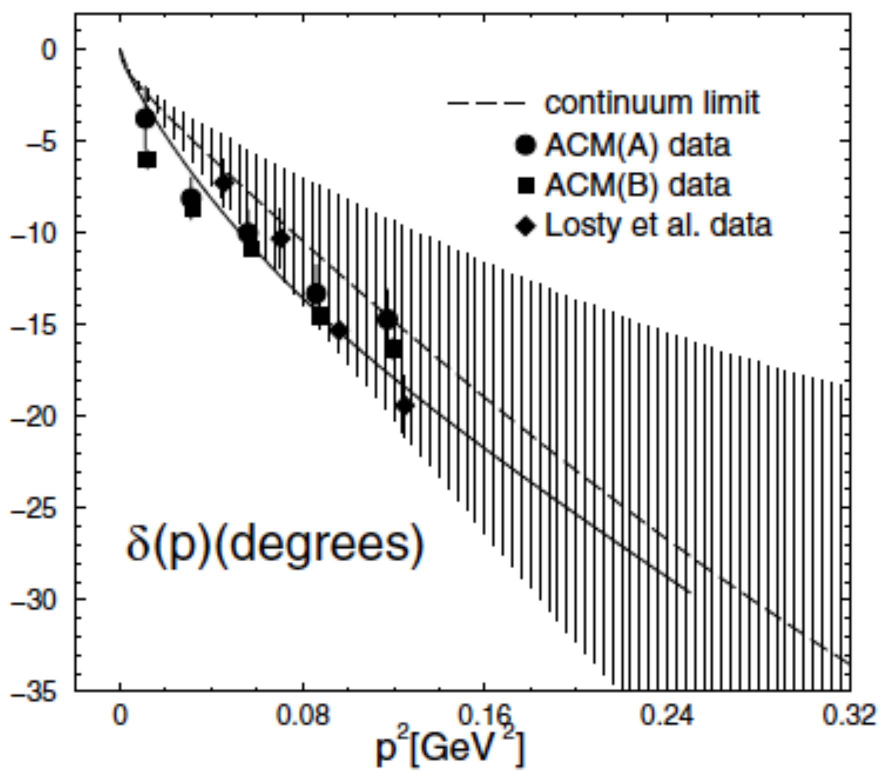
ππ scattering phase shifts

PHYSICAL REVIEW D, VOLUME 70, 074513

$I = 2\pi\pi$ scattering phase shift with two flavors of $O(a)$ improved dynamical quarks

T. Yamazaki,[1] S. Aoki,[1] M. Fukugita,[2] K-I. Ishikawa,[3] N. Ishizuka,[1,4] Y. Iwasaki,[1,4] K. Kanaya,[1] T. Kaneko,[5] Y. Kuramashi,[5] M. Okawa,[3] A. Ukawa,[1,4] and T. Yoshié[1,4]

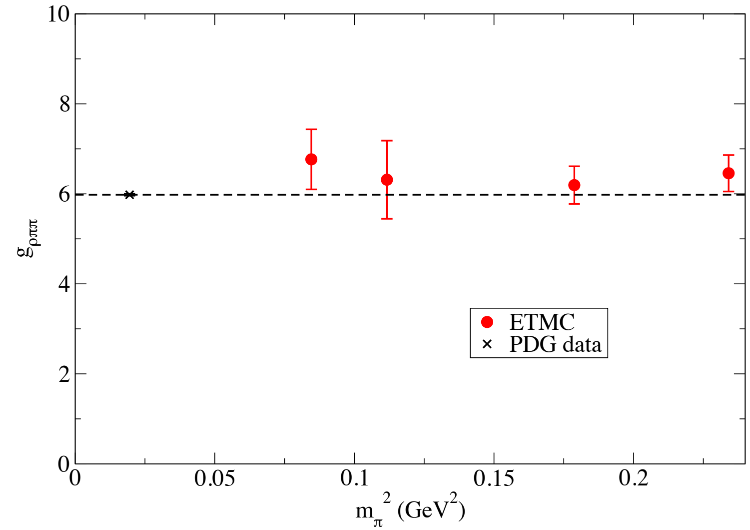
(CP-PACS Collaboration)

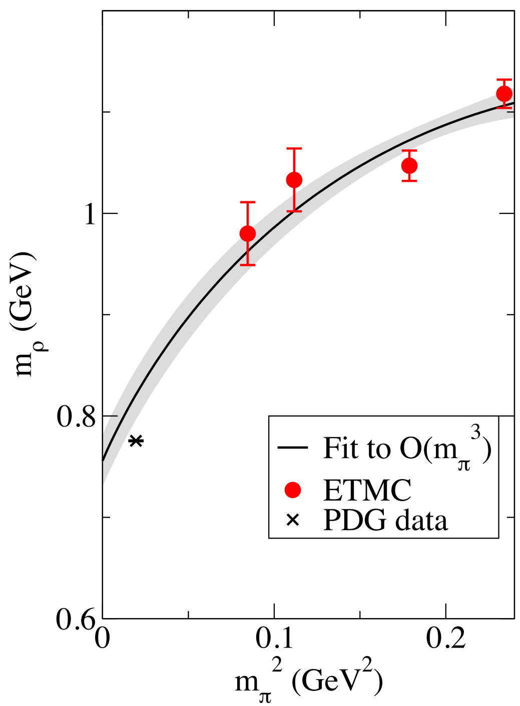


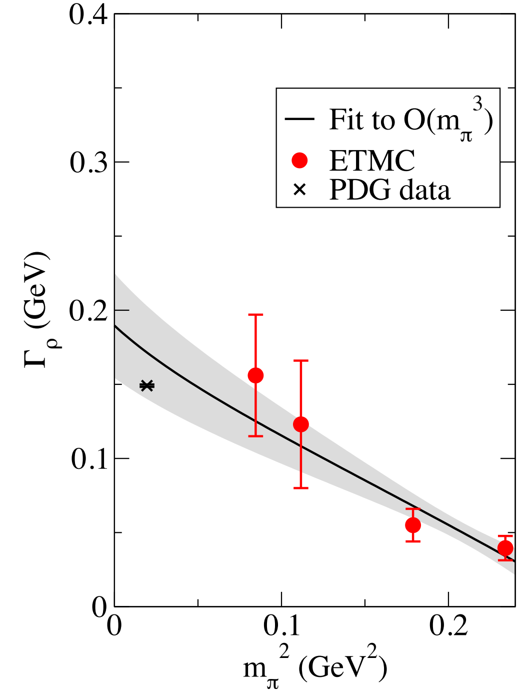
Resonance Parameters of the ρ-meson

X. Feng, K. Jansen D.B. Renner PRD 83 2011

$$\tan\delta_1 = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{E_{CM}(m_\rho^2 - E_{CM}^2)}\,, \quad p = \sqrt{E_{CM}^2/4 - m_\pi^2}$$





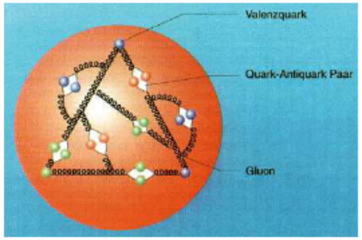


Nucleon-Nucleon Potential

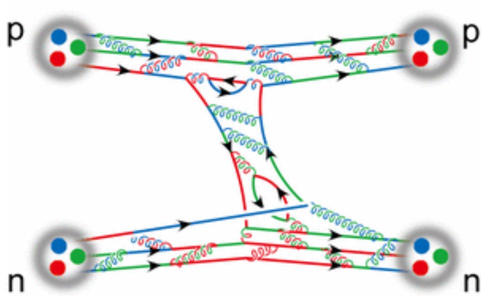
The most popular application of scattering in LQCD is the extraction of the NN potential
It deserves some remarks that could open further discussions

I. Concerning the interaction

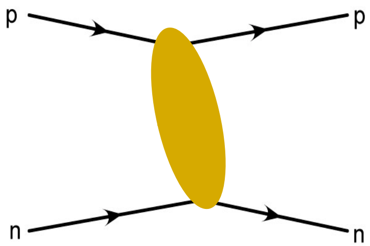
Nucleons are very complicate objects



What results into very complicated interactions



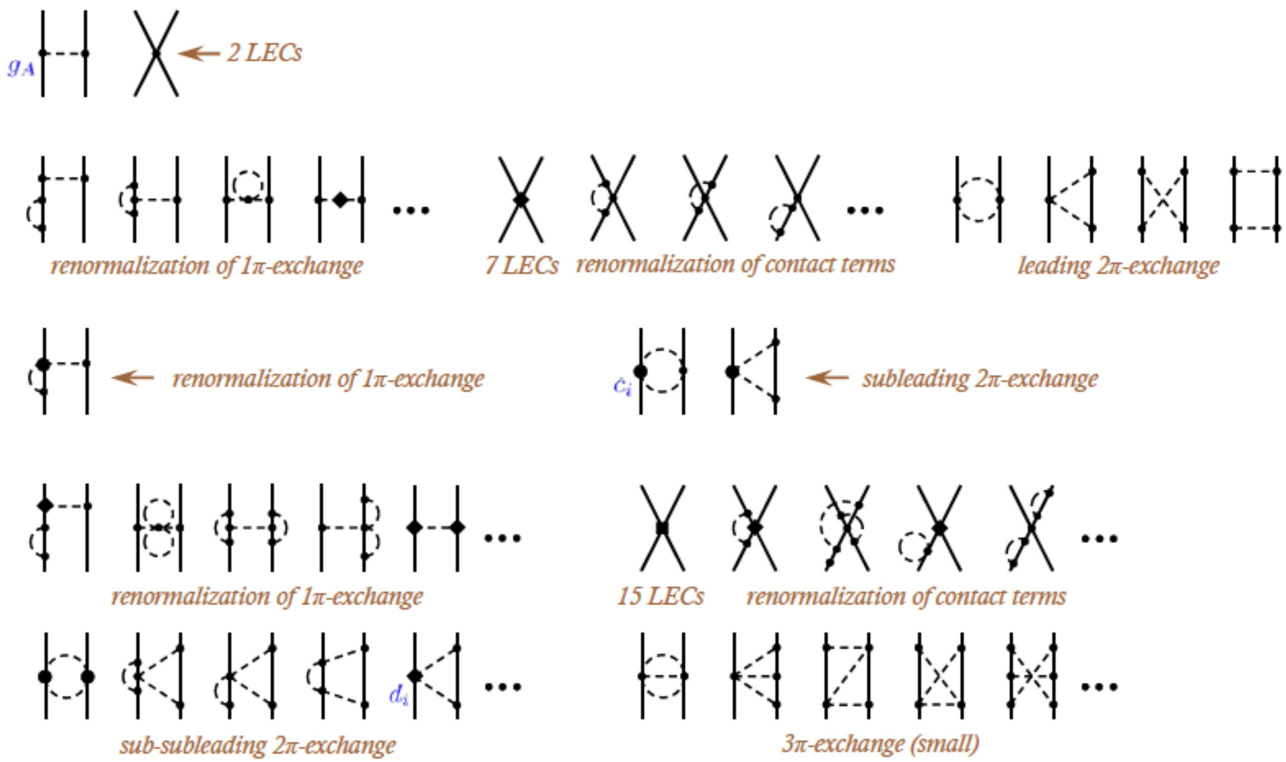
If we want to describe it by "potentials" V
between pointlike objects......



It is not astonishing to end up with "monsters" having many many parameters.....
ensuring an almost perfect description X^2/datum=1.01 of the very rich NN data (T_L<300MeV)

Example of NN "potential"

Epelbaum Joliot Curie School 2010, since it has been improved (N4LO)



+ isospin-breaking corrections...

Does it exist at all ?

How to get "a V_{NN}" from LQCD

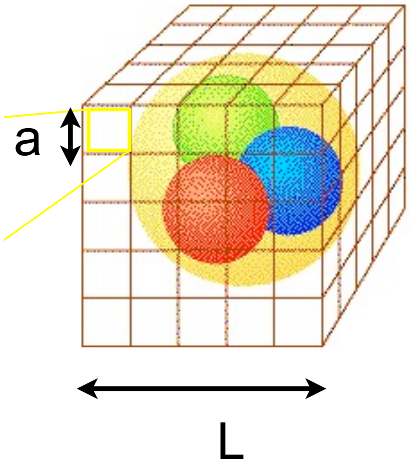
In NRQM $(H_0+V)\Psi=E\Psi$

Obtain V from (E,Ψ) is a very delicate problem (*)

Only solvable from the knowledge of (E,Ψ) for all E + some conditions (locality)

In QCD, V is not defined and there is no equivalent of Schrodinger eq.

So what ?



1. Compute $\Delta E=E-E_0$ and corresponding phase shift $\delta_0(E)$ (Luscher)
2. Compute the euclidean Bethe-Salpeter amplitude (well defined)

$$\Phi_{BS}(x-y) = \langle 0 \mid N(x)N(y) \mid NN \rangle$$

solution of $\Phi(k,P) = S_1(k,P)S_2(k,P) \int \frac{d^4k'}{(2\pi)^4} \, iK(k,k';P)\, \Phi(k',P)$

3. Identify Φ_{BS} to Ψ_{Sch} (eliminating k_0 dependence)
4. Insert it the Schrodinger and "deduce a V"... which depends on E

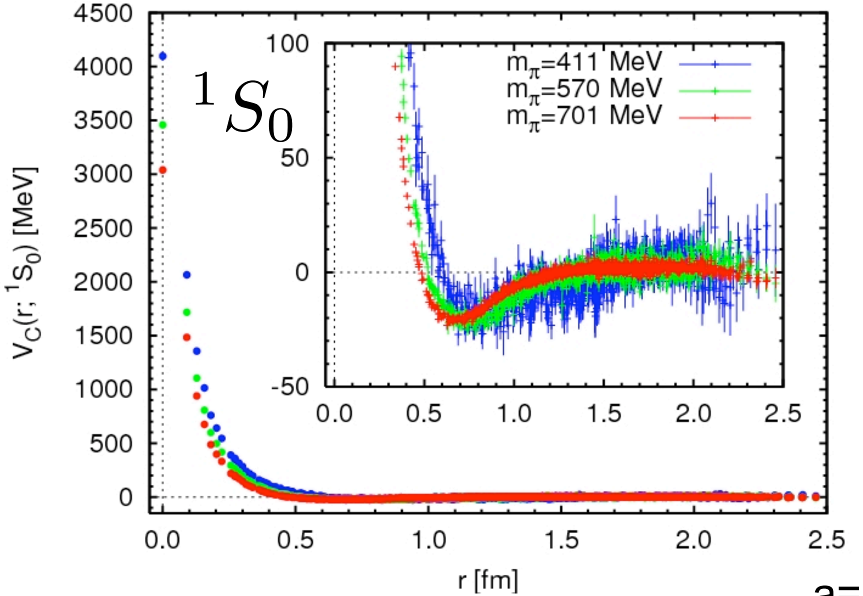
- Either by adjusting some parametrized form of V to give the same δ_0
- Either by $(-\Delta + V)\Psi = E\Psi \quad \Longrightarrow \quad V = \frac{(E+\Delta)\Psi}{\Psi}$

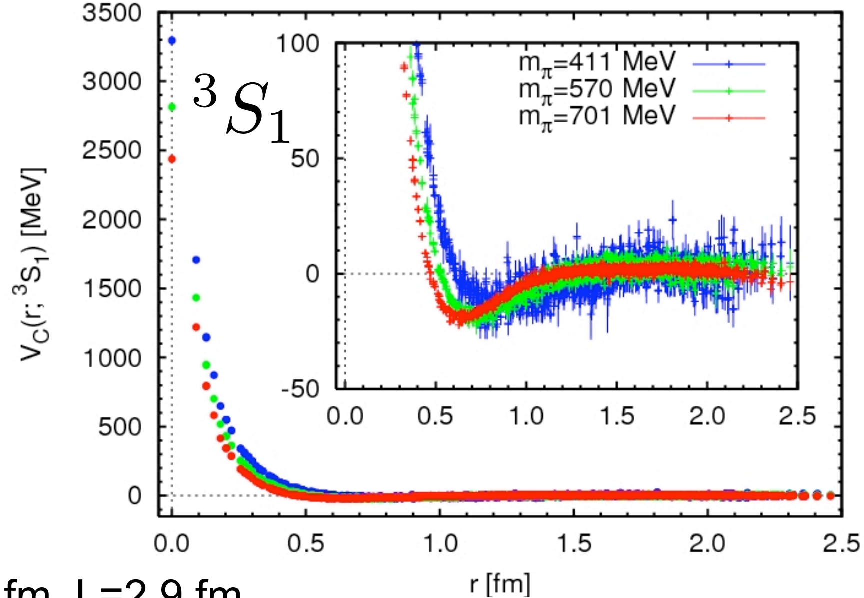
....E pur si muove !

(*) K. Shadan et P.C. Sabatier, Inverse Problem in Quantum Scattering Theory, Spinger Verlag 1977/2011

Central NN potentiel (S wave)

Coll. HAL QCD: Aoki, Doi, Hatsuda, Ikea, Ishii, Nemura, Sasaki, ...

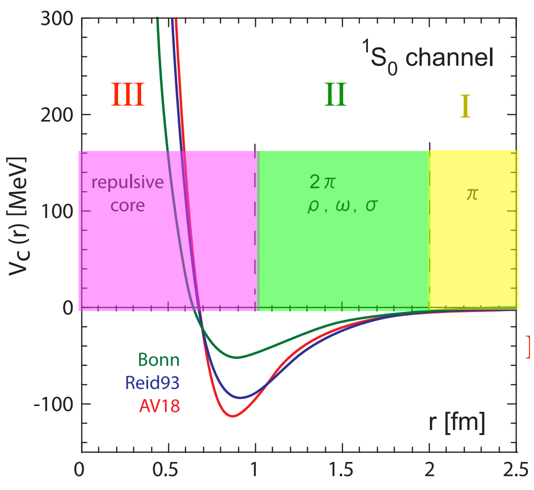




a=0.1 fm, L=2.9 fm

Not "physical" because of m_π
Not yet reliable because small L
Runs are in progress with BMW confs

None of these calculations found a Yukawa like
Rather an exponential which does'nt fit with any OBE model



Despite of several ambiguities in the protocol

- "inversion"
- identification BS-Schrodinger
- Euclidean / Minkowski metric
- ....

and the still rough approximations in LQCD (L,a,m_q,..)

It is a qualitatively important result: first trace of ”NN interaction” from a 2 parameter QCD

But will remain always qualitative, whatever the progress can be made

The ambiguities of the method would be always greater than the required accuracy in nuclear physics calculation (spectroscopy and reactions)

The V_{NN} – and they are badly needed in NRQM ! – would rather be provided by conventional boson-exchange or by QCD inspired EFT models

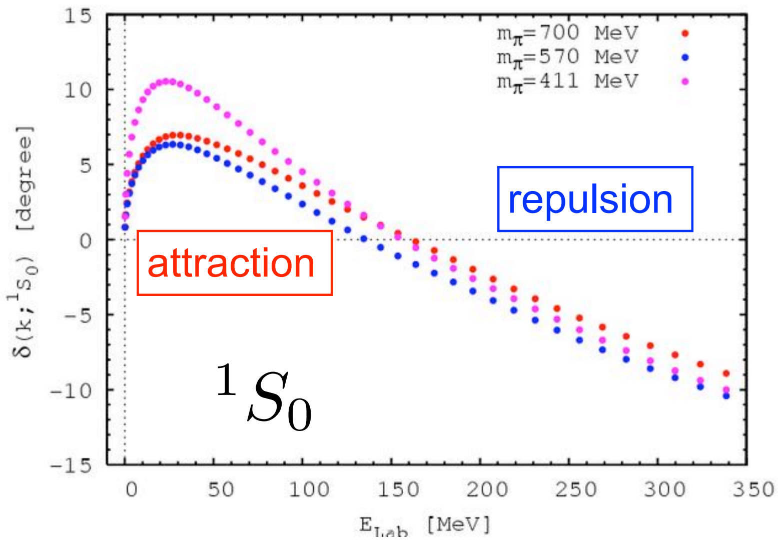
Even in NRQM V_{NN} is not “well defined”, in the sense that it is not unique
There are families of “phase equivalent V” (not an observable !)

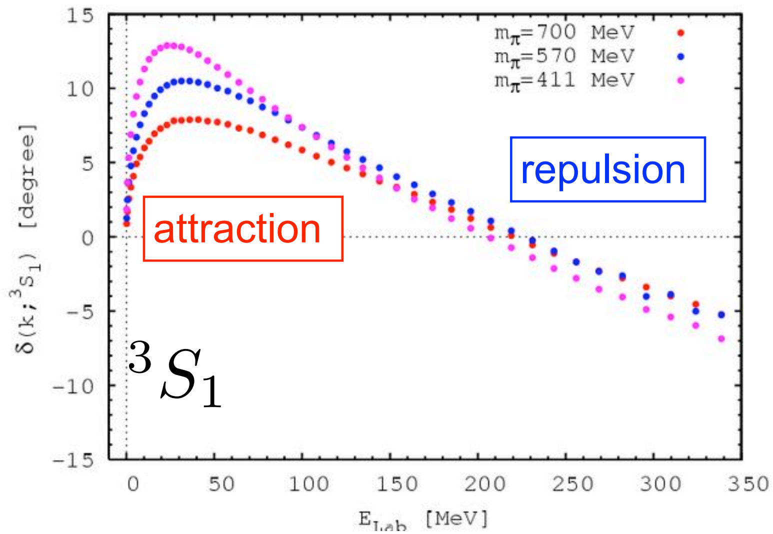
Another history are the NN phase shifts....

They are very well known experimentally

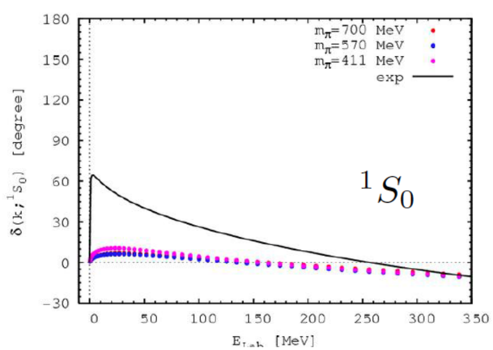
They are well defined in LQCD (some ambiguities in the coupled channel Luscher method ?)

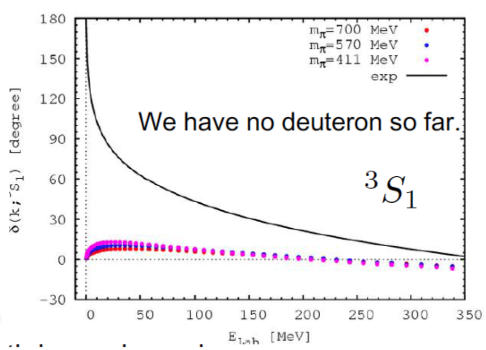
LQCD must be able to reproduce them accurately if: m_π=140 MeV, a "small", L large ($m_\pi L$=5)





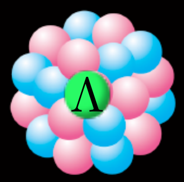
We are still far from that…. but they can come fast (5 years ?)

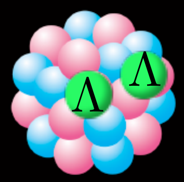




Hyperon-N and Hyperon-Hyperon Interaction

Rich experimental activity with hypernuclei and interest in understanding the S-role in n-stars





- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

However the Hyperon-N is poorly known
…and will remain “always” so, since low energy monokinetic hyperon beams will hardly come
Not to talk about Y targets…

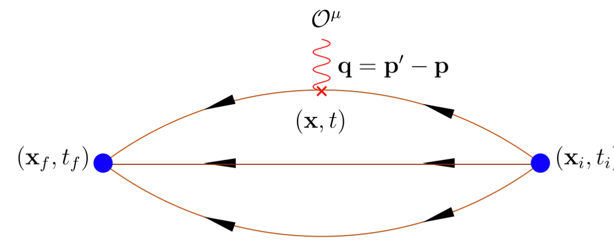
The Y-N phase shifts are well defined in LQCD and can be reliably calculated - even more than the NN ones - for the dominant states in low energy physics (L=0,1,2)
… provided **m_π**=140 MeV, **a** ”small” and **$m_\pi L=5$**

LQCD can soon supply this lack of experimental results....
One can always ”built” phase-equivalent V (OBE or EFT) to insert in Schrodinger equation and study more complex systems

The interplay of models (which remain necessary) and LQCD can be here very rich

HADRON STRUCTURE

Hadron Structure observables



During its propagation in euclidean time ($t_f \leftarrow t_i$) , N interacts with an external source. Computing this amplitude in the Lattice provides ”generalized form factors” and related quantities ($g_A \mu_N$, F1,F2/GE,GM ($\langle r^2 \rangle$), GPD)

Only 3 lattice groupes computed these quantities (unquenched) with « reasonable » m_π values

LHPC	last results in	Bratt et al, PRD82, 094502 (2010)
ETMC	Nf=2 results in	Alexandrou et al, PRD83, 114513 (2011)
	Nf=2+1+1 in	Alexandrou et al, arXiv:1303.5979
QCDSF-UKQCD	last results in	Collins et al, PRD84, 074507 (2011)

+ recent isolated works concentrated in particular problems g_A, $\langle r^2 \rangle$,$\langle x \rangle$

Untill now none of the results is fully "satisfactory"

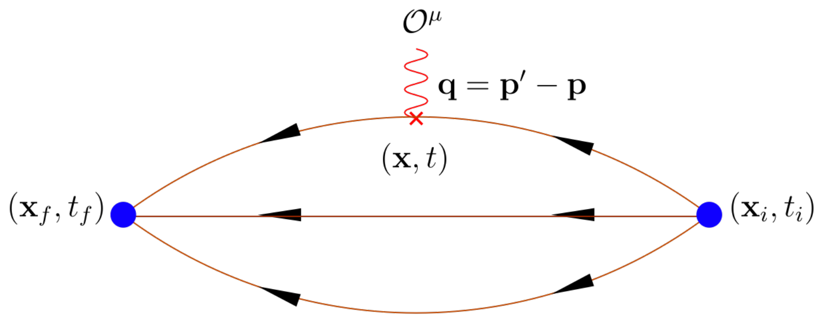
*** The computed quantities are the isovector (T=1) components, which are free from the disconnected contibutions
They are compared to the corresponding experimental data ***

How to compute form factors ?

Create N at x_i $\quad \overline{N}_a \equiv \epsilon^{ijk}(\bar{u}^i C\gamma_5 \bar{d}^j)\bar{u}^k_a$

Interact at x

Annihilate N at x_f $\quad N_a \equiv \bar{\epsilon}^{ijk}(u^i C\gamma_5 d^j)u^k_a$



Compute the 3-point Green function points (x_f, x, x_i)

if $\hat{O}^\mu = \bar{q}\gamma^\mu q \qquad C^\mu_{ab}(x_f, x, x_i) = <0 \mid N_a(x_f)\, \bar{q}(x)\gamma^\mu q(x)\, \bar{N}_b(x_i) \mid 0>$

$$\{\ldots\} = \bar{u}_b(p', s') \left\{ F_1(q^2)\, \gamma^\mu + F_2(q^2)\, \frac{i\sigma^{\mu\nu} q^\nu}{2M_N} \right\} u_a(p, s)$$

if $\hat{O}^\mu = \bar{q}\gamma^\mu\gamma_5 q$

$$\{\ldots\} = \bar{u}_b(p', s') \left\{ G_A(q^2)\, \gamma^\mu\gamma_5 + G_P(q^2)\, \frac{q^\mu\gamma_5}{2M_N} \right\} u_a(p, s)$$

$\{\ldots\}$ means TF

$t_i << t, t_f >> t$, to avoid excited states contamination, Lorentz….

With a well chosen combination of "traces and projections, FF are extracted

The simplest observable: g_A

- Axial form factor at Q=0

$$\langle N(p',s')|\mathcal{O}^{\mu}_{A^3}|N(p,s)\rangle =$$
$$\bar{u}_N(p',s')\left[G_A(q^2)\gamma_\mu\gamma_5 + \frac{q_\mu\gamma_5}{2m_N}G_p(q^2)\right]\frac{1}{2}u_N(p,s)$$

- No Q dependence (avoiding hypercubic artefacts)

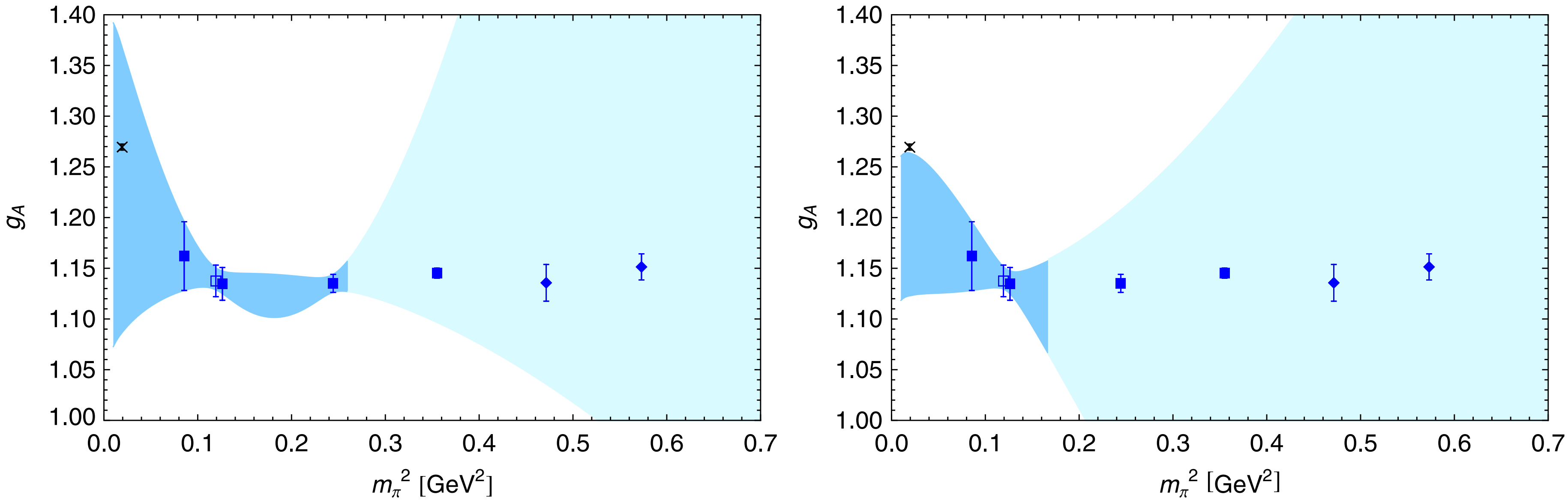
- Renormalization constant Z_A well determined non pertubatively

- No disconected diagrams

…. Well known experimentaly g_A=1.267

Exemple: Axial charge g_A=1.27 (LHPC)

$g_A = G_A(q^2=0)$

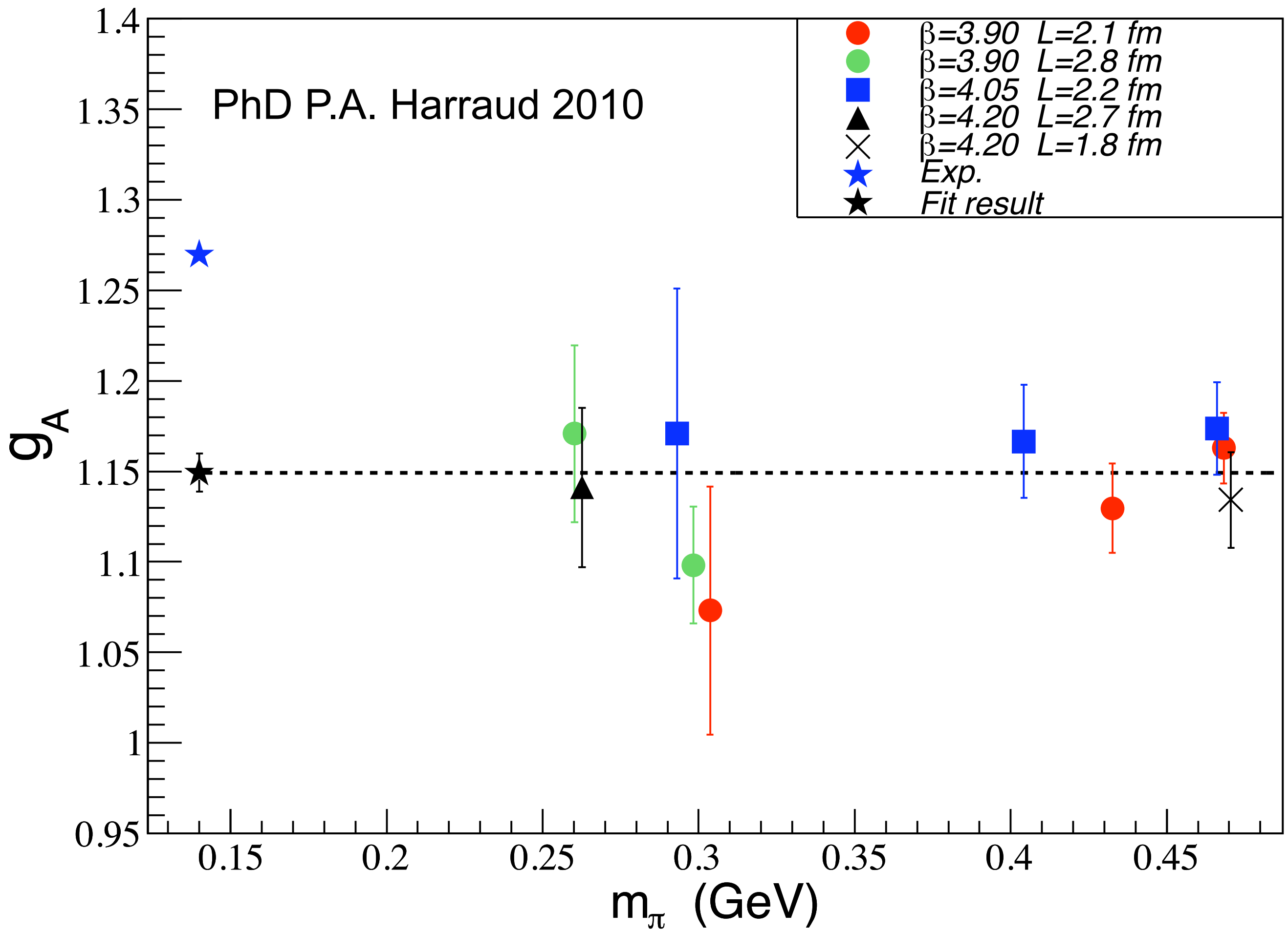


The computed values are practicaly independent of m_π

« Naive » linear extrapolation gives 1.153(28)

Using chiral extrapolations (2-3 parameters) the value is compatible with experimental data
… in fact extrapolation is out of control !

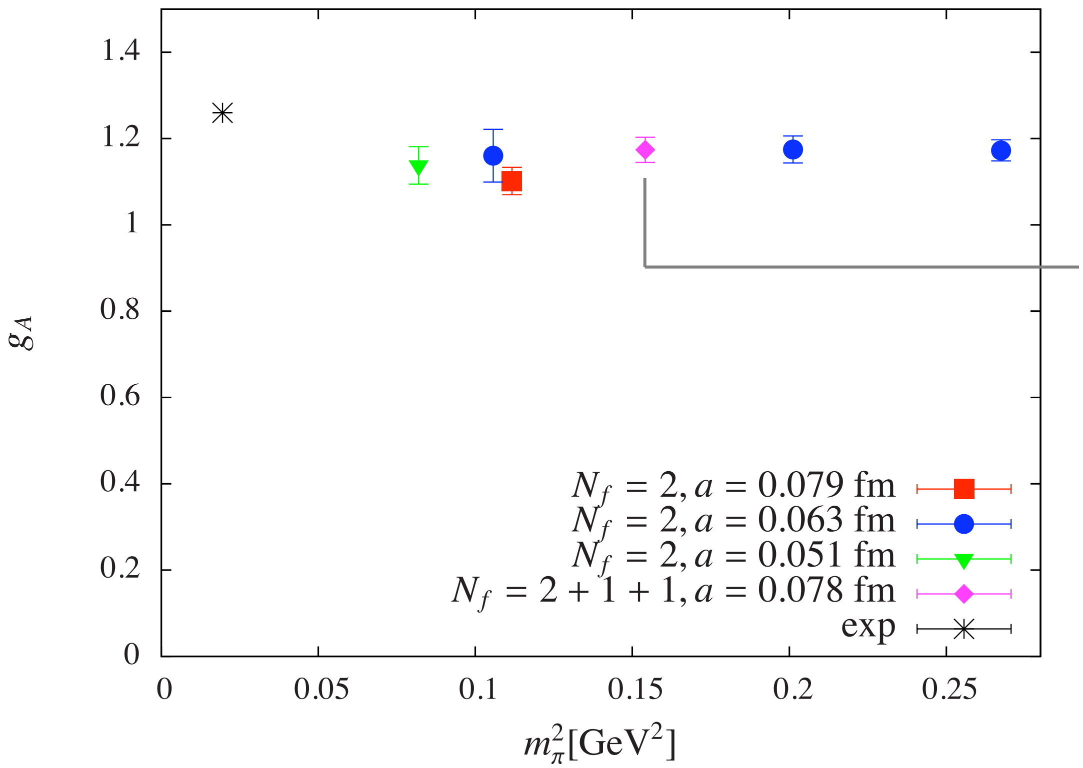
ETMC axial forme factor g_A



Raisonnable 1.15(1) …but 10% of (uncontrolled) error

Including nf=2+1+1 ETMC results (Alexandrou et al 2013 arXiv:1303.5979)

Looking from far enough it looks nice !

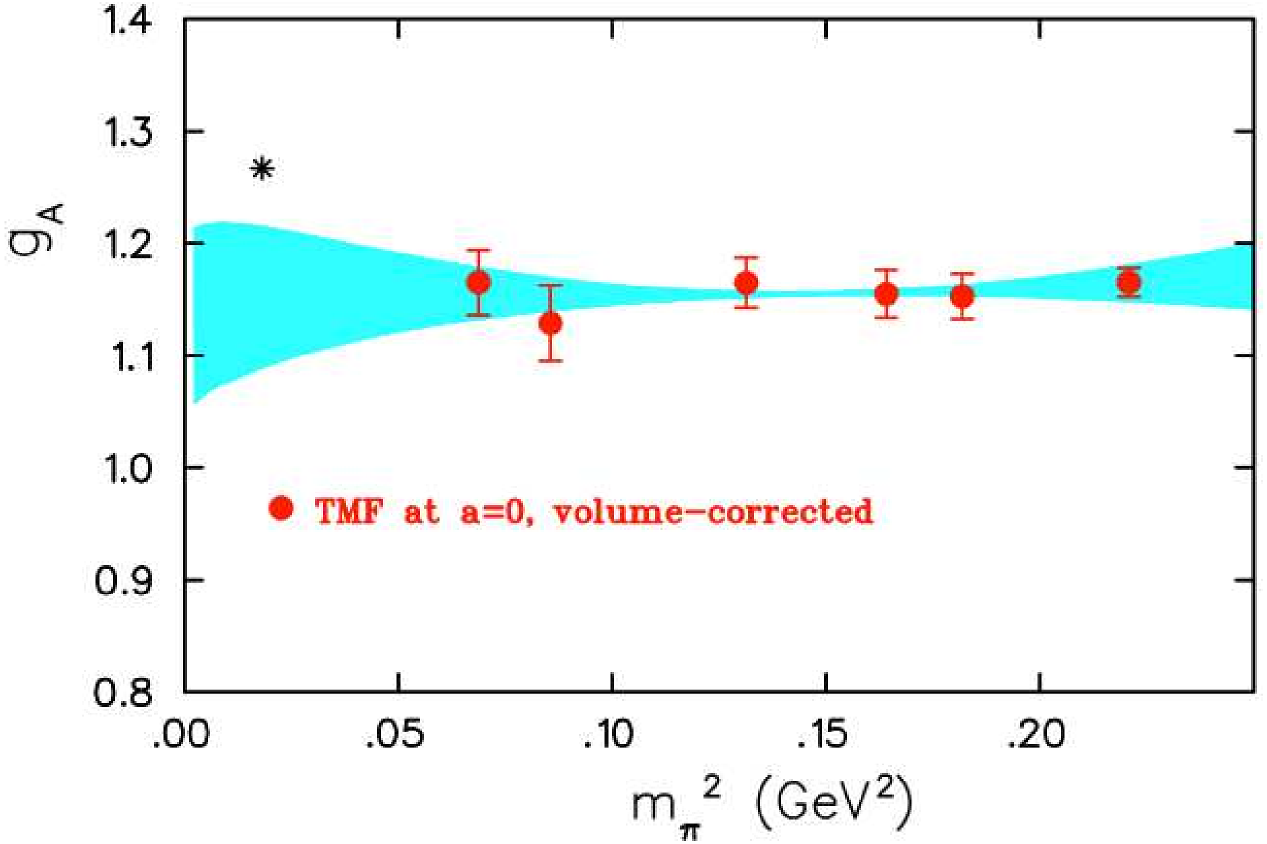


But g_A=1 is somehow trivial

ETMC: Taking continuum limit and V-corrected results

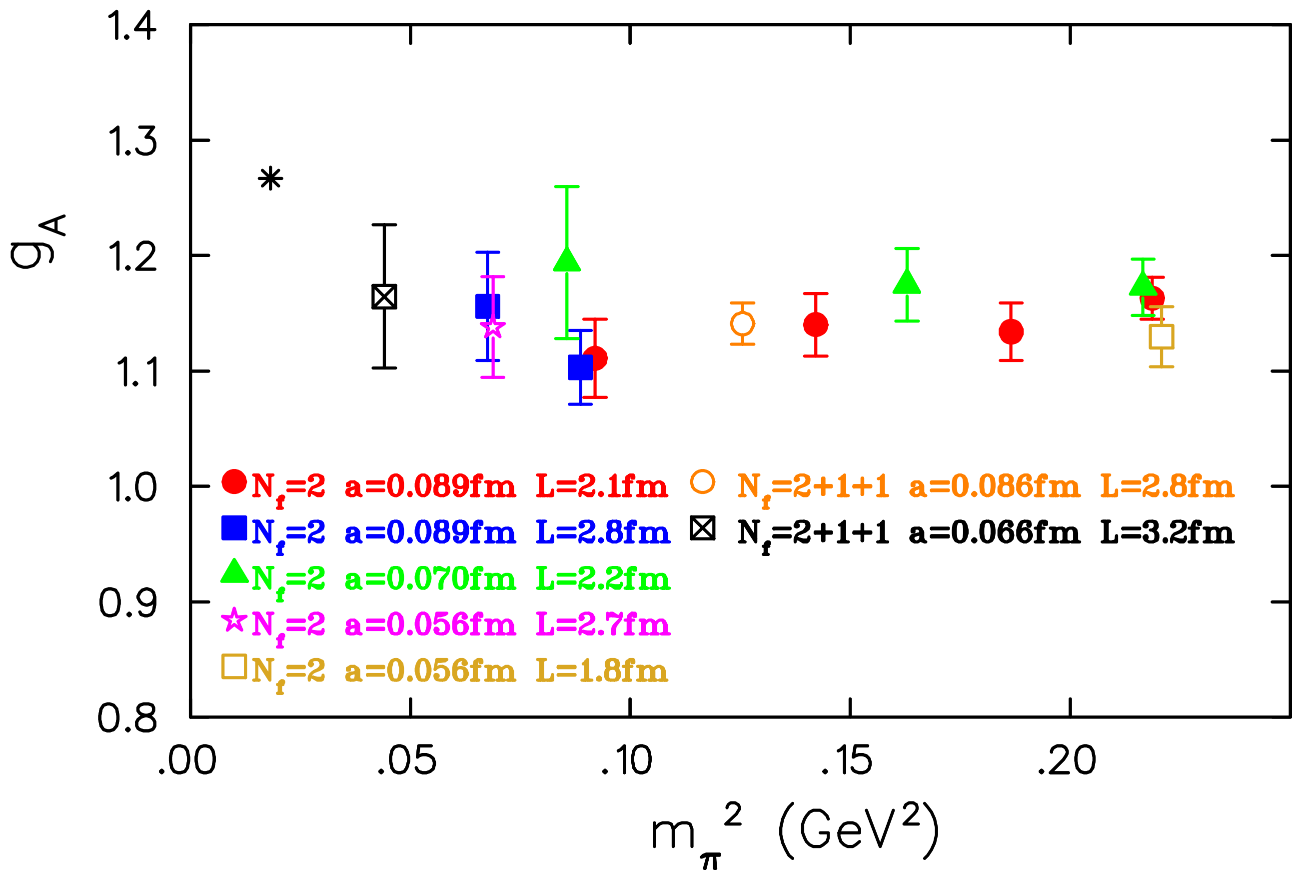
Z_A determined non perturbatively (RI-MOM)

$$Z_A = 0.757(3), 0.776(3), 0.789(3)$$



g_A=1.12(8)

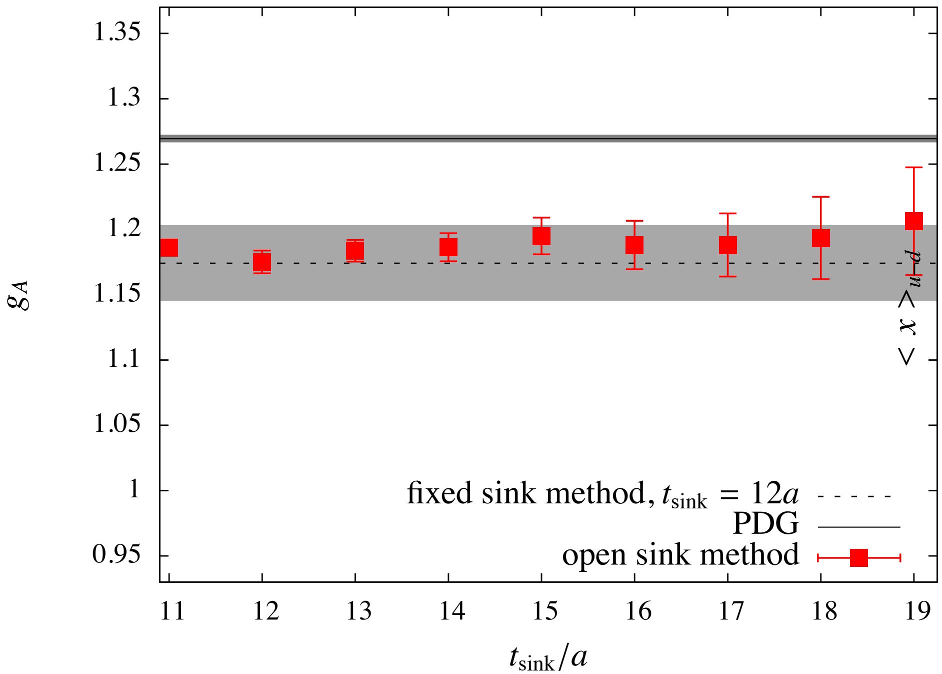
Last ETMC result N_f=2 and 2+1+1 $m_\pi \geq 210$ a=0.066 fm

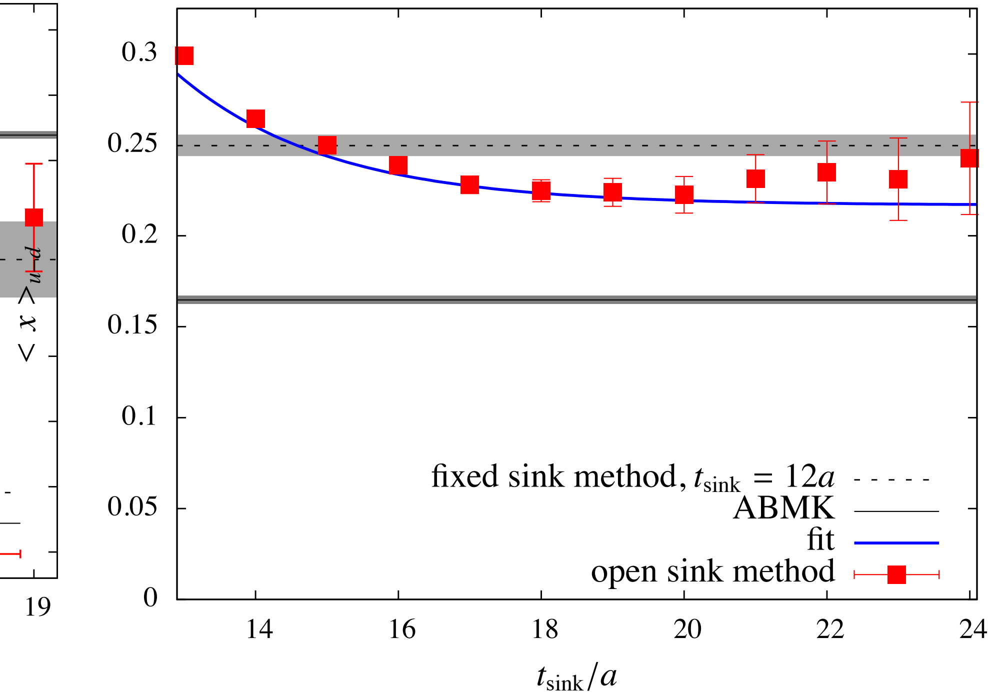


No « cut-off » or V-effects
No contamination

Some heroic work work (25000 confs!) was dedicated to find a possible explanation of the <x> and g_A discrepancies
Done in ETMC Nf=2+1+1 and m_π=380 MeV

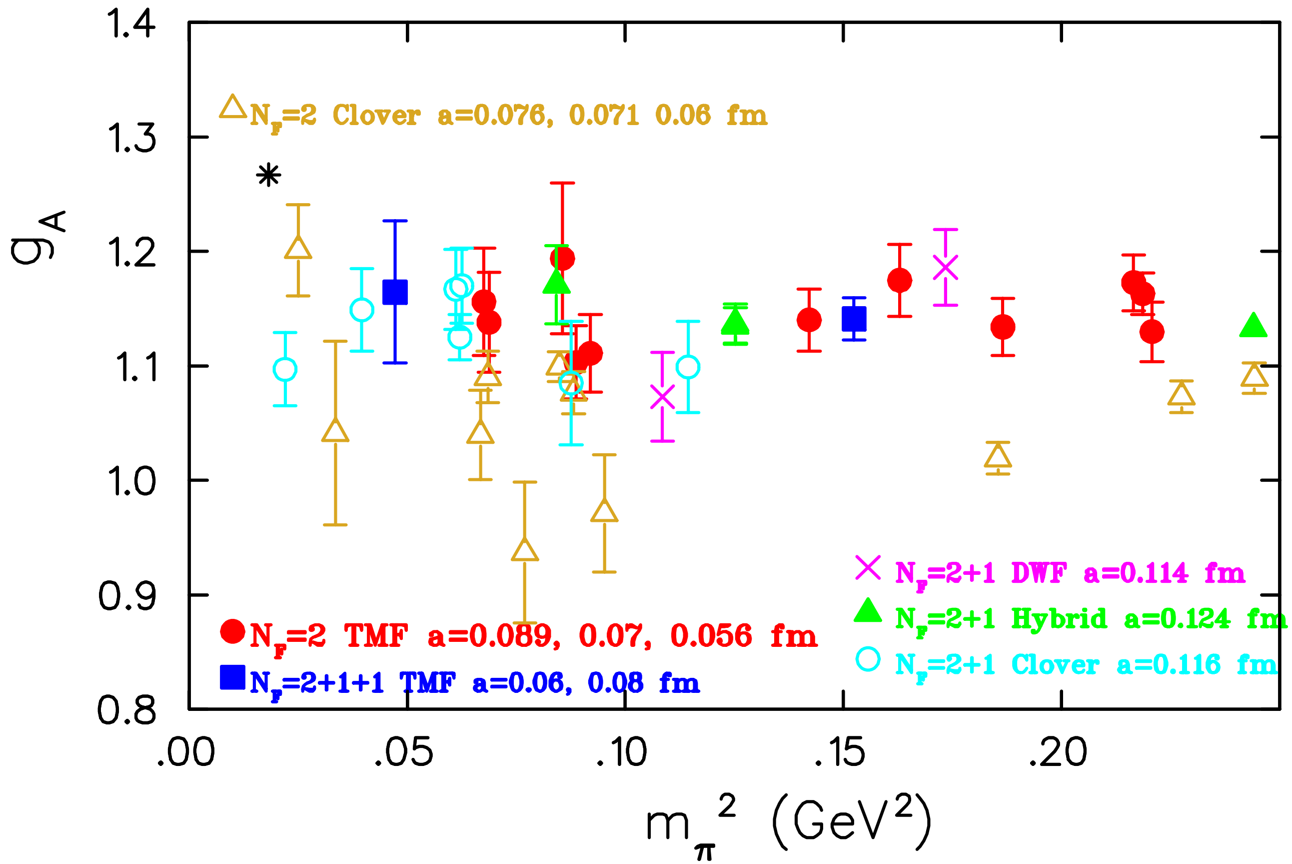
- No effect of excited state contamination in g_A
- 10% improvement on <x>





S. Dinter et al. , Phys.Lett. B704 (2011) 89-93

Summary of the last LQCD results (from Alexandrou et al)



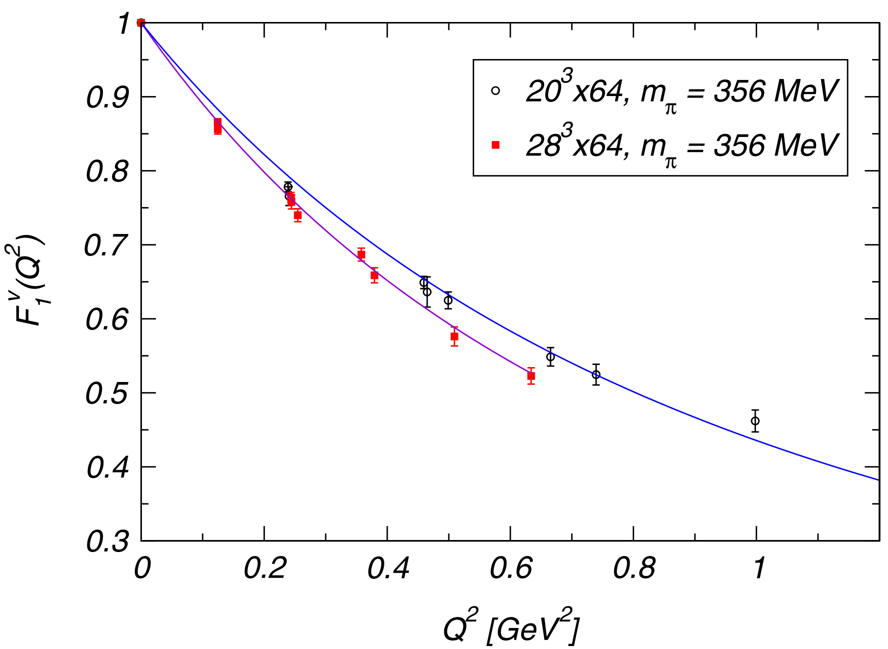
« Something is rotten in the state of Denmark »…. But what ?

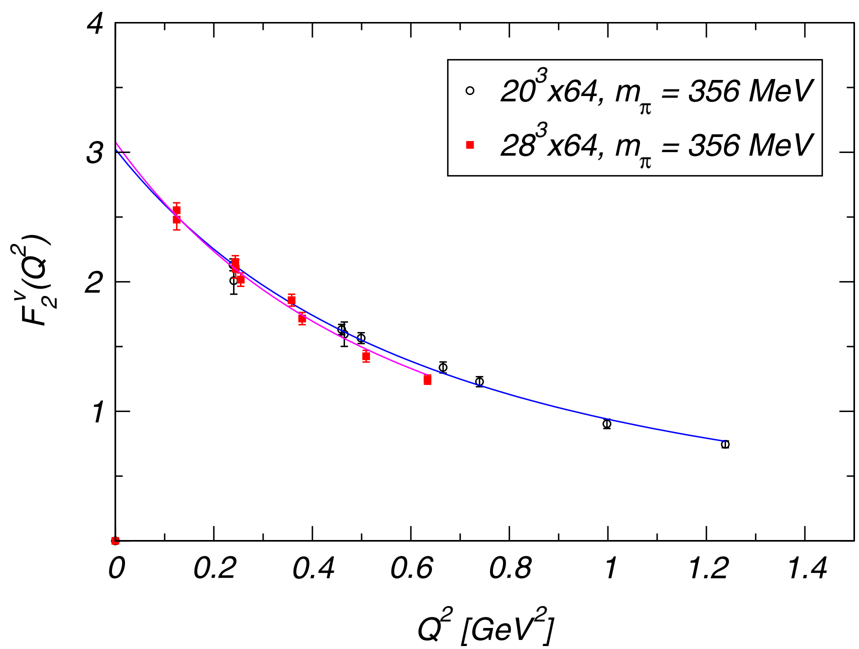
Electromagnetic Form Factors (LHPC)

$$G_E(Q^2) = F_1^v(Q^2) - \frac{Q^2}{(2m_N)^2} F_2^v(Q^2),$$

$$G_M(Q^2) = F_1^v(Q^2) + F_2^v(Q^2).$$

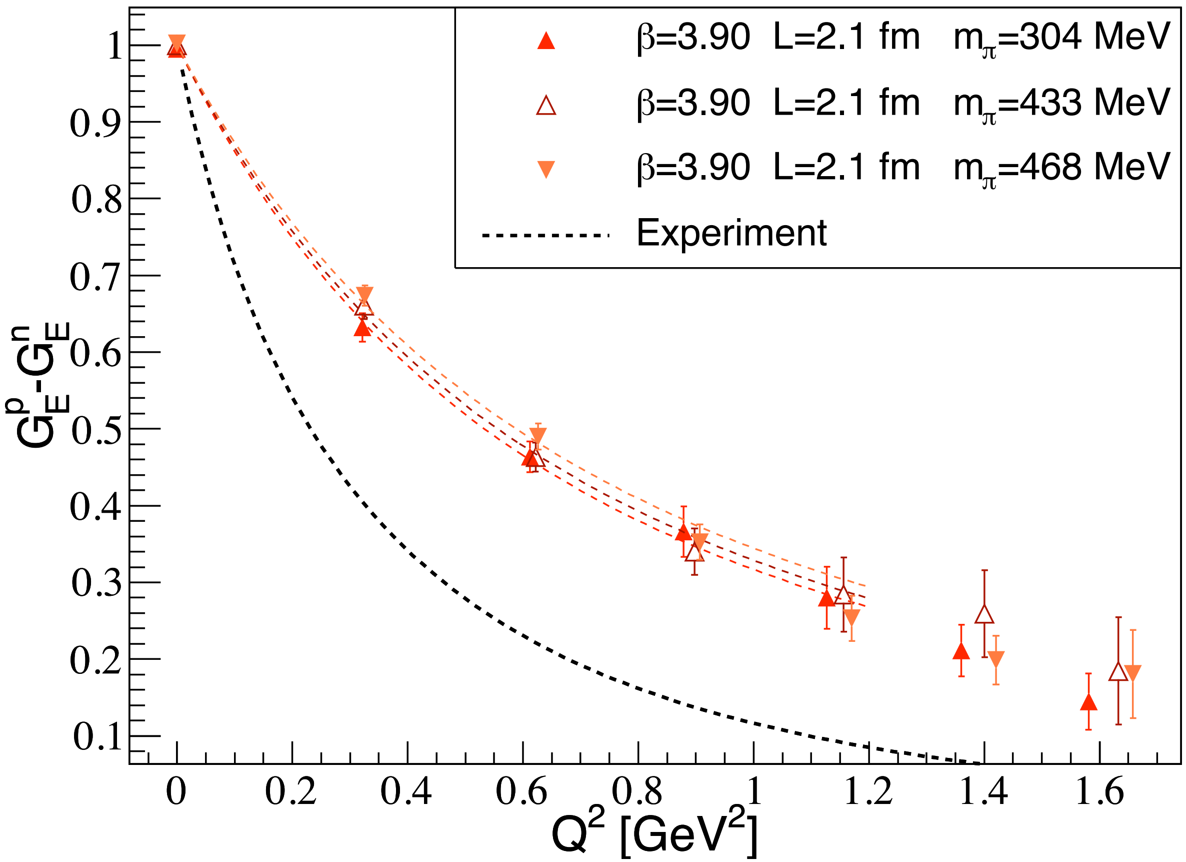
and corresponding radii $F_i(Q^2) = F_i(0)(1 - \frac{1}{6}Q^2 \cdot \langle r_i^2 \rangle + \mathcal{O}(Q^4)).$



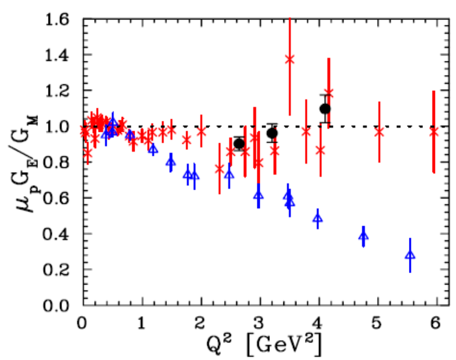


The computed values are compatible with a dipole form … but with a too compact N !

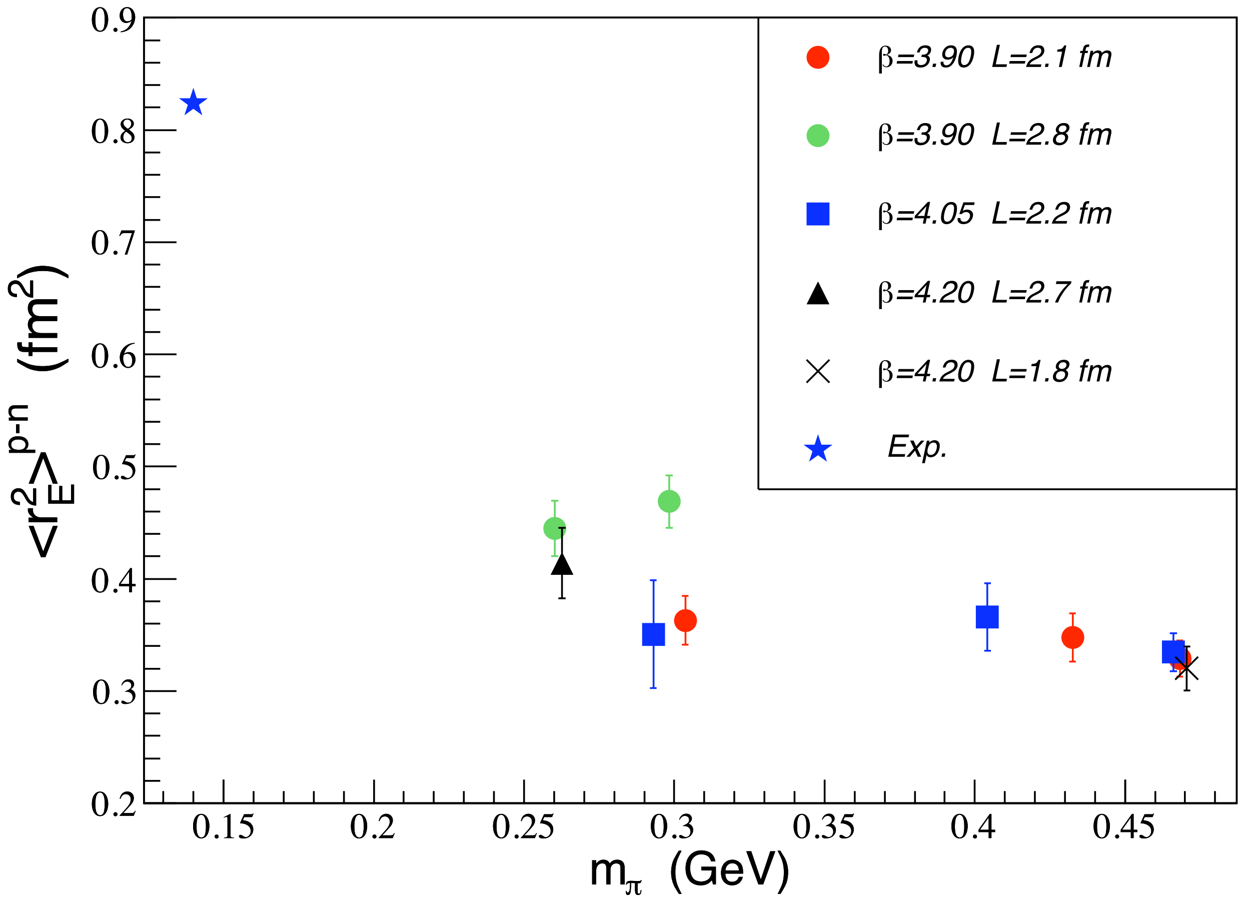
r^2_1(exp)=0.64 fm^2 r^2_1(calc)=0.273(15) fm^2



A typical behaviour… up to $Q^2 \approx 1.5$ GeV^2
Beyond, the form factors becomes noisy, as well as the E(Q) relation
Unable to state about the $G_E/G_M(Q^2)$ measurment at JLab



Everything looks fine … but with a much smaller Nucleon !

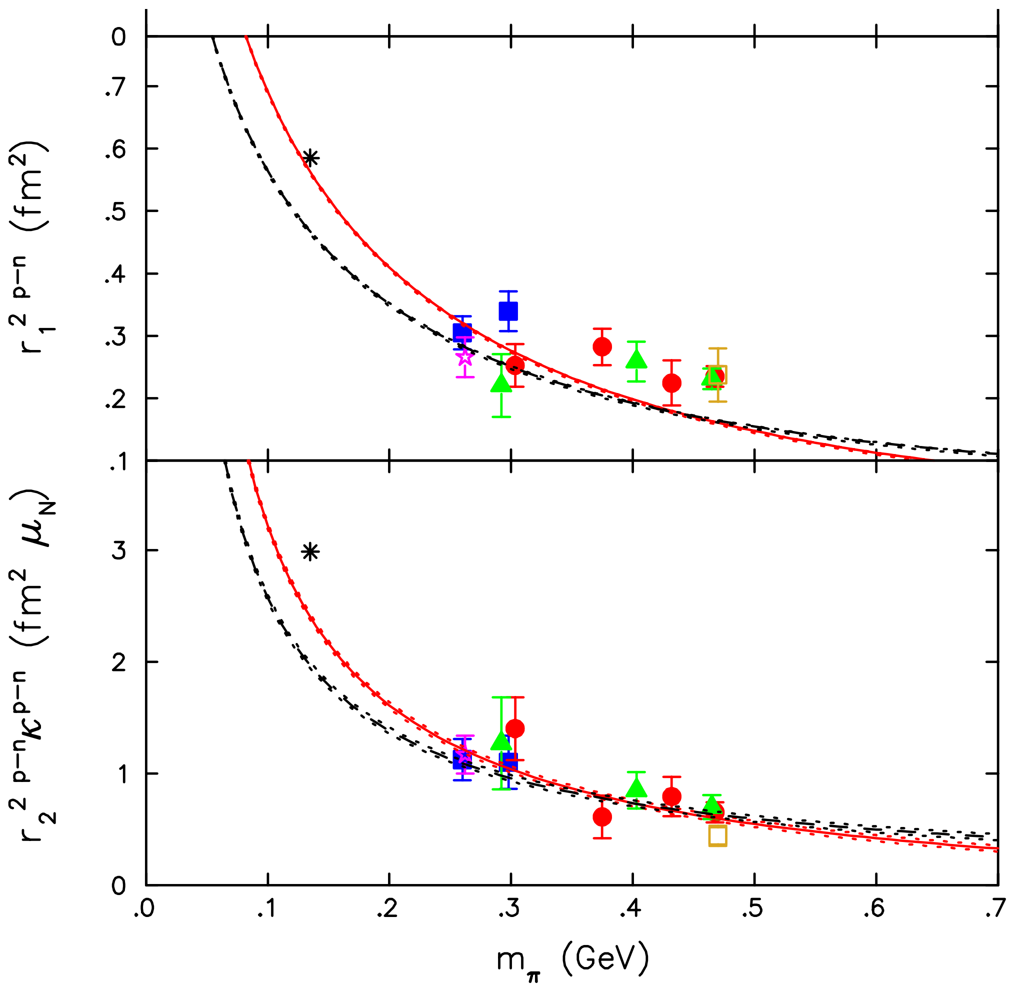


Using « naive » linear extrapolation (so what ?) one gets $r^2_E \simeq 0.40$
wrong by a factor 2 !

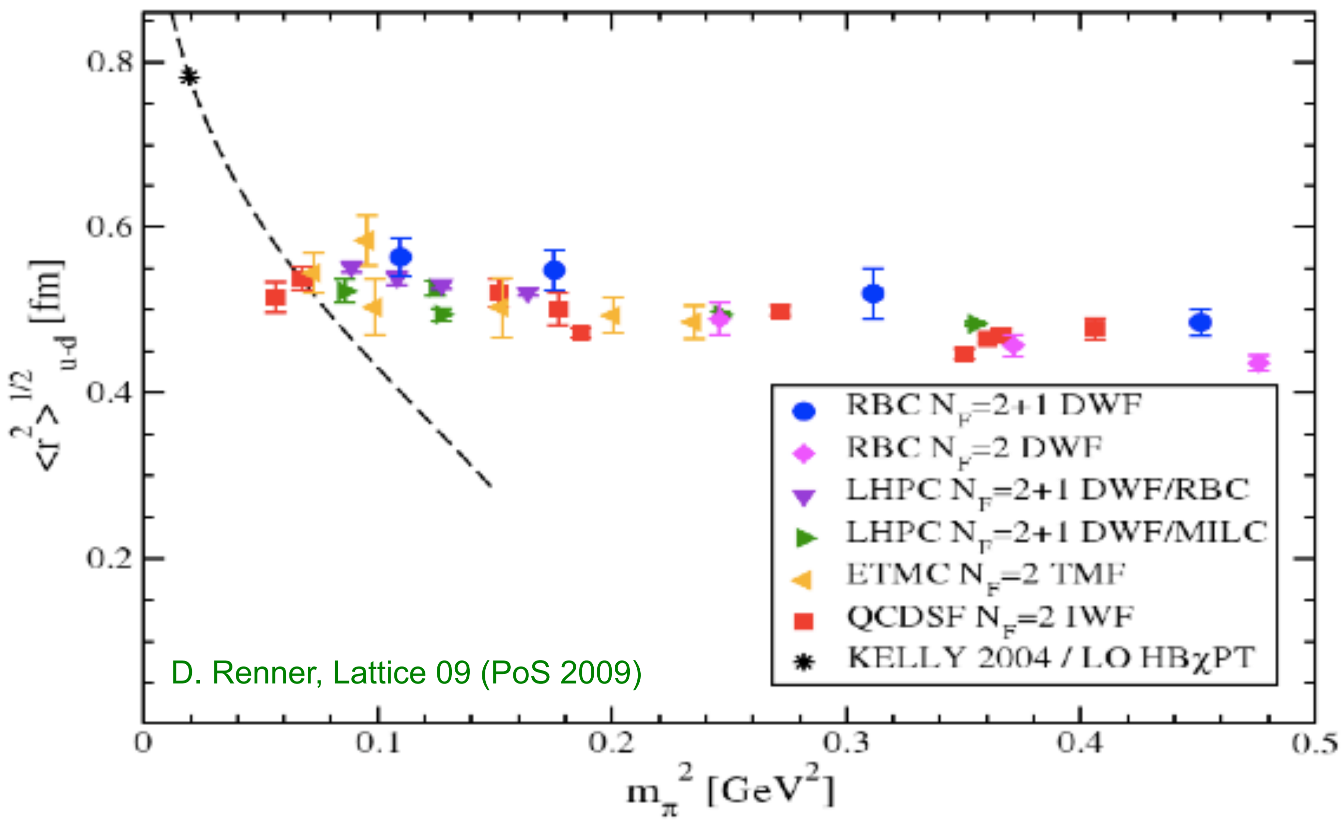
If the extrapolation is « sufficiently not naive » , everything « could » work

$$r_2^2 = \frac{1}{\kappa_v(m_\pi)}\left\{\frac{g_A^2 m_N}{8 f_\pi^2 \pi m_\pi} + \frac{c_A^2 m_N}{9 f_\pi^2 \pi^2 \sqrt{\Delta^2 - m_\pi^2}} \log R(m_\pi) + 24 m_N B_{c2}\right\}. \quad ($$

$$r_1^2 = -\frac{1}{(4\pi f_\pi)^2}\left[1 + 7g_A^2 + (10g_A^2 + 2)\log\left(\frac{m_\pi}{\lambda}\right)\right] - \frac{12B_{10}}{(4\pi f_\pi)^2} + \frac{c_A^2}{54\pi^2 f_\pi^2}\left[26 + 30\log\left(\frac{m_\pi}{\lambda}\right) + 30\frac{\Delta}{\sqrt{\Delta^2 - m_\pi^2}}\log R(m_\pi)\right]$$



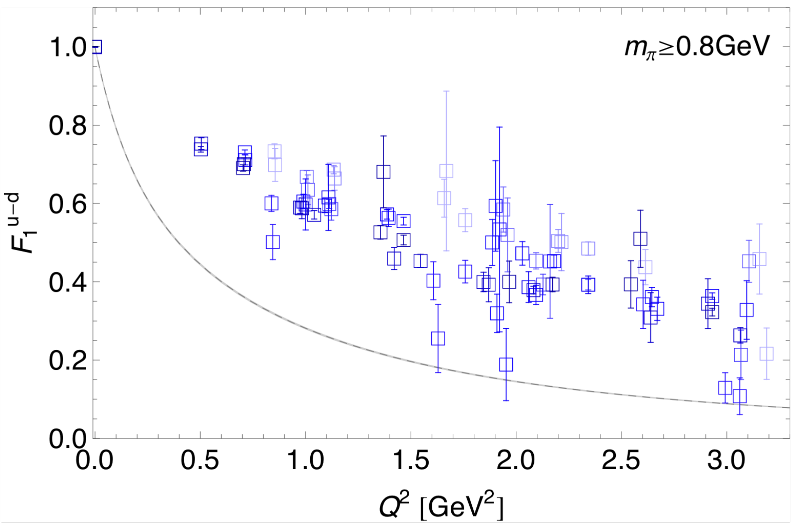
« World-wide » evaluation

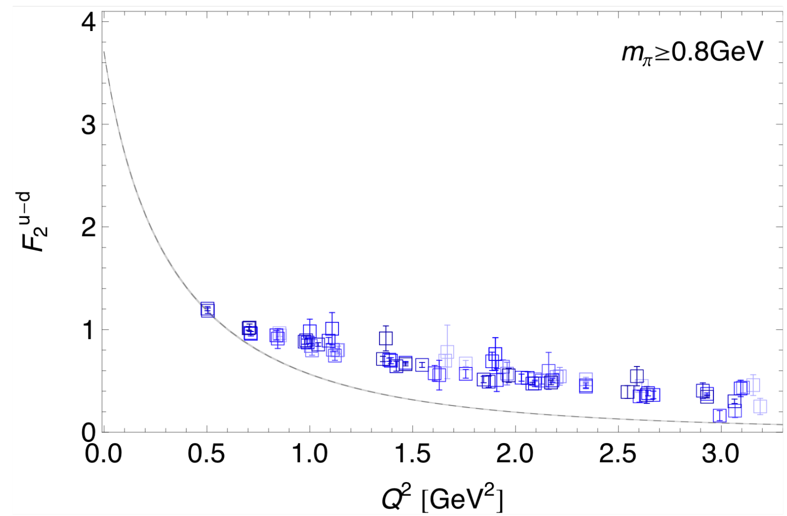


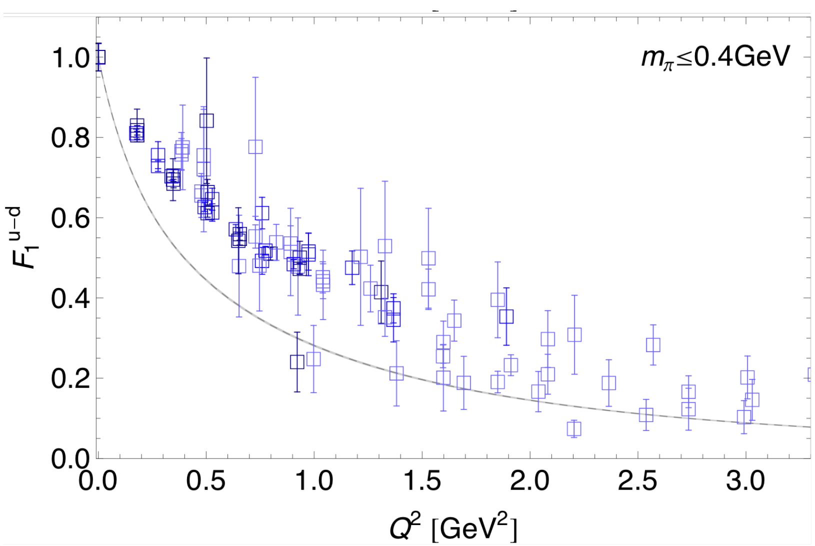
Maybe everybody is wrong… (one can suspect a strong L-dependence at the physical point)

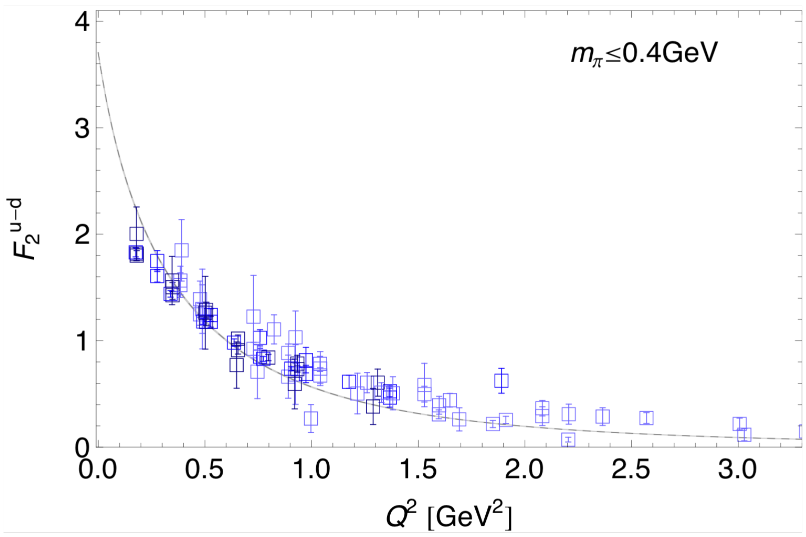
Maybe something is missing …

RESULTS QCDSF-UKQCD

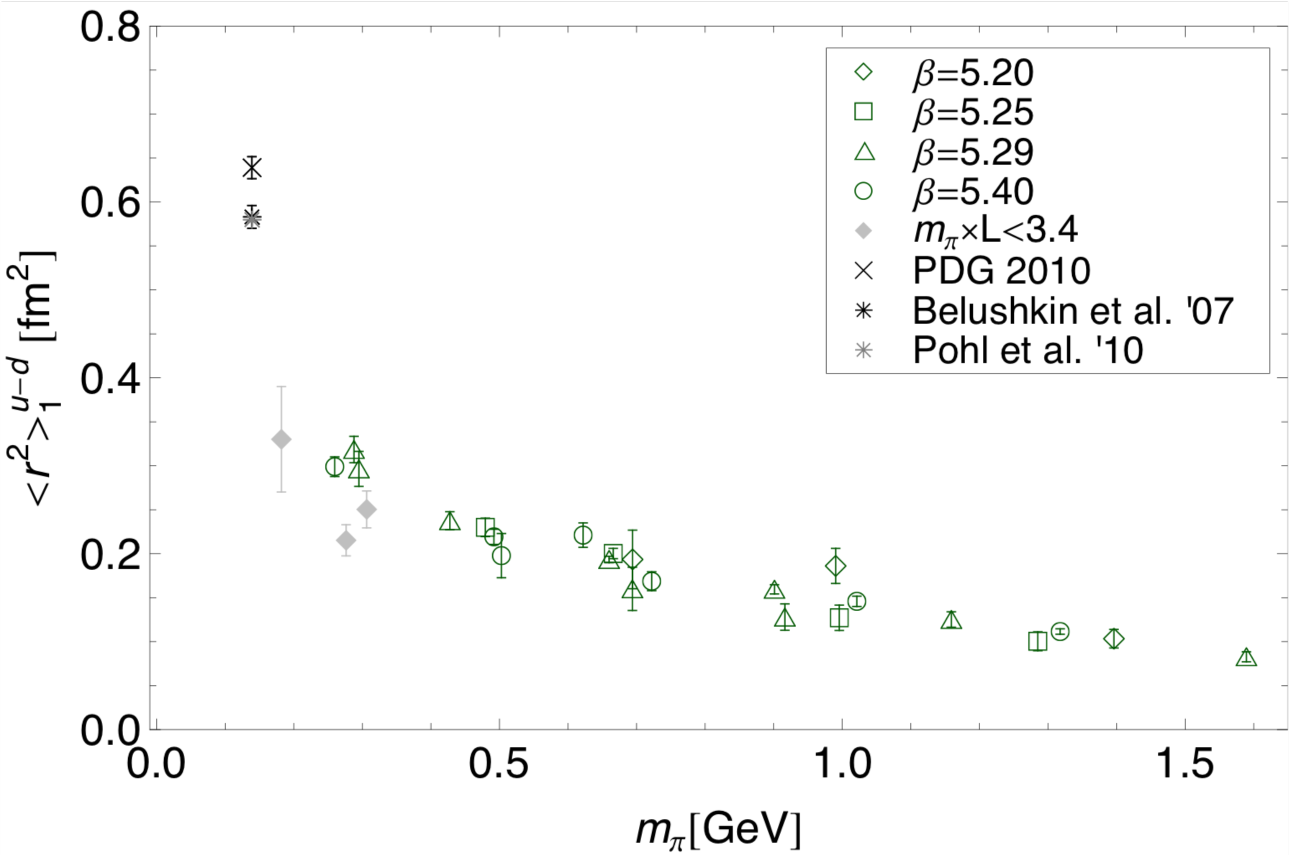








Results QCDSF-UKQCD



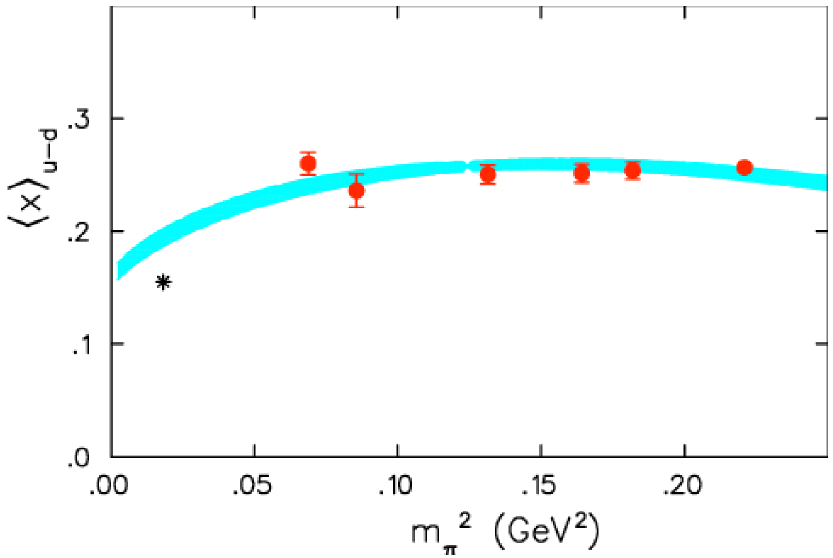
A 180 MeV pion does not solve the problem… but $m_\pi L=2.8$

ETMC also computed a bulk of more elaborate observables (GFF: pdf, GPD)

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} x^{n-1} q(x) dx,$$

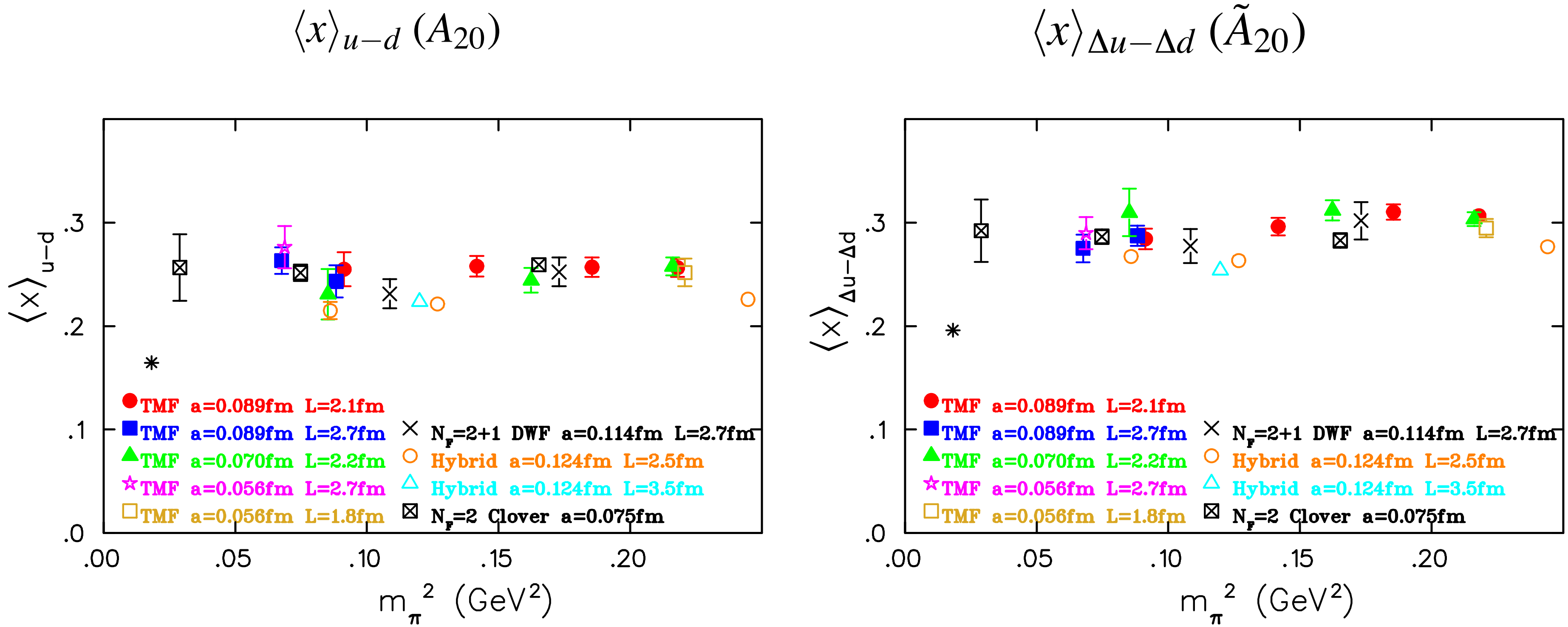
$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} x^{n-1} \Delta q(x) dx.$$

$< x >= A_2(0)$



More difficult to compute, less well known experimentally
…but the results presented above were illustrative of the general situation

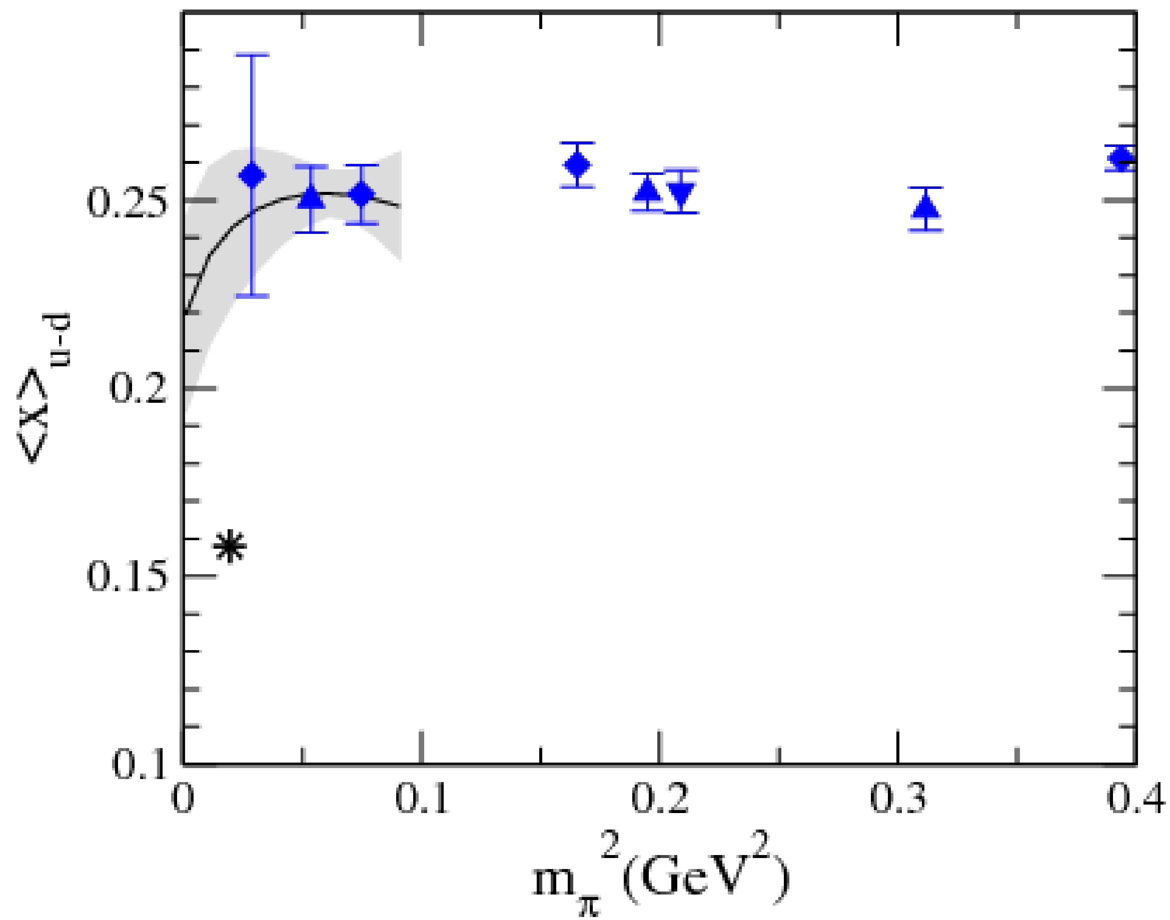
Everythig is “almost fine” … but something is missing



$\langle x \rangle_{u-d}$ (A_{20})
$\langle x \rangle_{\Delta u-\Delta d}$ ($\tilde{A}_{20}$)
$\langle x \rangle_{u-d}$
$\langle x \rangle_{\Delta u-\Delta d}$
.0
.1
.2
.3
.00
.05
.10
.15
.20
m_π^2 (GeV^2)
TMF a=0.089fm L=2.1fm
TMF a=0.089fm L=2.7fm
TMF a=0.070fm L=2.2fm
TMF a=0.056fm L=2.7fm
TMF a=0.056fm L=1.8fm
N_F=2+1 DWF a=0.114fm L=2.7fm
Hybrid a=0.124fm L=2.5fm
Hybrid a=0.124fm L=3.5fm
N_F=2 Clover a=0.075fm

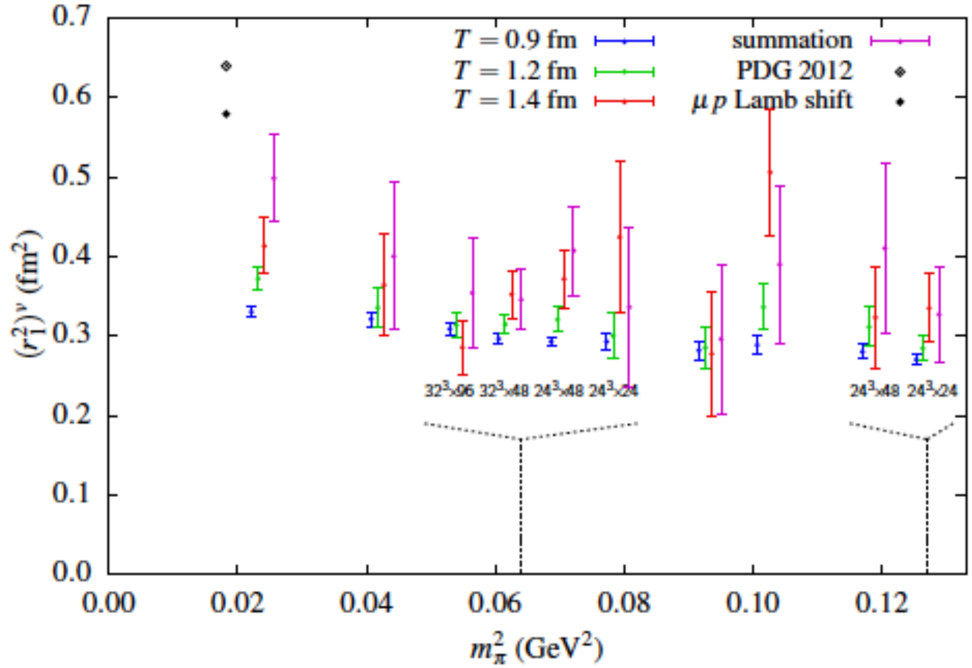
Momentum fraction from QCDSF

$<x>=A_2(0)$ smallest pion mass : 170MeV !



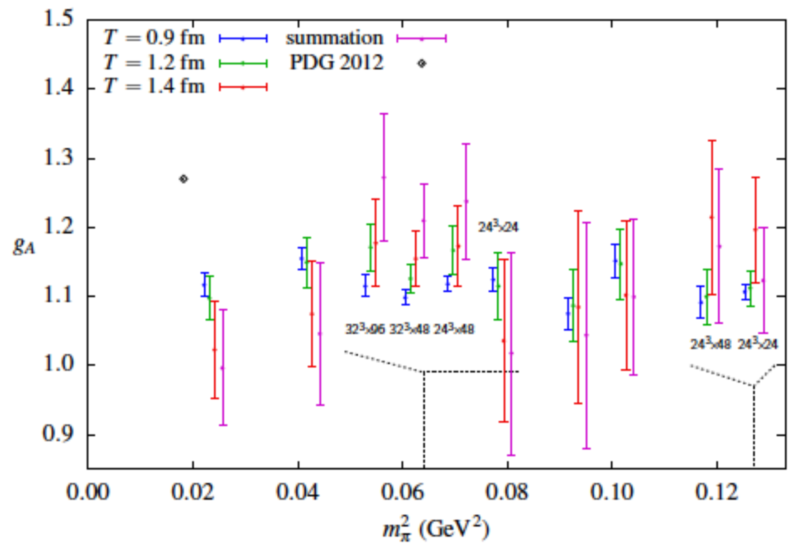
A recent work at the pion mass (MIT + BMW)

J. Green et al PoS LATTICE2012 (2012) 170



When r^2 is improved g_A ….goes down !!!

Uncontrolled erros



Lattice Non-QCD (it exists!!!)

There are (at least) three ways of doing Nuclear Physics on the Lattice

1. "Ab initio" from LQCD, in the sense presented above (NN, ^{2}H,^{3}He,^{4}He..)

2. Using lattice techniques to solve the many-nucleon problem
"Nuclear Lattice Simulations" group: Bochum, Bonn, Juelich, North-Caroline SU
It is a non relativistic QF approach, strictly equivalent (by construction) to Fadeev-Yakubovsky equations.
It has been very successful and is able to go well beyond

3. Putting the conventional NN interaction lagrangians on the lattice
… and see how much the full QFT contents differ from the underlying V

I'll talk about 3 with the simplest meson-fermion coupling: Yukawa model

The Yukawa model in QFT

Understand the full QFT content of the simplest fermion-fermion interaction model

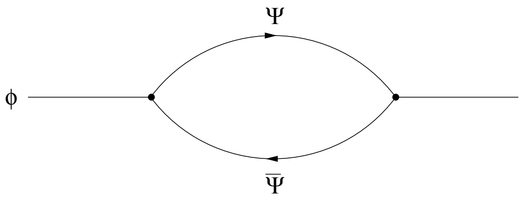
$$\mathcal{L}_{int}(x) = g_0 \bar{\Psi}(x)\, \Gamma\, \Phi(x)\, \Psi(x) \qquad \Gamma = 1, \gamma_5$$

…. 70 years after his formulation by Yukawa (1935) !

How far is V from L ? $V(\vec{r}) = -\frac{g^2}{4\pi} \frac{e^{-\mu r}}{r}$

- Compute the low energy observables (B,a,..)
- Compare them to the potential results, in different dynamical equations

We worked in the "quenched approximation"
i.e. neglect NN pairs in meson propagator



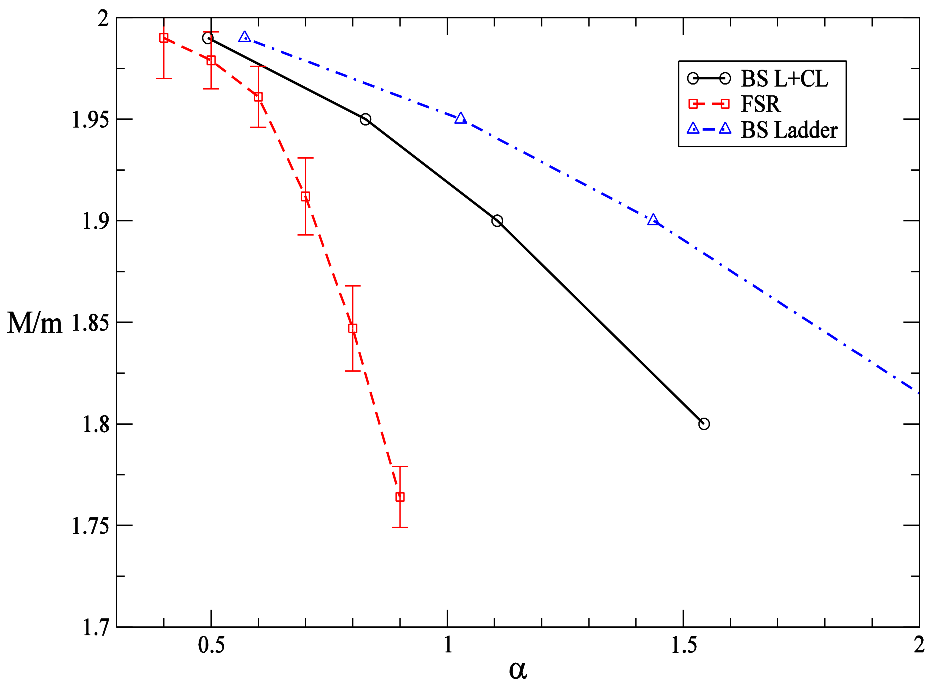
A "reasonable" approximation given the N mass and
in any case implicit in ALL nuclear models

F. De Soto et al, Eur. Phys. J. A47 (2011) 57; arXiv 1104.1907

Motivation: a pionneer result for scalar ($\varphi^2\chi$) theory

Cross ladder effects are big, very expensive to compute and only the first correction to ladder
J.C. and V.A. Karmanov, Eur. J. Phys A27 (2006)11

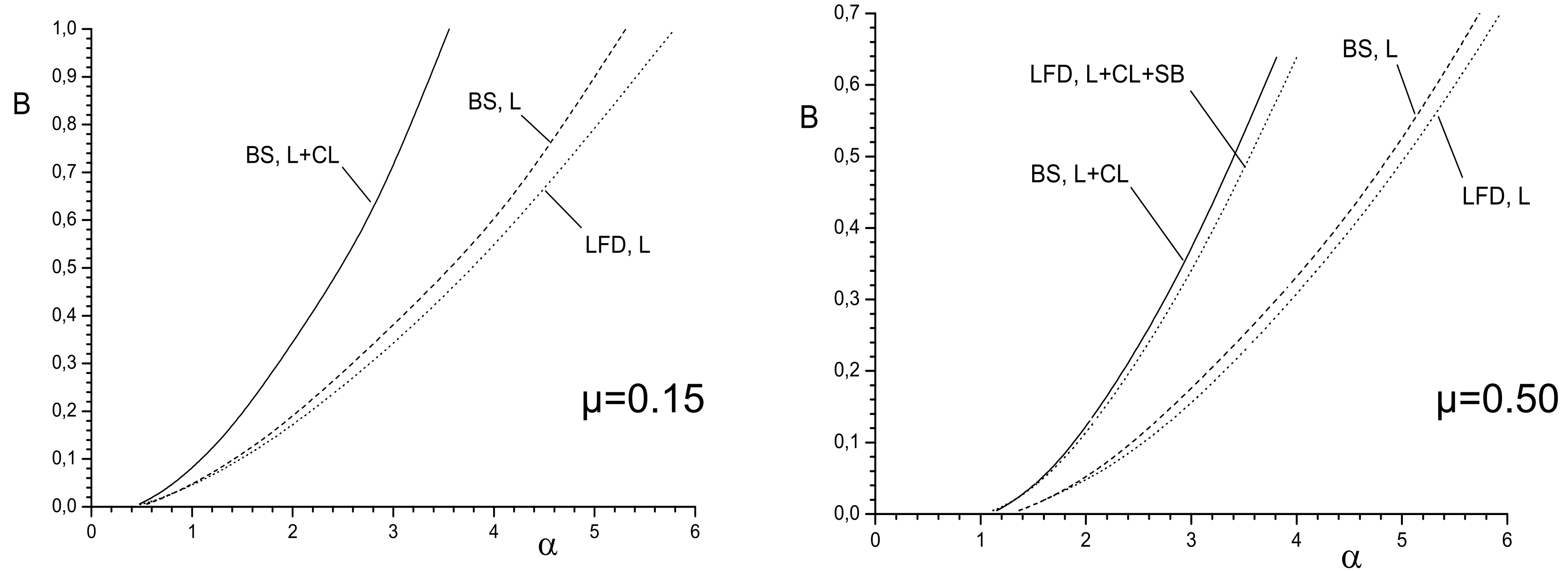
T.Nieuwenhius and J. Tjon sumed all exchange diagrams for scalar $\varphi^2\chi$ theory (Feynman-Schwinger representation) and found spectacular changes PRL77 (1996) 814



However the scalar $\varphi^2\chi$ theory is not bounded by below (Baym 60's).
We thus looked at the simplest - well defined - theory with ladder counterpart ("potential")

This is the fermion-fermion Yukawa model (apart from "triviality")

Cross ladder effects in $\varphi^2\chi$ theory with BS and LFD equation



$\alpha = g^2/4\pi$

One looses a factor 2 in B, even at small B, idependently of μ

J.C. and V.A. Karmanov, Eur. J. Phys A27 (2006)11

All these results motivated us to consider a full QFT solution of the problem…

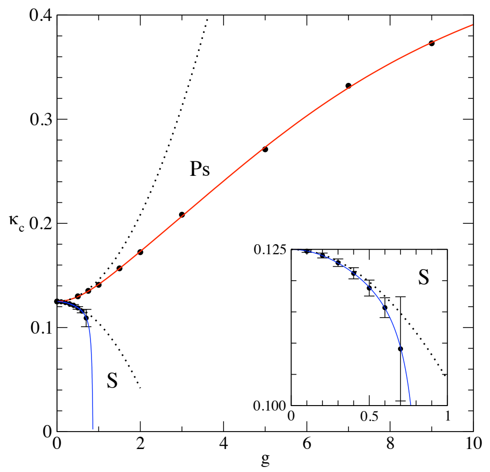
Results

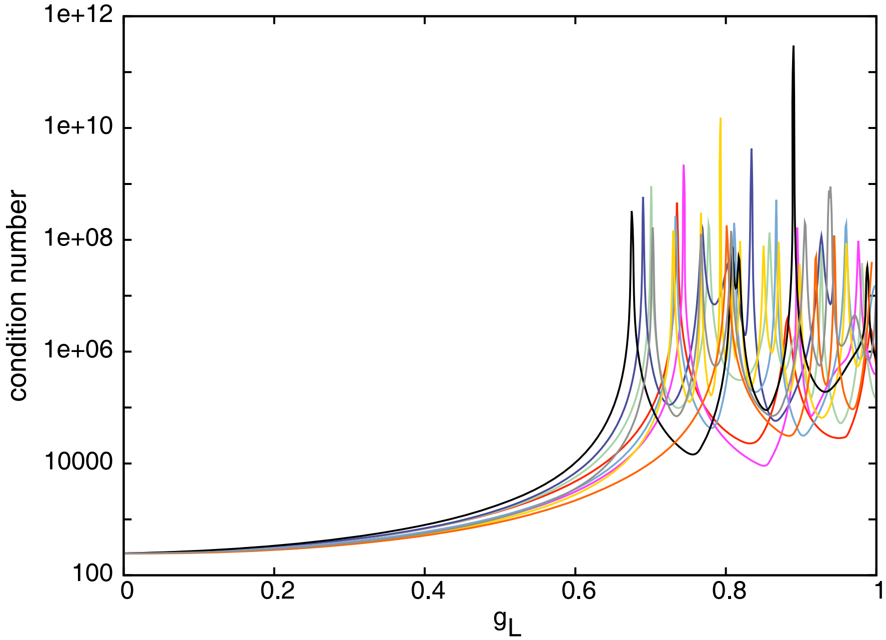
Absence of any bound state !!!

Fermion propagator is obtained as solution of a linear system with Dirac operator

$$D_{zx}(\phi)S_x(\phi) = \delta_{z0}. \quad (1)$$

When g increases (g≈0.8), D has an increasing number of very small eigenvalues which makes system (1) ill-conditioned

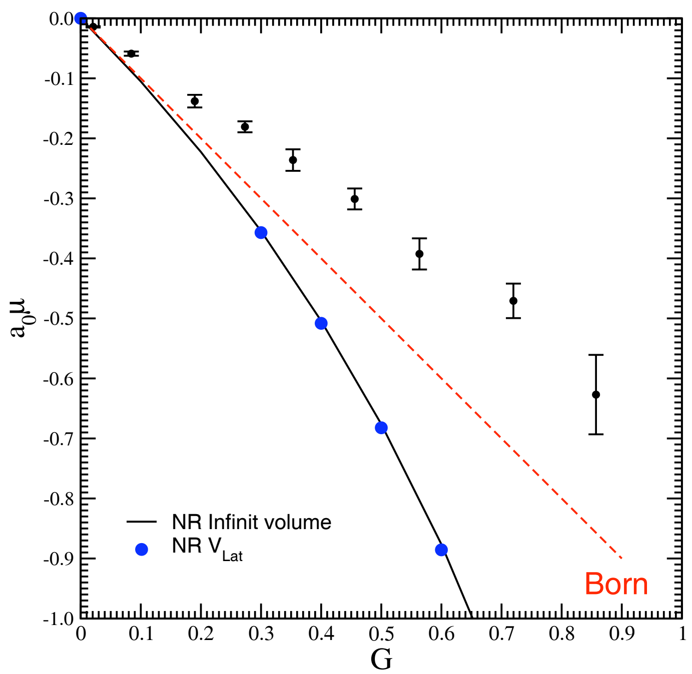




As a QFT, the Yukawa model “does not exist” without $N\bar{N}$ loops…
In “nuclear models” who cares about that ?

Results

In the "small g" region we computed the scattering length a_0 and compare to non relativistic limit

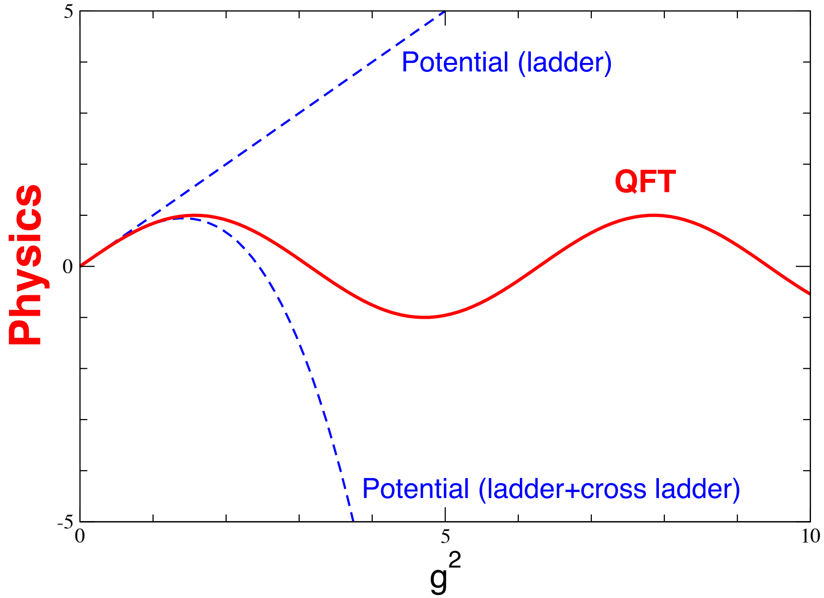


Results

The Nuclear Yukawa model - in its full glory - remains to be solved !

Although the NN potentials are « inspired » by QFT, the link is far from obvious

For strong interactions (large g^2), using « potentials » is like using a Taylor expansion in trigonometric functions



With 1st and 2ond order perturbation on can go everywhere… except to the right place !

Conclusion

After going "through impossible walls", and faced to "irreducible difficulties", LQCD started a "Rennaissance"

- Inclusion of quarks (u,d,c,s) dynamically N_f=2, N_f=2+1, N_f=2+1+1
- Physical masse $m_\pi \approx 140$ MeV reached par 3 collaborations (BMW,QCDSF)
- God control of discretisation ($a \approx 0.05$ fm) and finite volume effects (???)

Several collaborations (discretisations!) produce interesting results:

Spectroscopy

GS are 1-3% and entering a precision era

Significant differences remain in the "N sigma terms" $\sigma_{\pi N}$ and y_N

Excited (resonant) states display sizeable and systematic disagreements, maybe due to thresholds effects

The way to take them into account is clear and in progress

First unquenched glueball spectrum appeared: the lowest 0^{++} mass OK, the other ones much less !

Multibaryon systems (H) : no clear conclusion but seems unbound…in the real world !

First bound nuclei from a 2 parameter QCD !!!

Scattering

Used to obtain NN potential but remains qualitatively

YN and YY phasehifts would be reliable and welcome !

N structure

Everything is "almost correct"…but "the devil is hidden in the small details"

The simplest "hot points" (g_{A}, r^2_p, G_E/G_M) are no yet in agreement with data

Only simulations at the physical point can point out an eventual disagreement