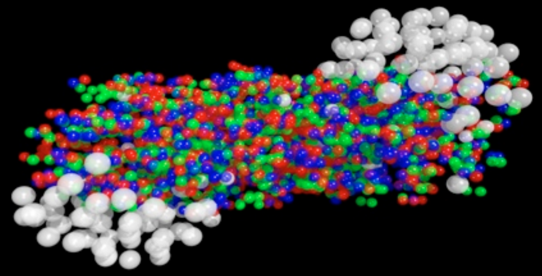
Universiteit Utrecht



Heavy-ions soft probes

Ecole Internationale Joliot-Curie

Raimond Snellings

La Villa Clythia, Frejus
(France) 29-09-2013

Content (lecture 1-3)

1) Heavy Ion Collisions

2) Thermal Model and Collective Flow

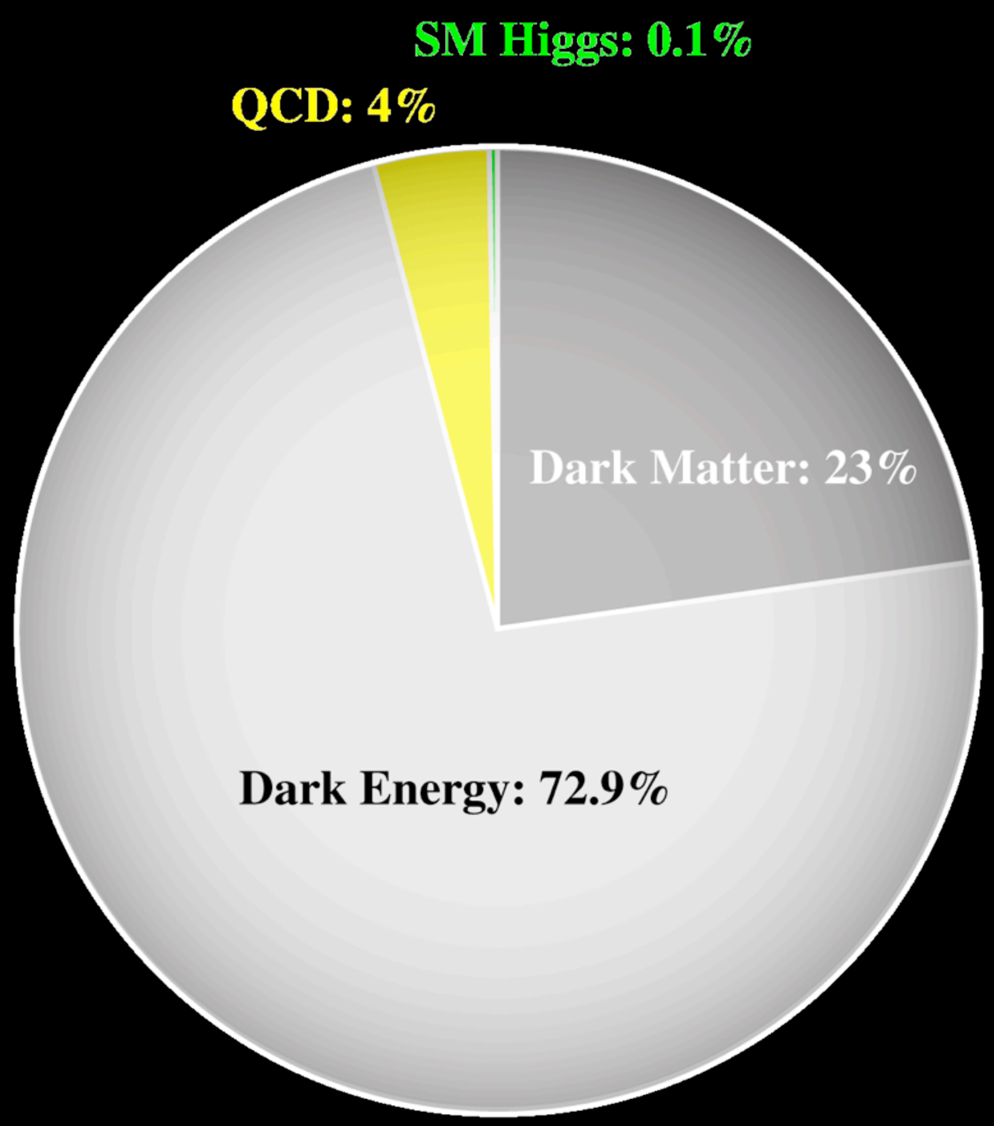
3) Current results

The Standard Model

QCD the theory of the strong force

What is the universe made of?

- Elementary particles make up 0.1% of the mass of the universe
  - SM Higgs mechanism
- Composite particles can account for ~4%
  - QCD chiral symmetry breaking
- Dark Matter 23%
- Dark Energy 72.9%
- The ~4% are still not understood very well, and the other 95% a complete mystery!



QCD; Quarks and Gluons

- In the world around us quarks and gluons do not exist as free particles
    - confined in hadrons by the strong interaction (QCD)
- At $T \rightarrow \infty$ asymptotic freedom tells us that quarks and gluons are the relevant degrees of freedom and this phase of QCD is called the Quark Gluon Plasma
- We think that this state of matter permeated the early universe until the first microseconds after the Big Bang
- After expanding and cooling down the universe goes through a phase transition in which the quarks and gluons become confined
- This phase transition is poorly understood from first principles but some theoretical understanding of the complex features can be obtained from lattice QCD

The QCD vacuum

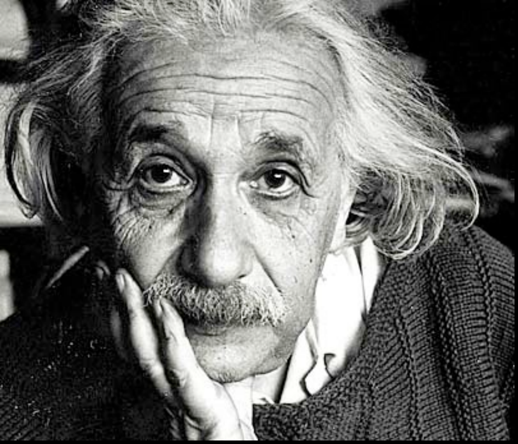
"In high-energy physics we have concentrated on experiments in which we distribute a higher and higher amount of energy into a region with smaller and smaller dimensions

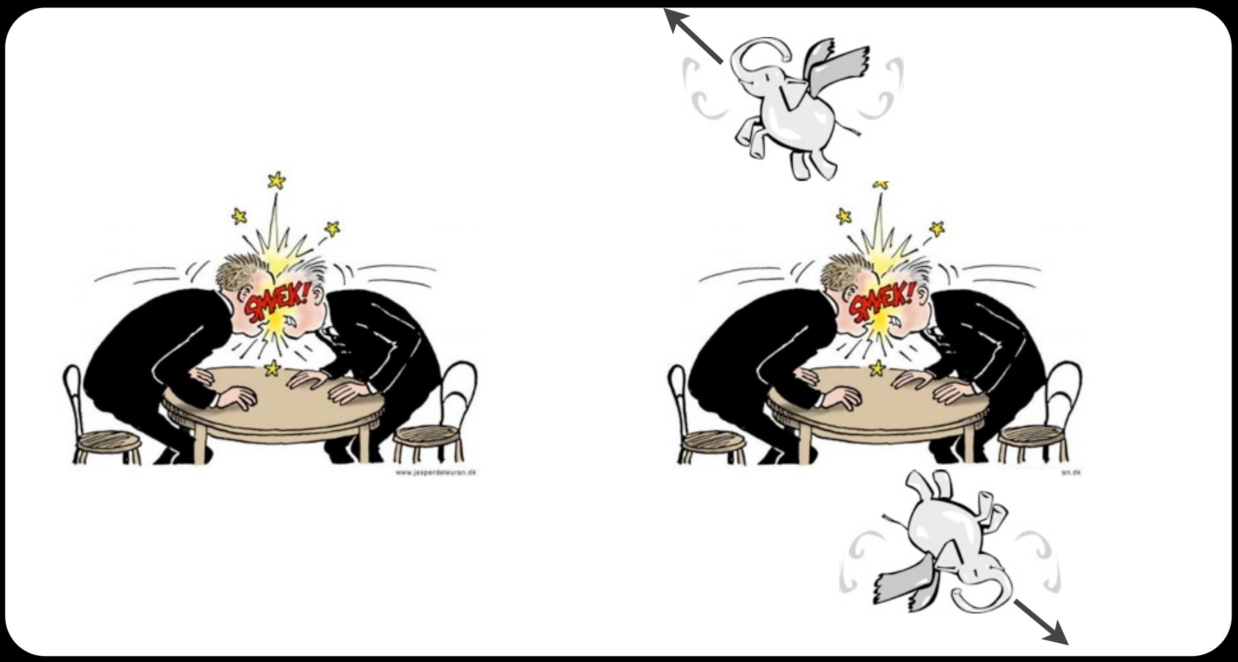
In order to study the question of 'vacuum', we must turn to a different direction; we should investigate some bulk phenomena by distribution high energy over a relatively large volume"

T.D. Lee

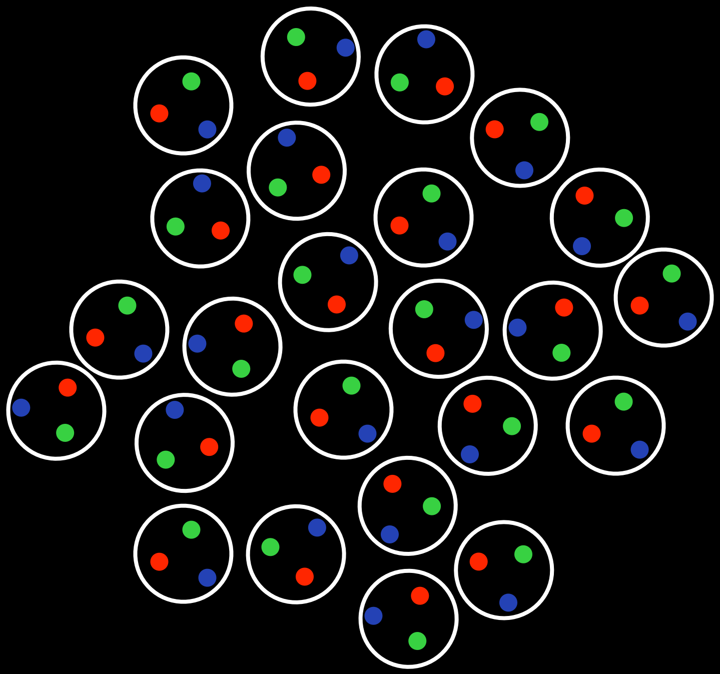
Rev. Mod. Phys. 47 (1975) 267.

How?



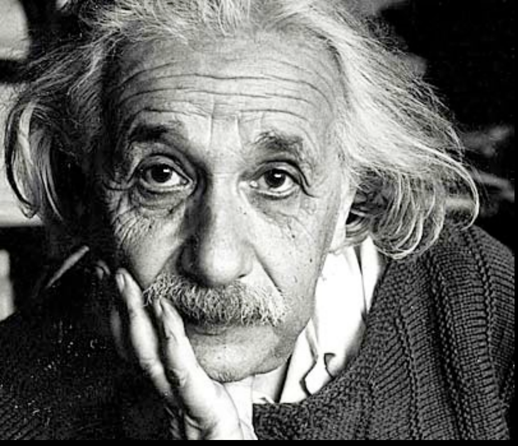


$$E = mc^2$$

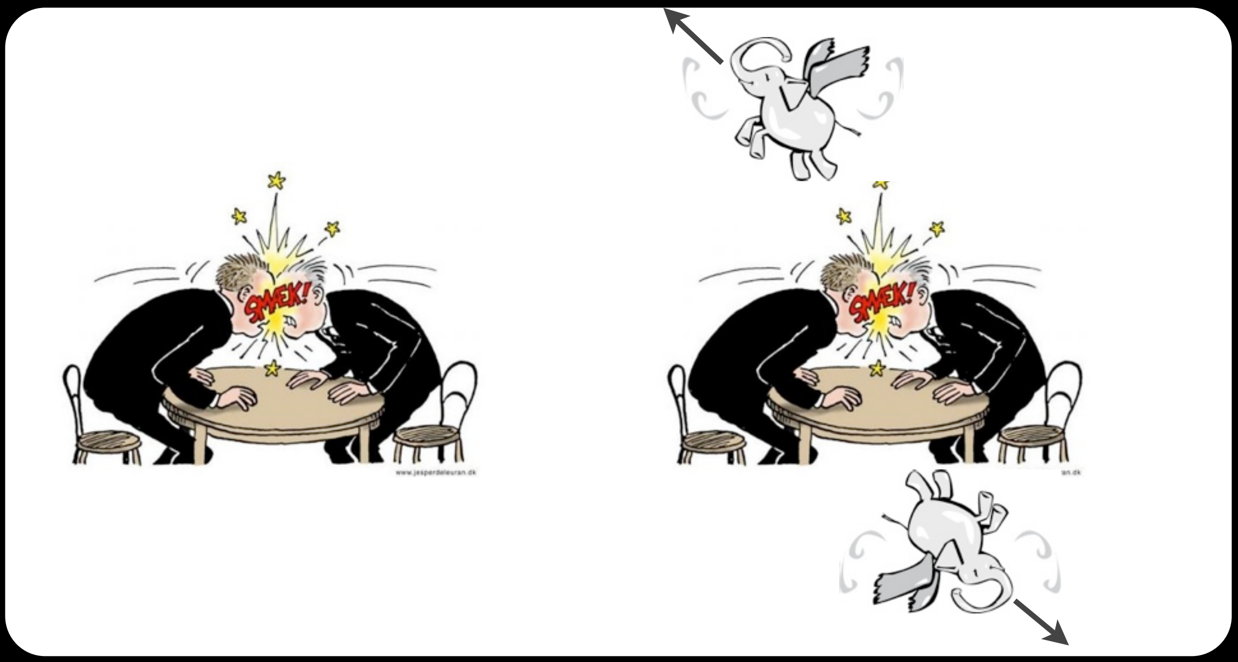


Nuclear Matter
(confined)

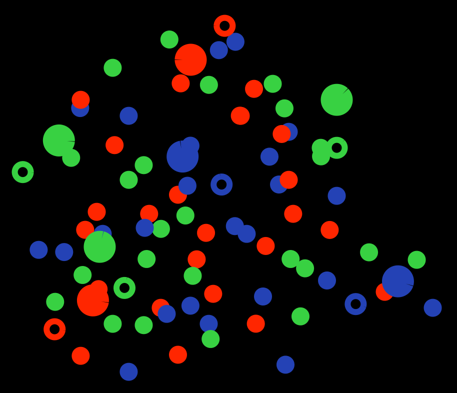
collisions at high energy allow us to create new heavy particles and in collisions of heavy-ions a "little bang"



How?

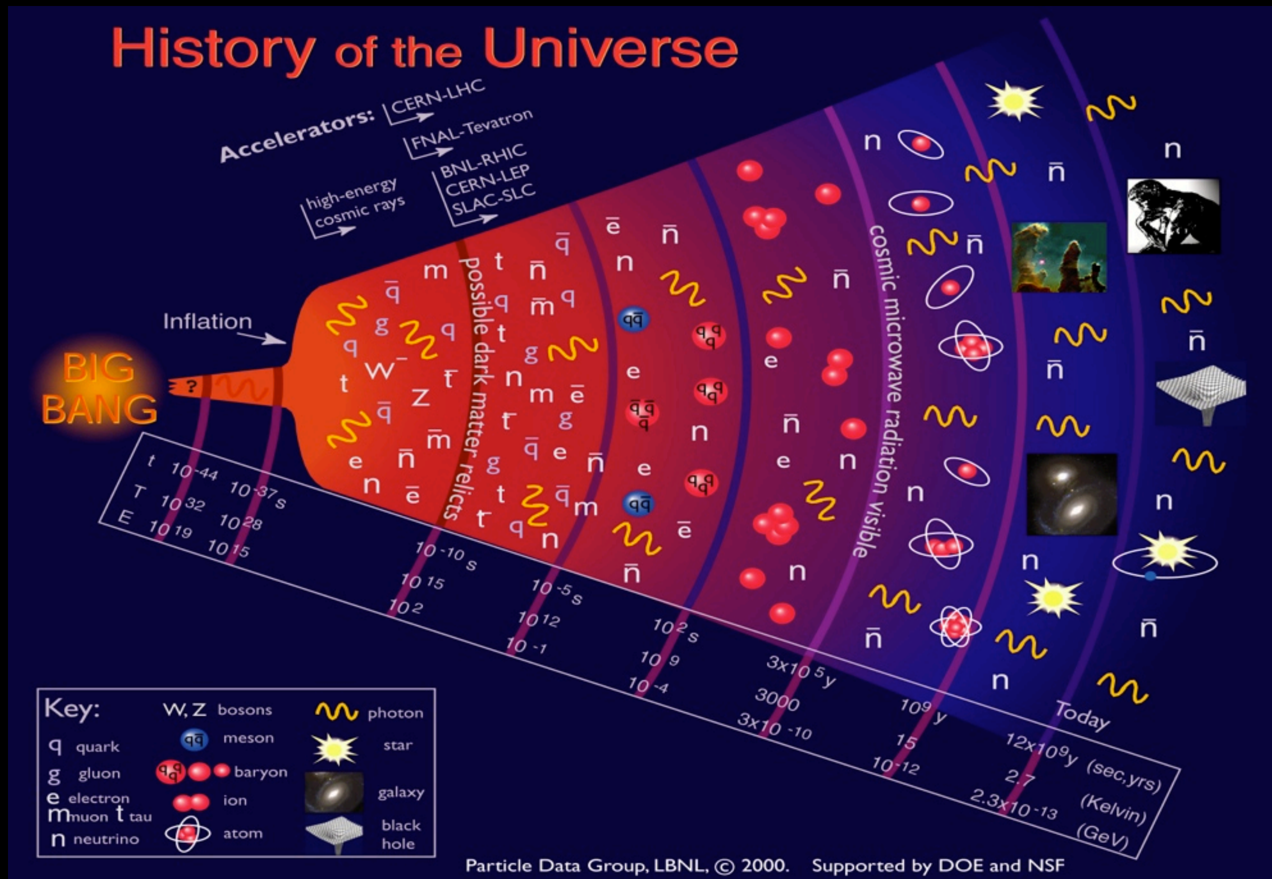


$$E = mc^2$$

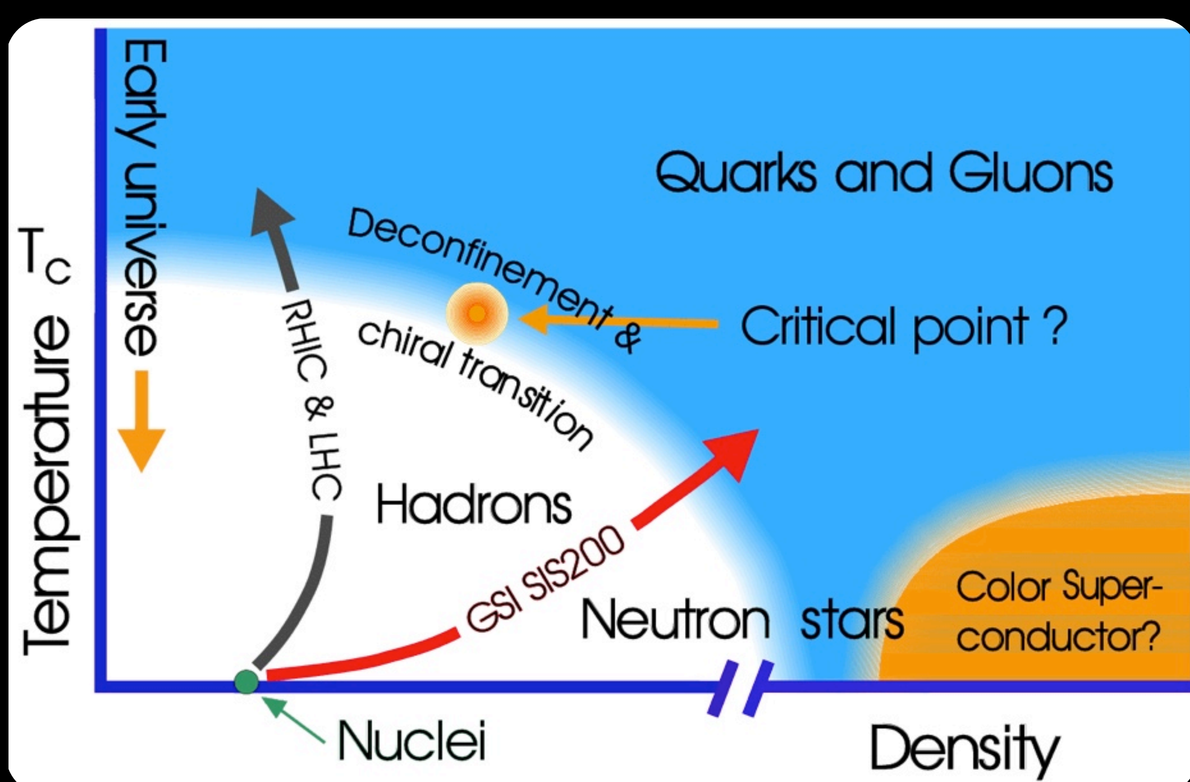


Quark Gluon Plasma
deconfined !

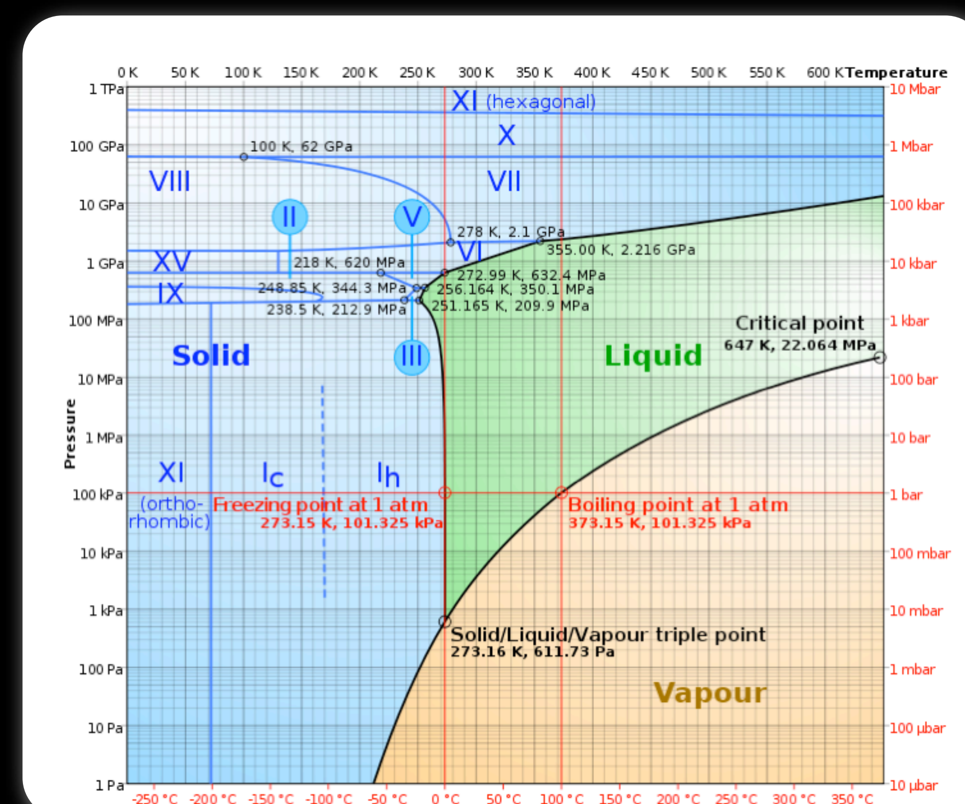
collisions at high energy allow us to create new heavy particles and in collisions of heavy-ions a “little bang”



- heavy-ion collisions provide experimental access to the properties of the QGP
  - better understand the evolution of our universe
  - better understanding of QCD in the non-perturbative regime



QCD phase diagram



phase diagram of water

rough estimate: EoS and degrees of freedom

ideal gas Equation of State: $$p = \frac{1}{3}\varepsilon = g\frac{\pi^2}{90}T^4$$

$$\frac{\varepsilon}{T^4} = g\frac{\pi^2}{30}$$

➡ energy density of g massless degrees of freedom

$$\frac{\varepsilon}{T^4} = 3\frac{\pi^2}{30}$$

➡ hadronic matter dominated by lightest mesons (π^+, π^-, and π^0)

➡ deconfined matter, quarks and gluons

$$g = 2_{\mathrm{spin}} \times 8_{\mathrm{gluons}} + \frac{7}{8} \times 2_{\mathrm{flavors}} \times 2_{\mathrm{quark/anti\text{-}quark}} \times 2_{\mathrm{spin}} \times 3_{\mathrm{color}}$$

$$\frac{\varepsilon}{T^4} = 37\frac{\pi^2}{30}$$

➡ during phase transition large increase in degrees of freedom !

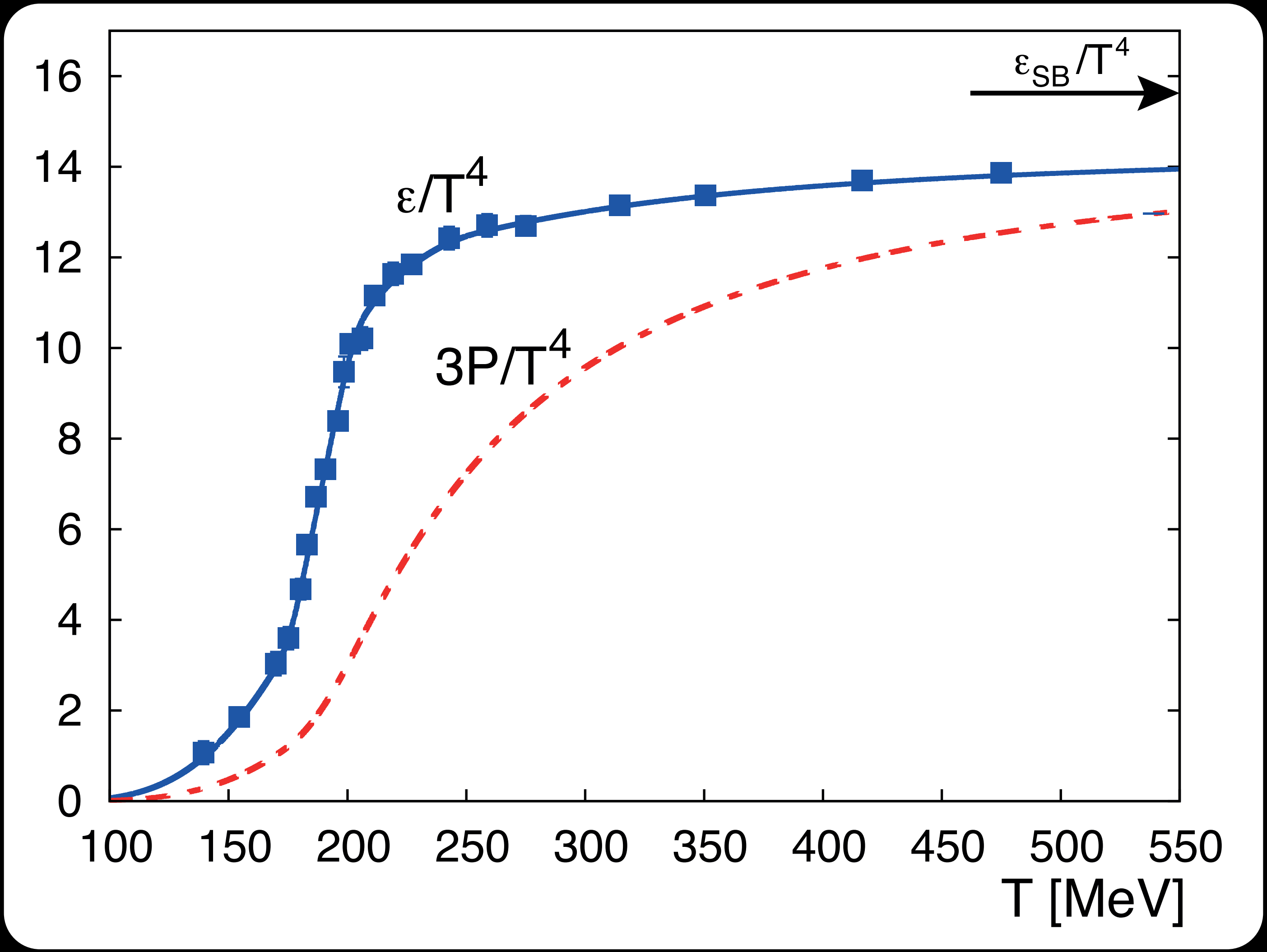
rough estimate: QCD phase transition temperature

- confinement due to bag pressure B (from the QCD vacuum)
  - $B^{1/4} \sim 200$ MeV
- deconfinement when thermal pressure is larger than bag pressure

$$
\begin{aligned}
p &= \frac{1}{3}\epsilon = g\frac{\pi^2}{90}T^4 \\
T_c &= (\frac{90B}{37\pi^2})^{1/4} = 140 \text{ MeV}
\end{aligned}
$$

crude estimate!

QCD on the Latice



T ~ 170 MeV, ε ~ 1 GeV/fm^3

at the critical temperature a strong increase in the degrees of freedom

✓ gluons, quarks & color!

not an ideal gas!?

✓ residual interactions

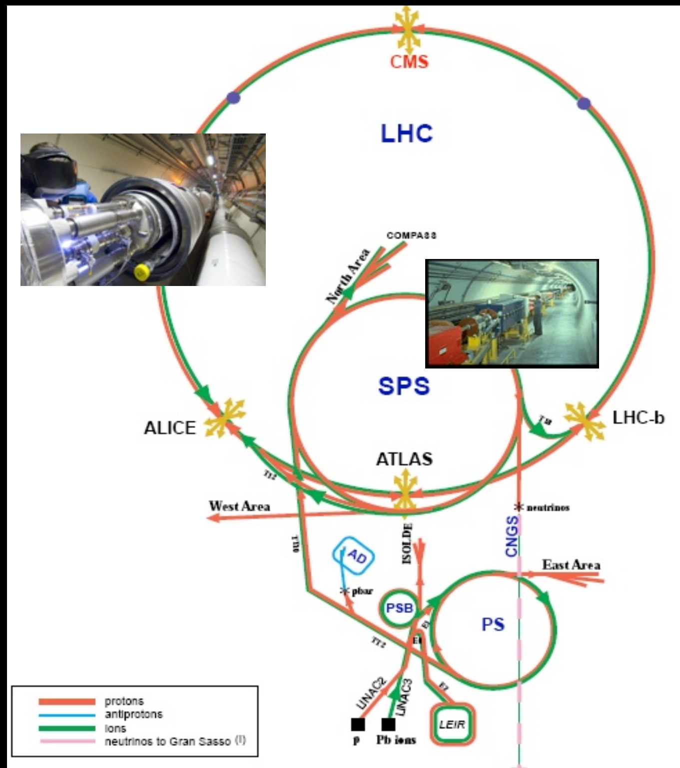
at the phase transition dp/dε decreases rapidly

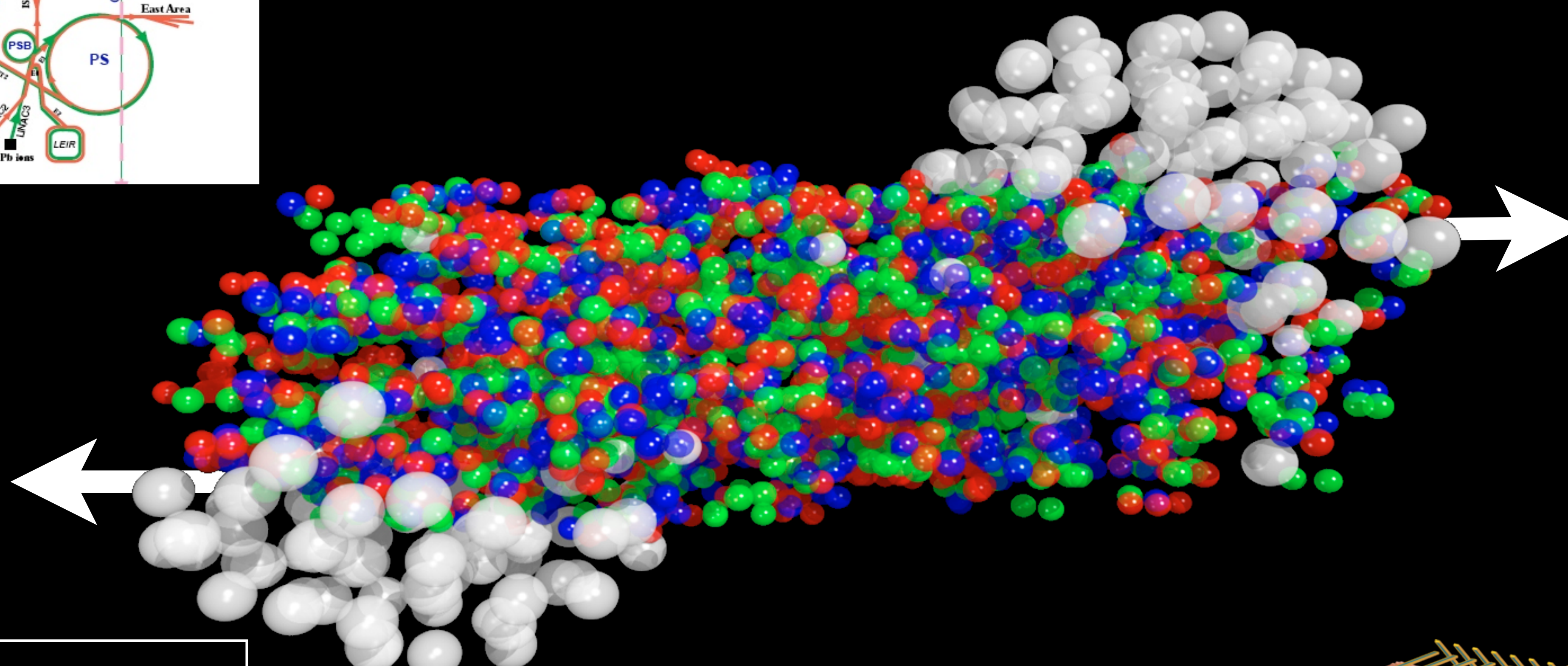
$$p = \frac{1}{3}\epsilon = g\frac{\pi^2}{90}T^4$$

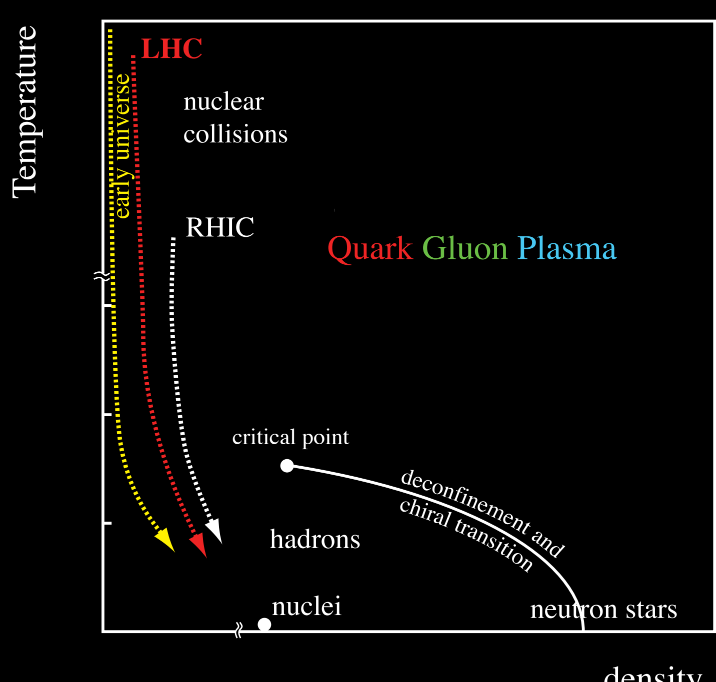
$$g_{\rm H} \approx 3 \quad g_{\rm QGP} \approx 37$$

$$g = 2_{\rm spin} \times 8_{\rm gluons} + \frac{7}{8} \times 2_{\rm flavors} \times 2_{q\bar{q}} \times 2_{\rm spin} \times 3_{\rm color}$$

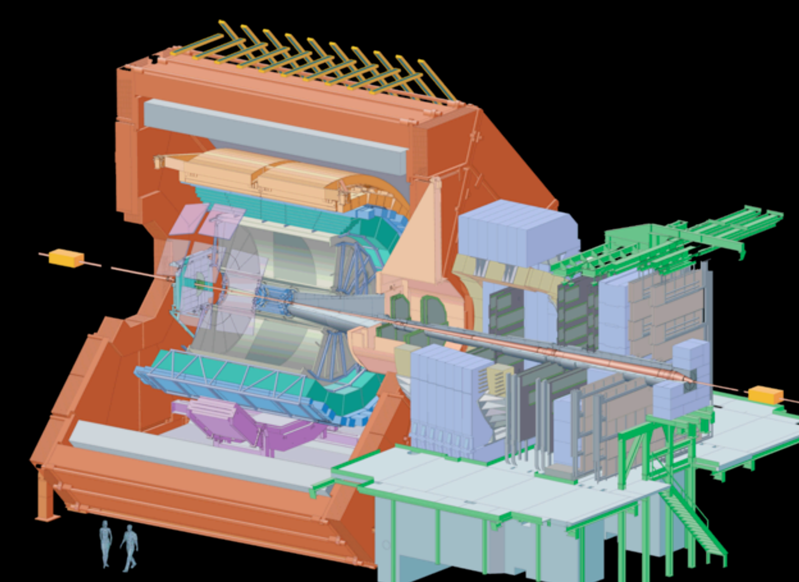
Experiment?



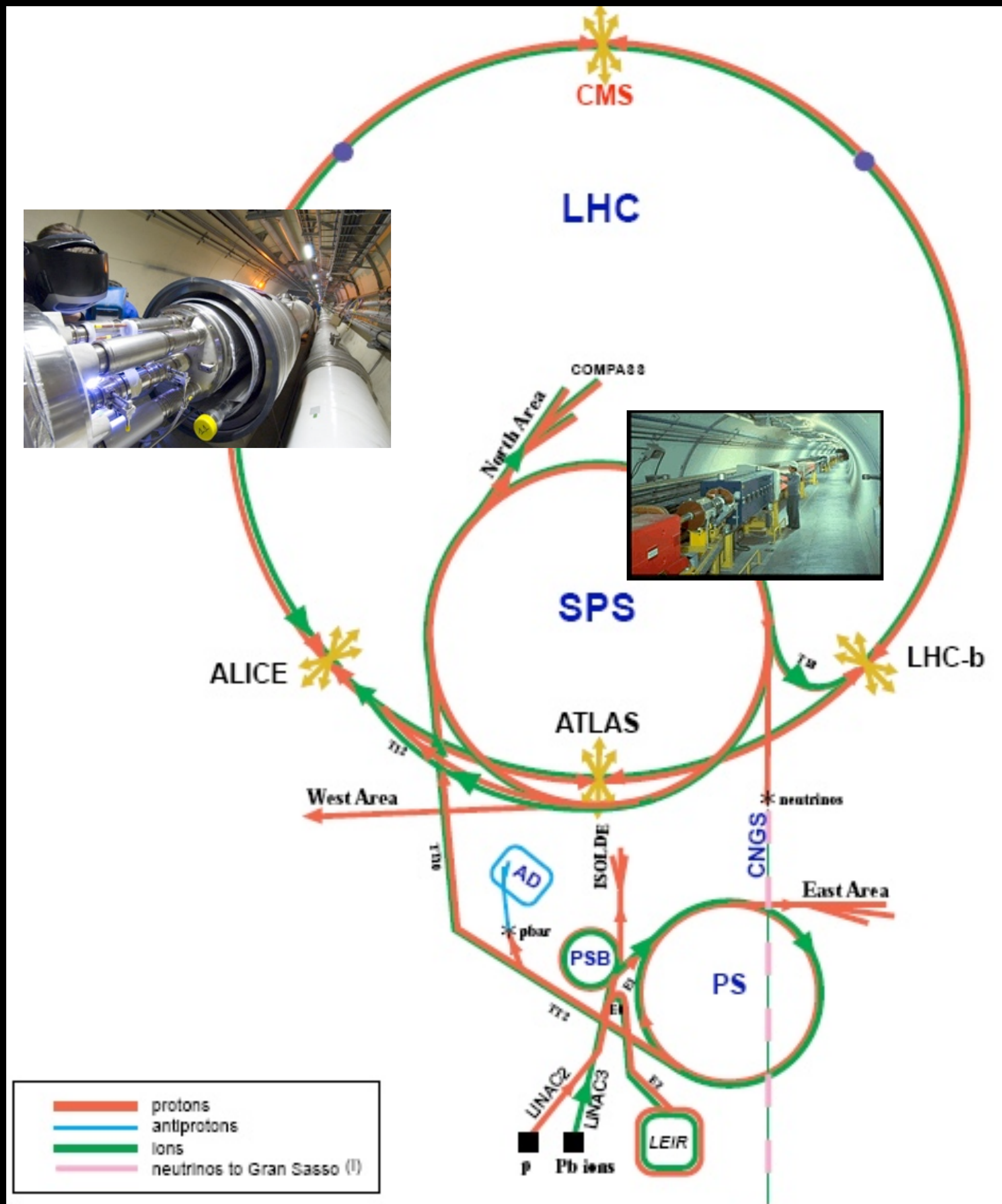


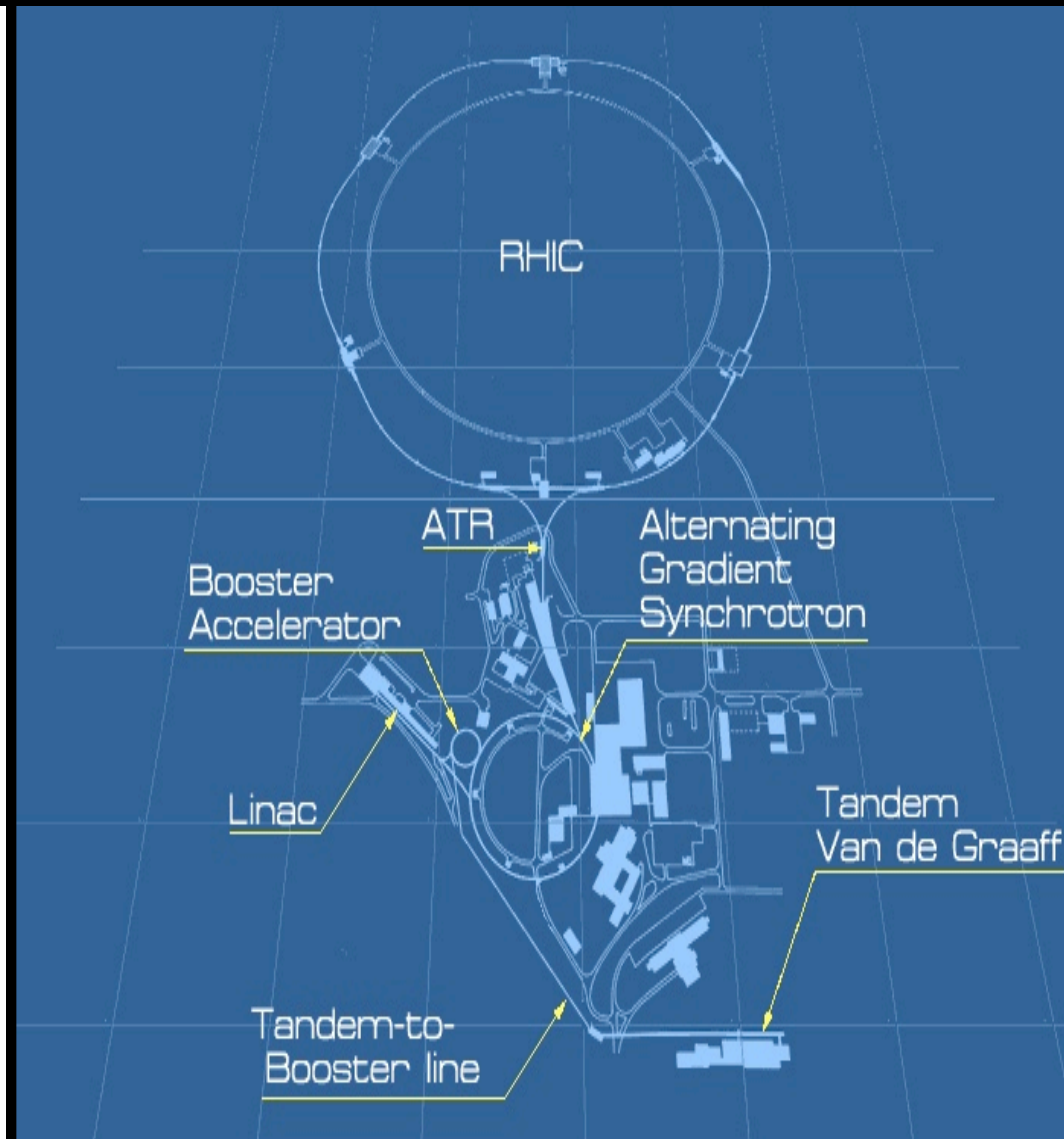


study phase transition in controlled lab conditions by colliding heavy-ions

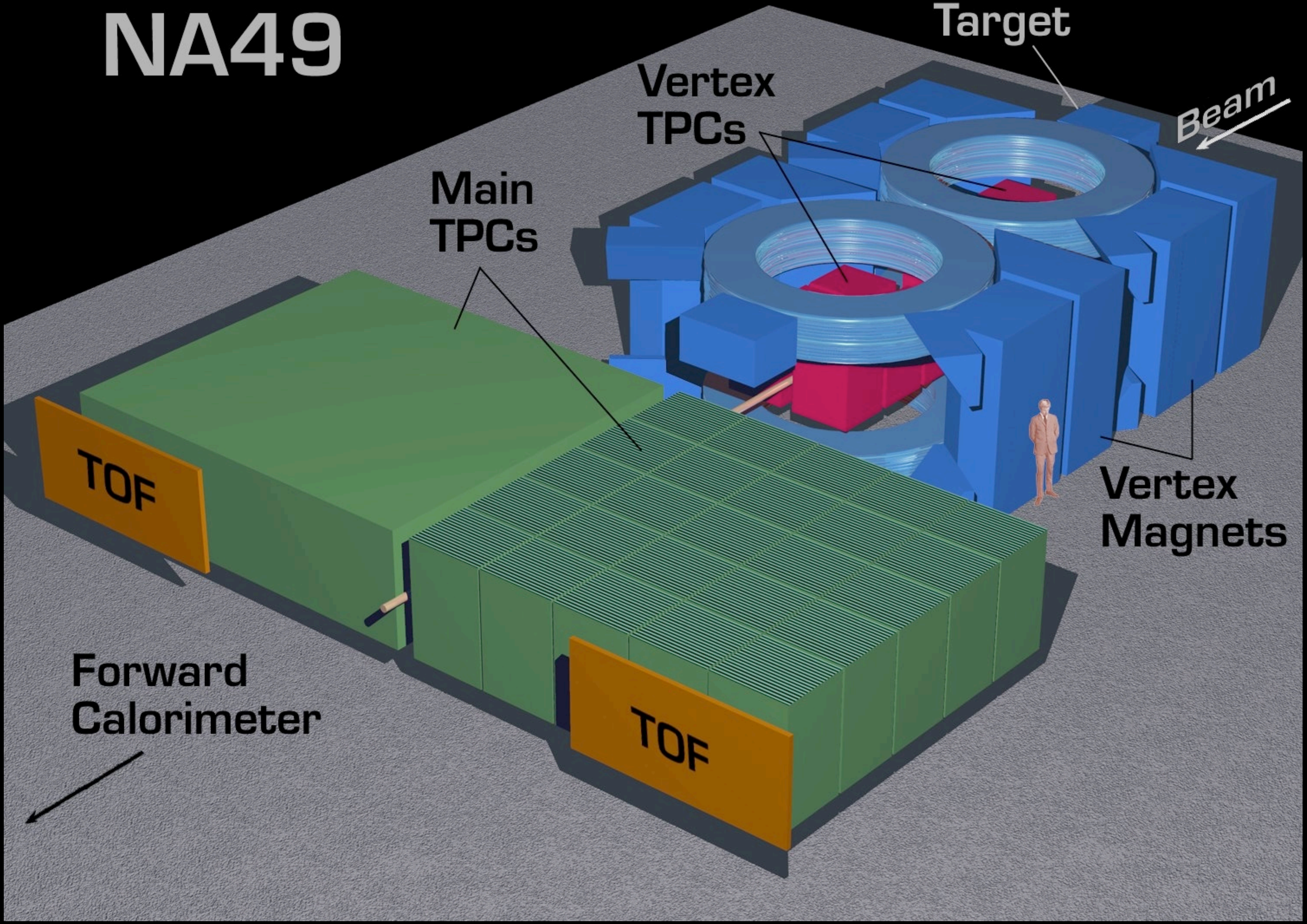


CERN and BNL

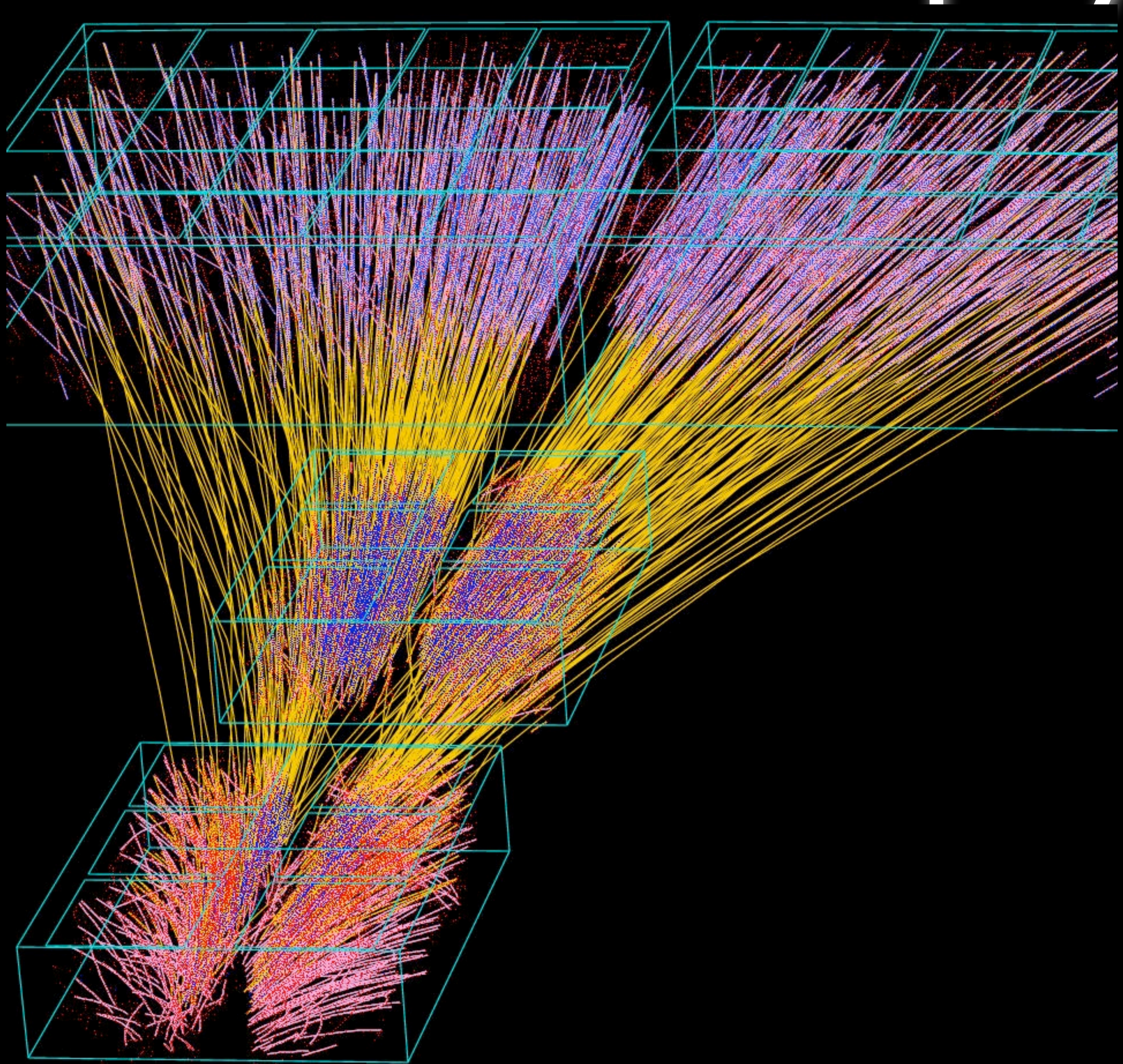

CMS
LHC
COMPASS
North Area
SPS
ALICE
LHC-b
ATLAS
TI2
TI8
West Area
neutrinos
CNGS
ISOLDE
AD
pbar
East Area
PSB
PS
TT2
LINAC2
LINAC3
p
Pb ions
LEIR
protons
antiprotons
ions
neutrinos to Gran Sasso (I)


RHIC
ATR
Alternating Gradient Synchrotron
Booster Accelerator
Linac
Tandem Van de Graaff
Tandem-to-Booster line

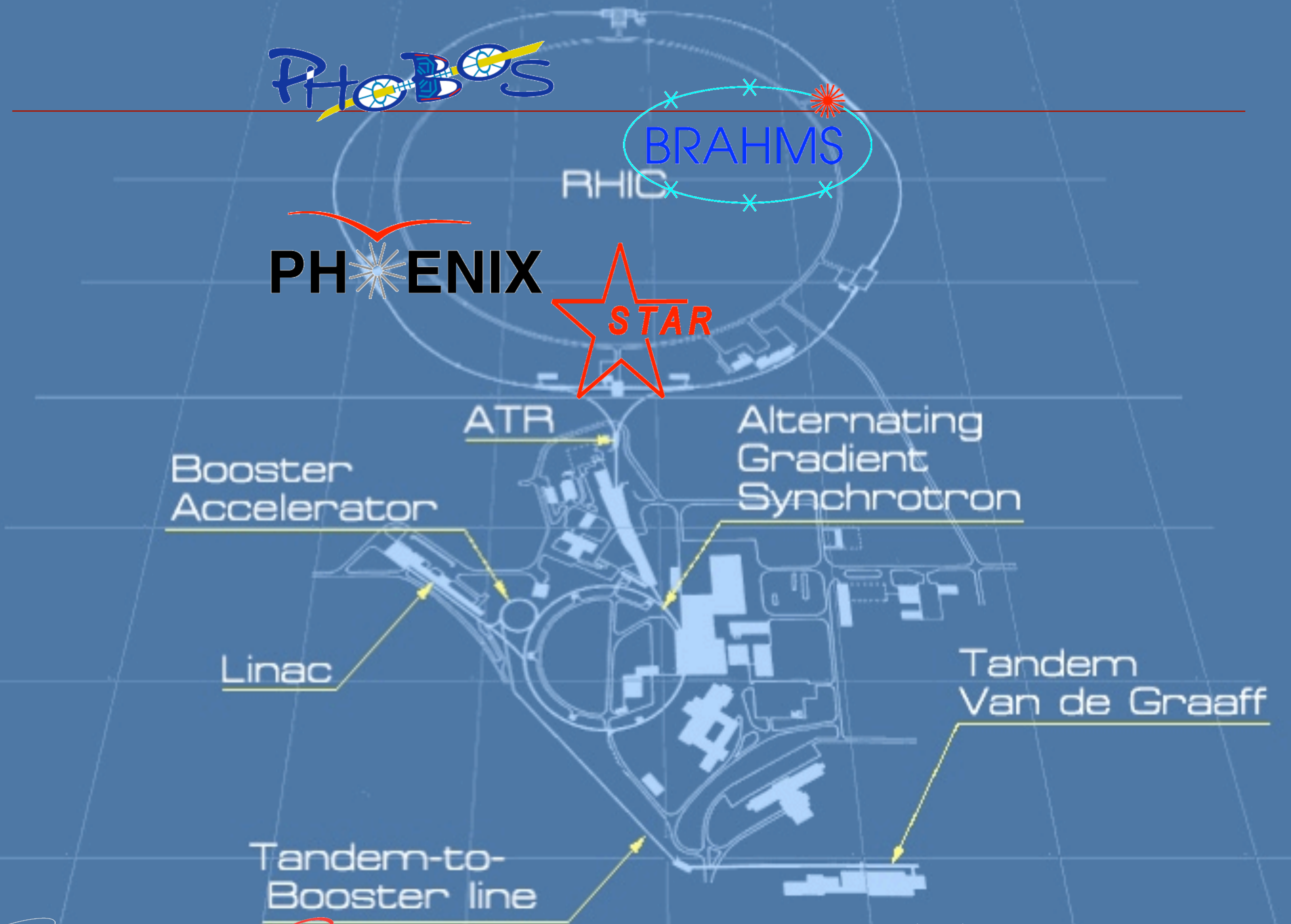
SPS detector example: NA49



NA49 event display

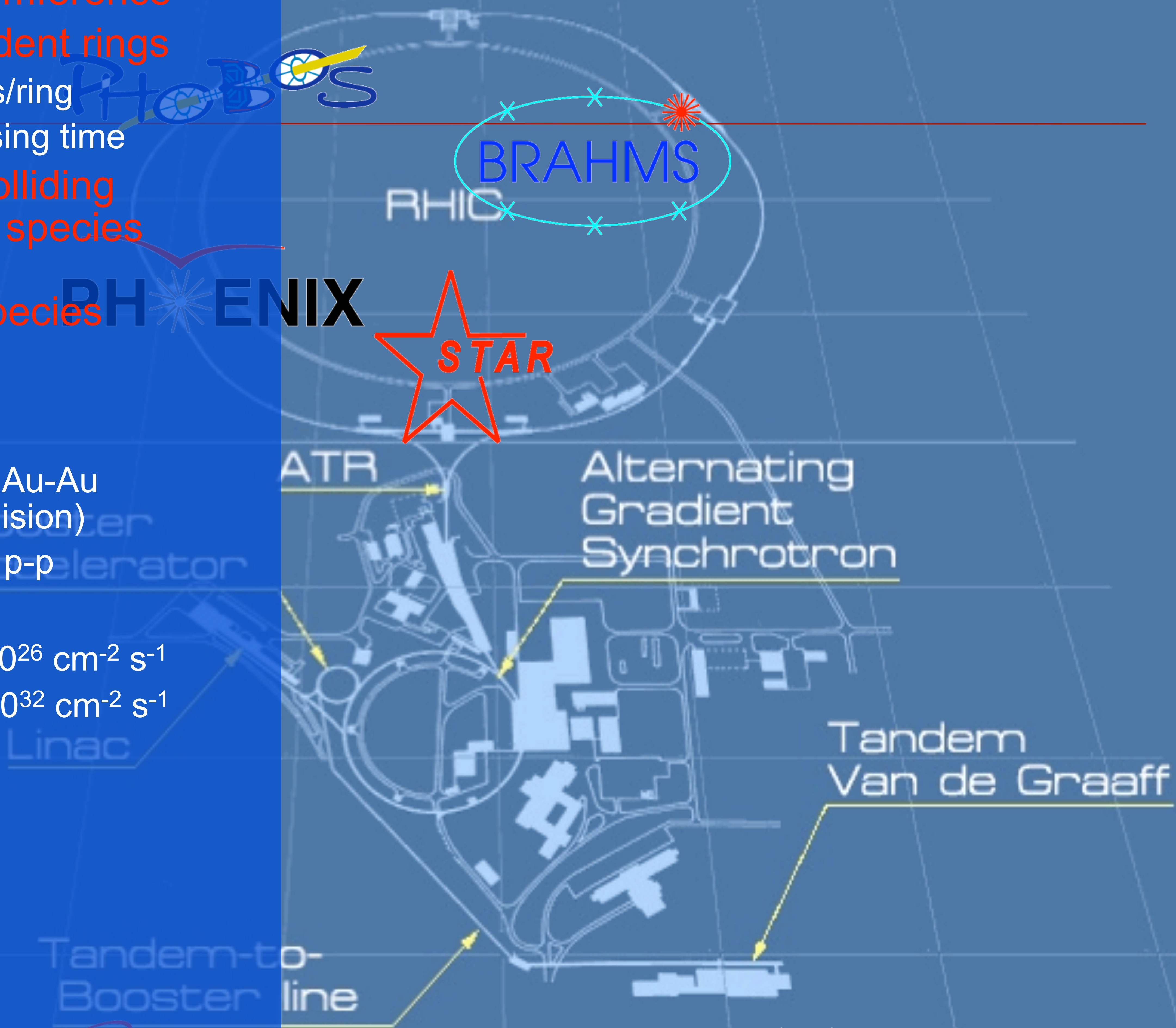


The Relativistic Heavy Ion Collider

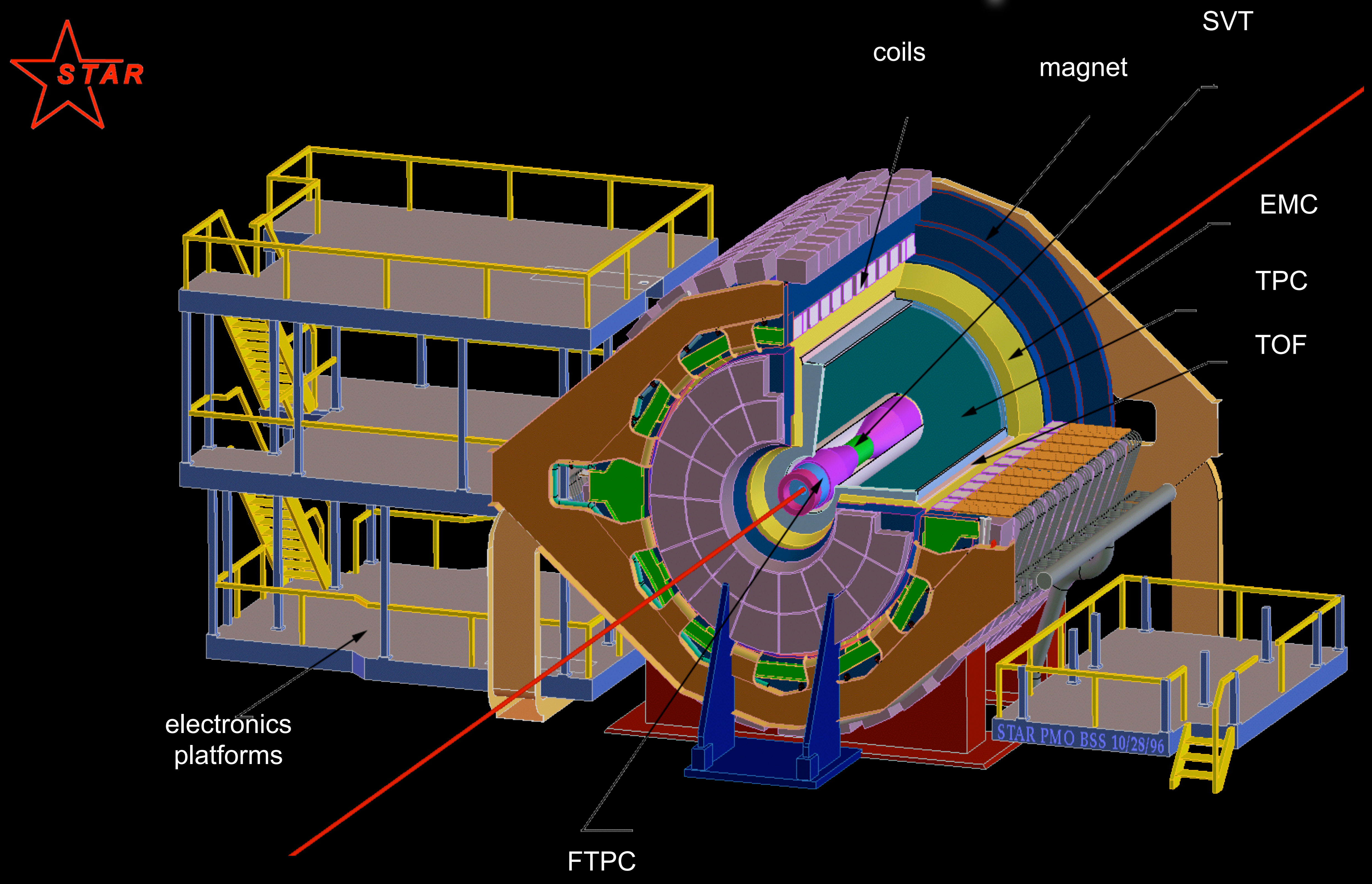

PHOBOS
BRAHMS
RHIC
PHENIX
STAR
ATR
Booster
Accelerator
Alternating
Gradient
Synchrotron
Linac
Tandem
Van de Graaff
Tandem-to-
Booster line

- 3.83 km circumference
- Two independent rings
  - 120 bunches/ring
  - 106 ns crossing time
- Capable of colliding
  ~any nuclear species
  on
  ~any other species

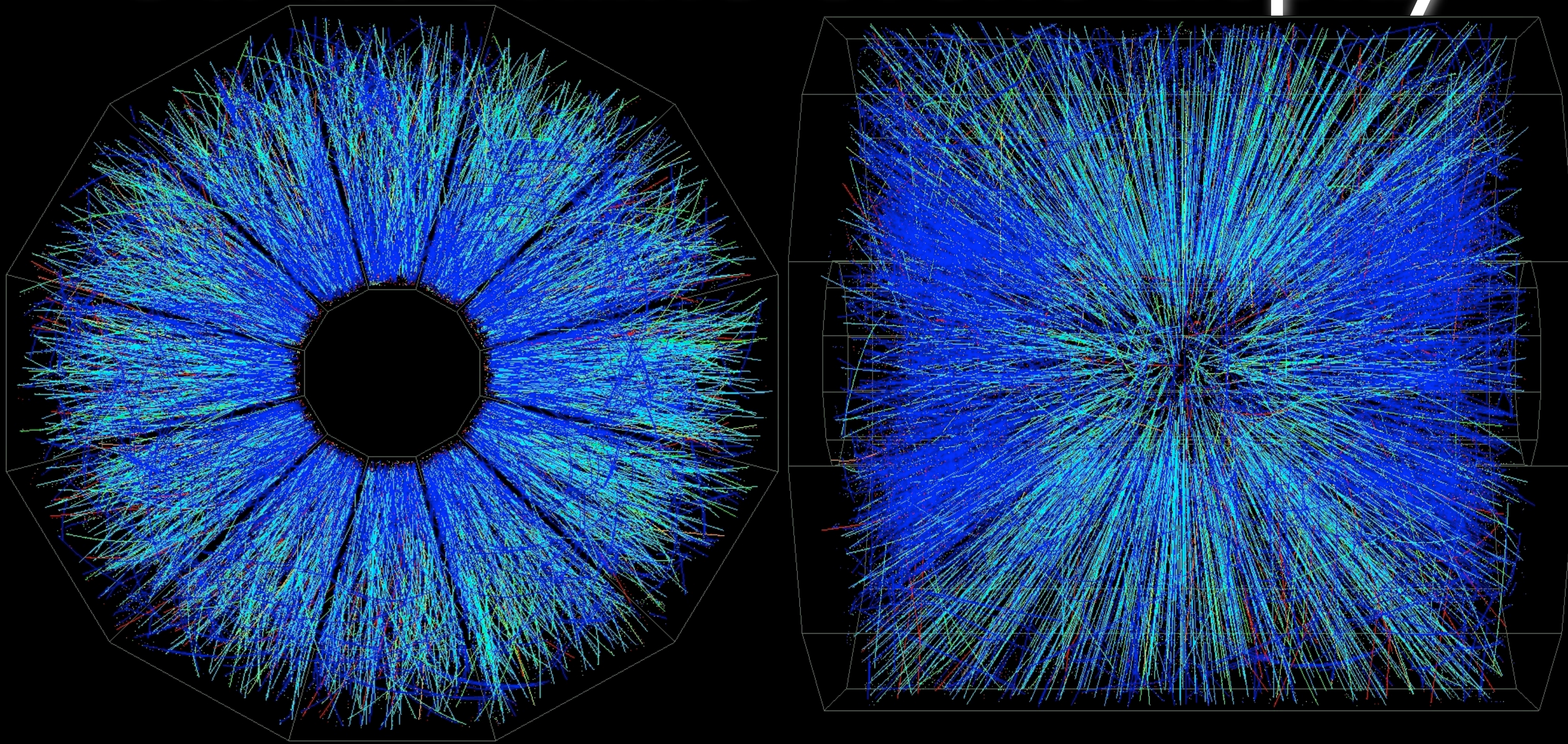
- Energy:
  - 200 GeV for Au-Au
    (per N-N collision)
  - 500 GeV for p-p
- Luminosity:
  - Au-Au: 2×10^{26} cm^{-2} s^{-1}
  - p-p : 2×10^{32} cm^{-2} s^{-1}
    (*polarized*)



RHIC detector example: STAR

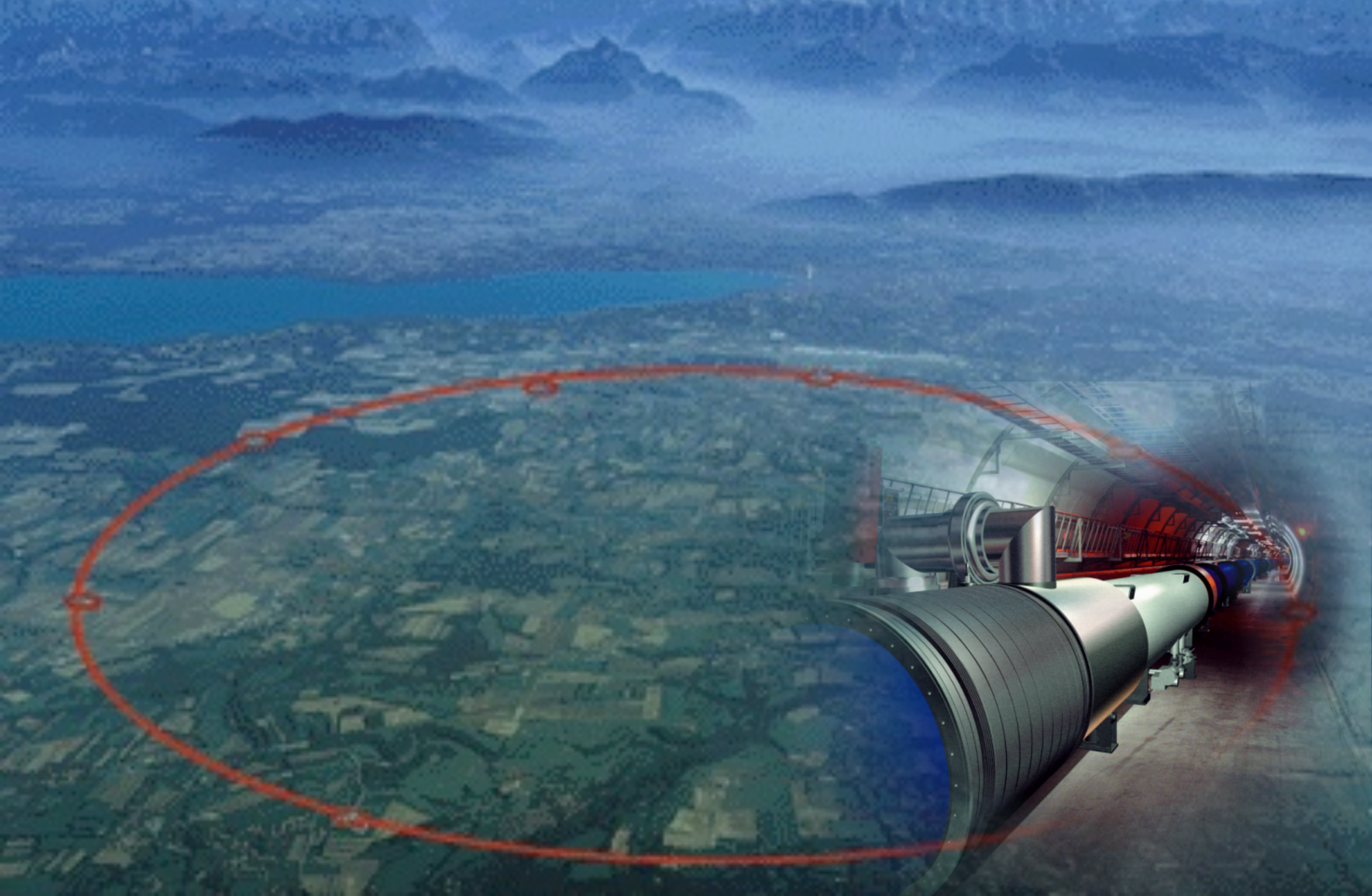


STAR online event display

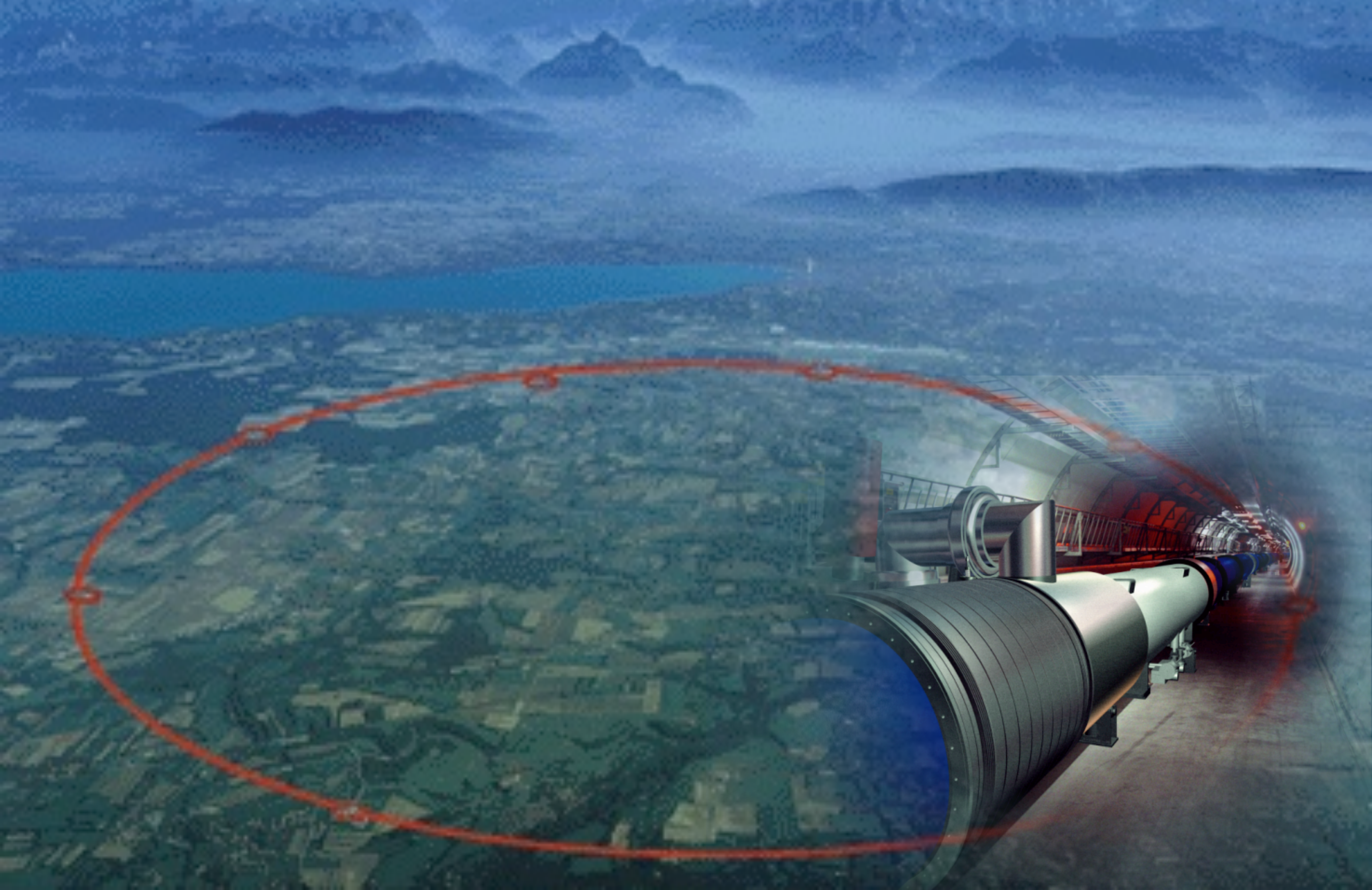


Online Level 3 Trigger Display

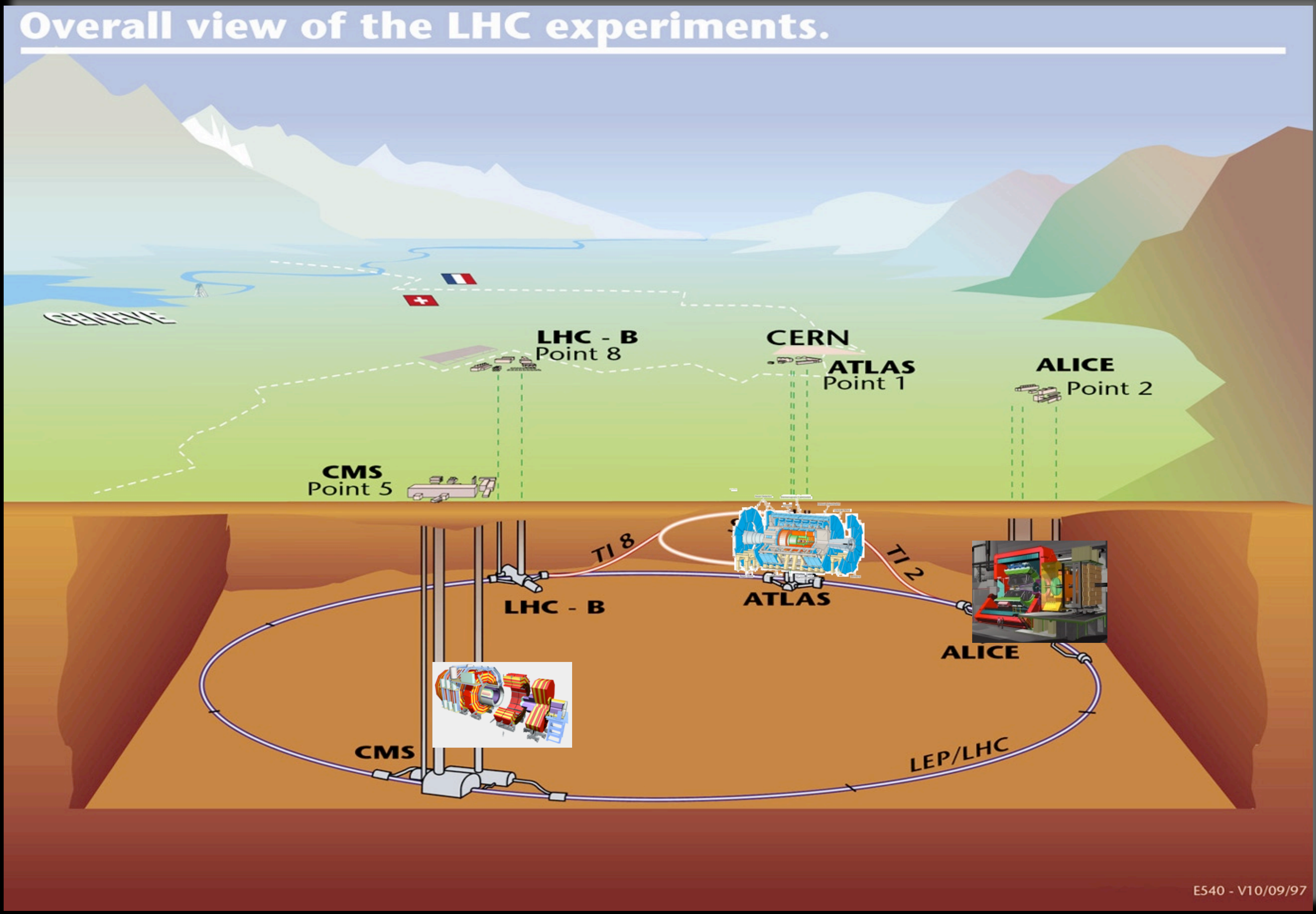
The Large Hadron Collider (LHC)



The Large Heavy ion Collider (LHC)

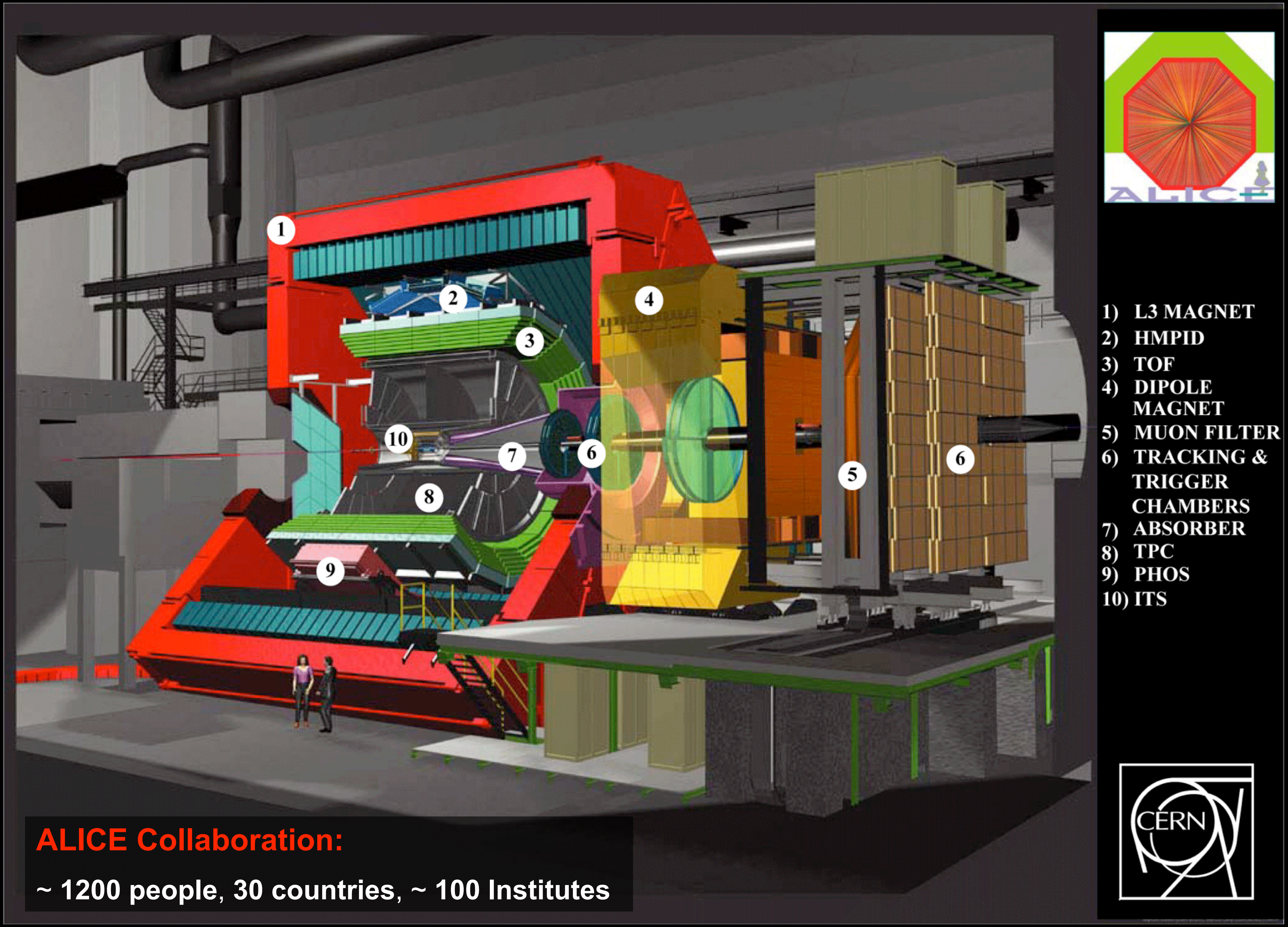


LHC



Overall view of the LHC experiments.
GENEVE
LHC - B
Point 8
CERN
ATLAS
Point 1
ALICE
Point 2
CMS
Point 5
TI 8
TI 2
LHC - B
ATLAS
ALICE
CMS
LEP/LHC
E540 - V10/09/97

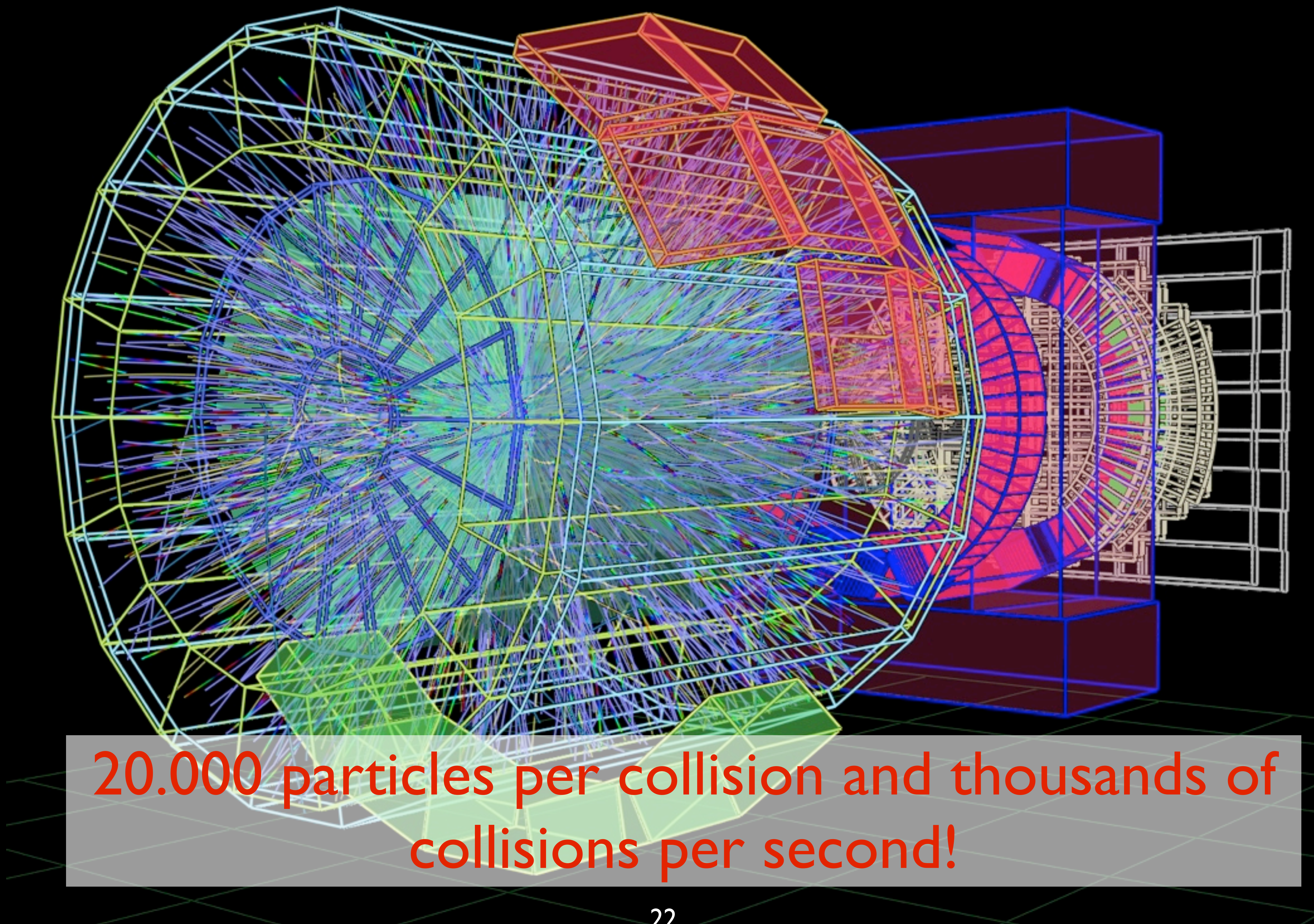
The ALICE Detector



ALICE Collaboration:

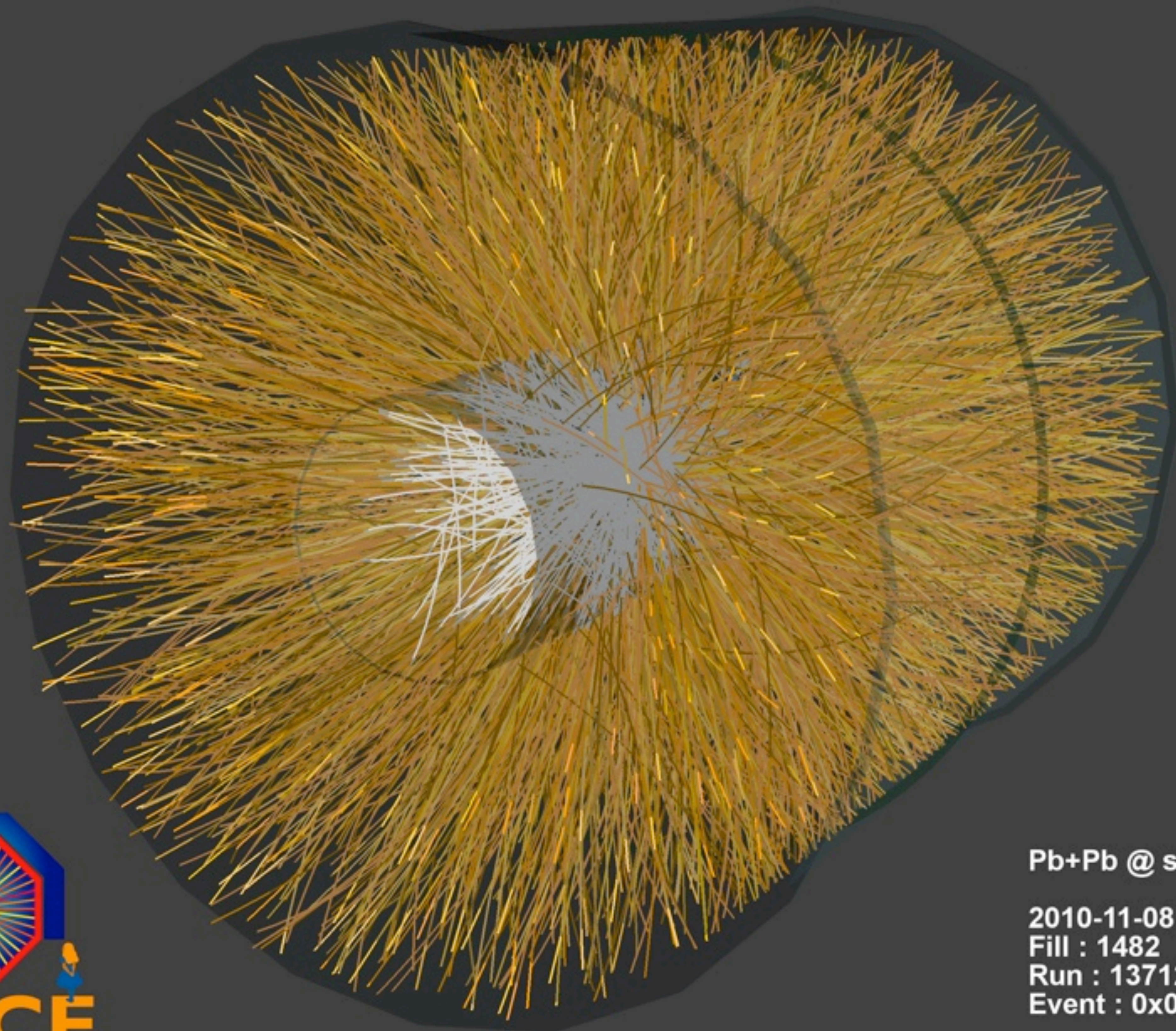
~ 1200 people, 30 countries, ~ 100 Institutes

ALICE



20.000 particles per collision and thousands of collisions per second!

The first Pb-Pb collision!

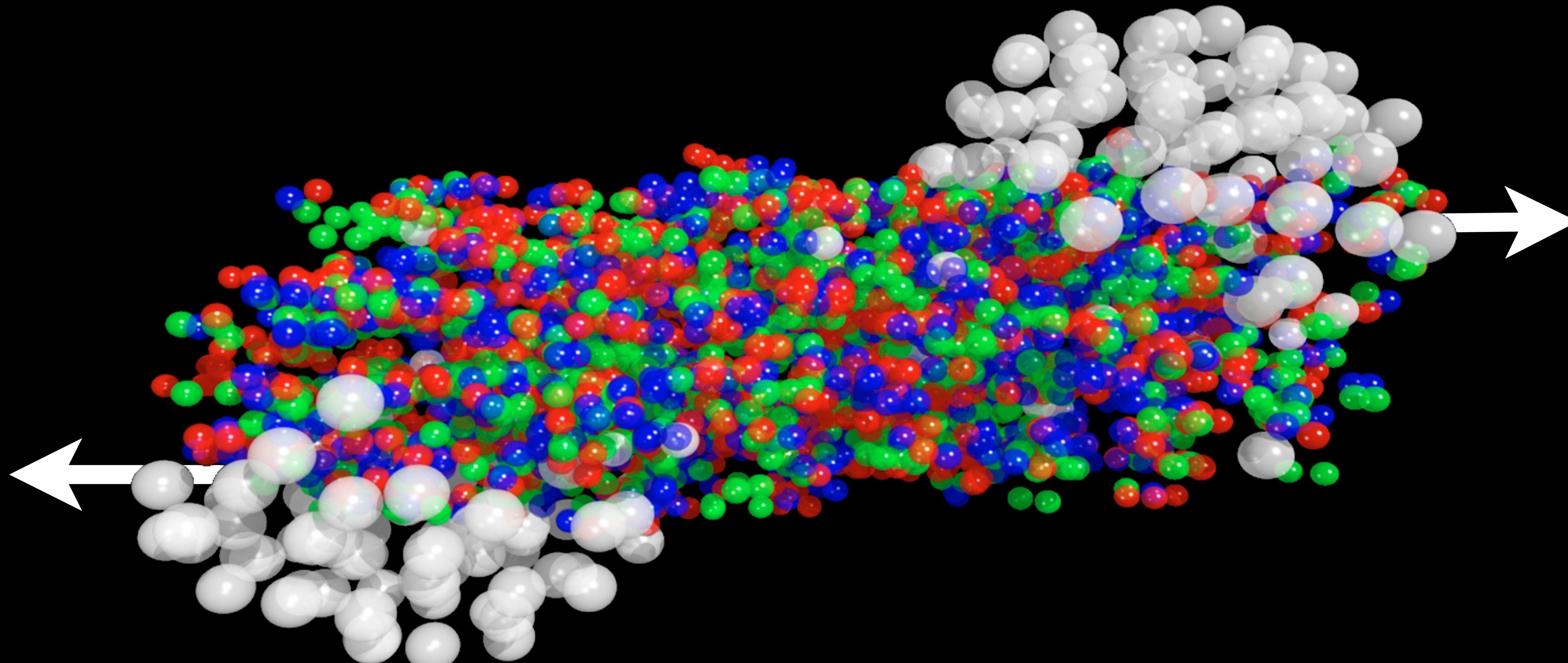


Pb+Pb @ sqrt(s) = 2.76 ATeV

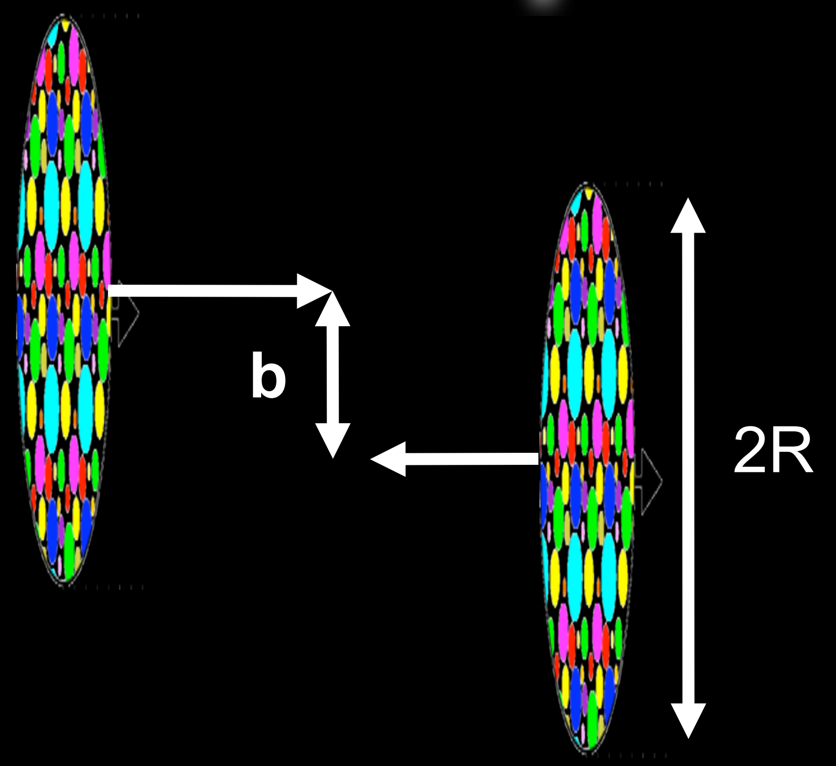
2010-11-08 11:30:46
Fill : 1482
Run : 137124
Event : 0x00000000D3BBE693

ALICE

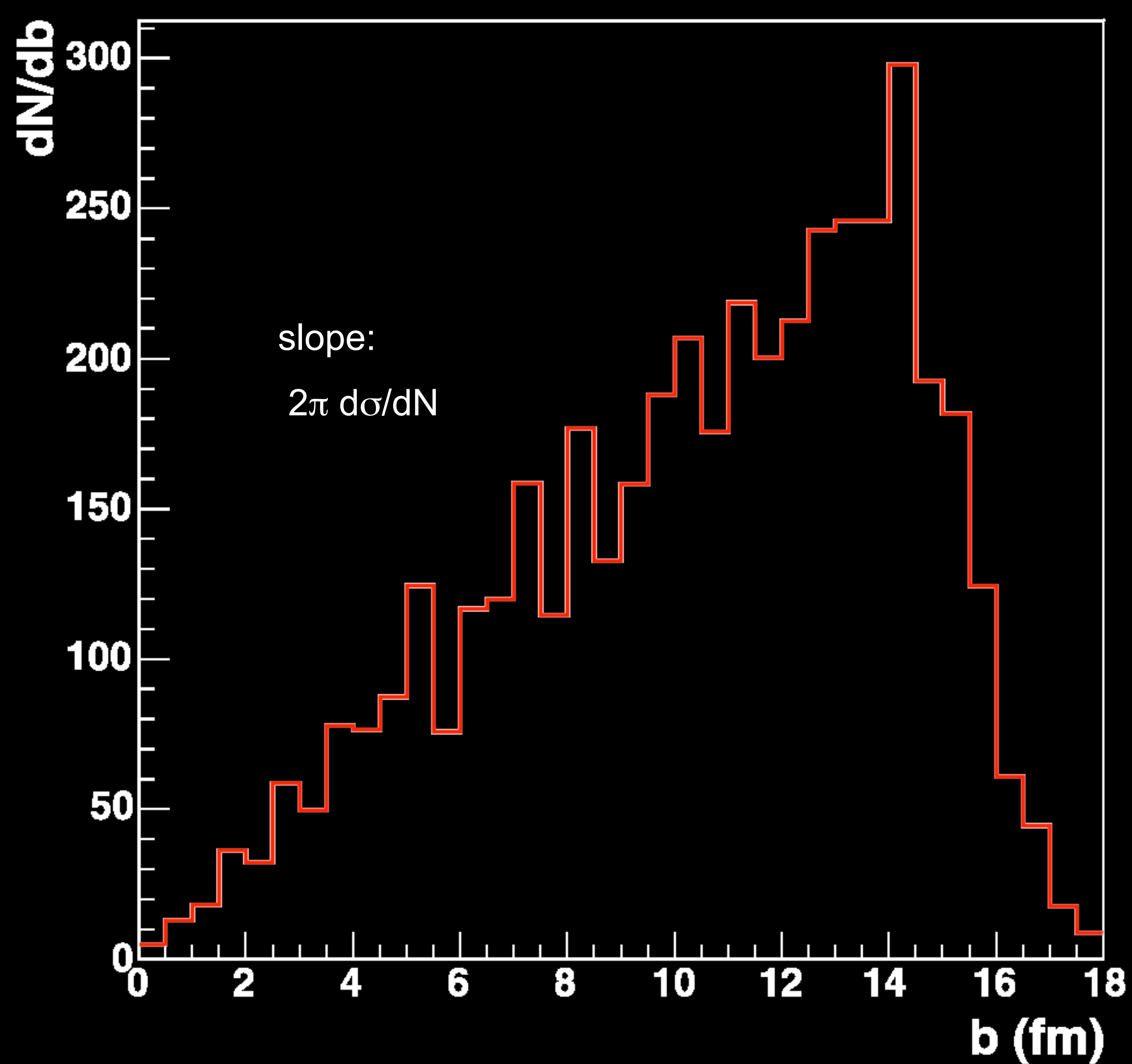
Event Characterization



Impact Parameter

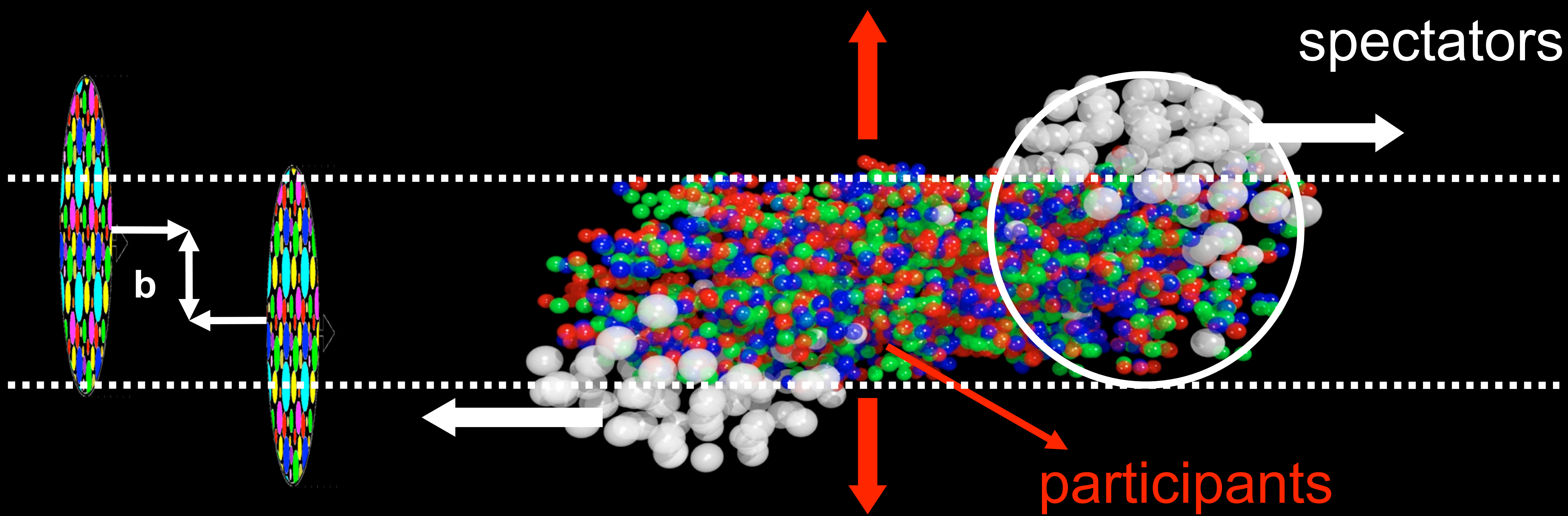


- impact parameter **b**
  - perpendicular to beam direction
  - connects centers of the colliding ions



$$d\sigma = 2\pi b\, db$$

Centrality Determination (I)



centrality characterized by:

1. N_{part}, $N_{wounded}$: number of nucleons which suffered at least one inelastic nucleon-nucleon collision
2. N_{coll}, N_{bin}: number of inelastic nucleon-nucleon collisions

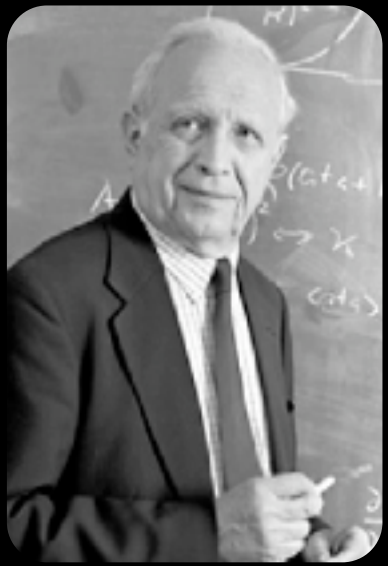
Glauber Model Calculations

- ✓ nuclear density from Wood-Saxon distribution

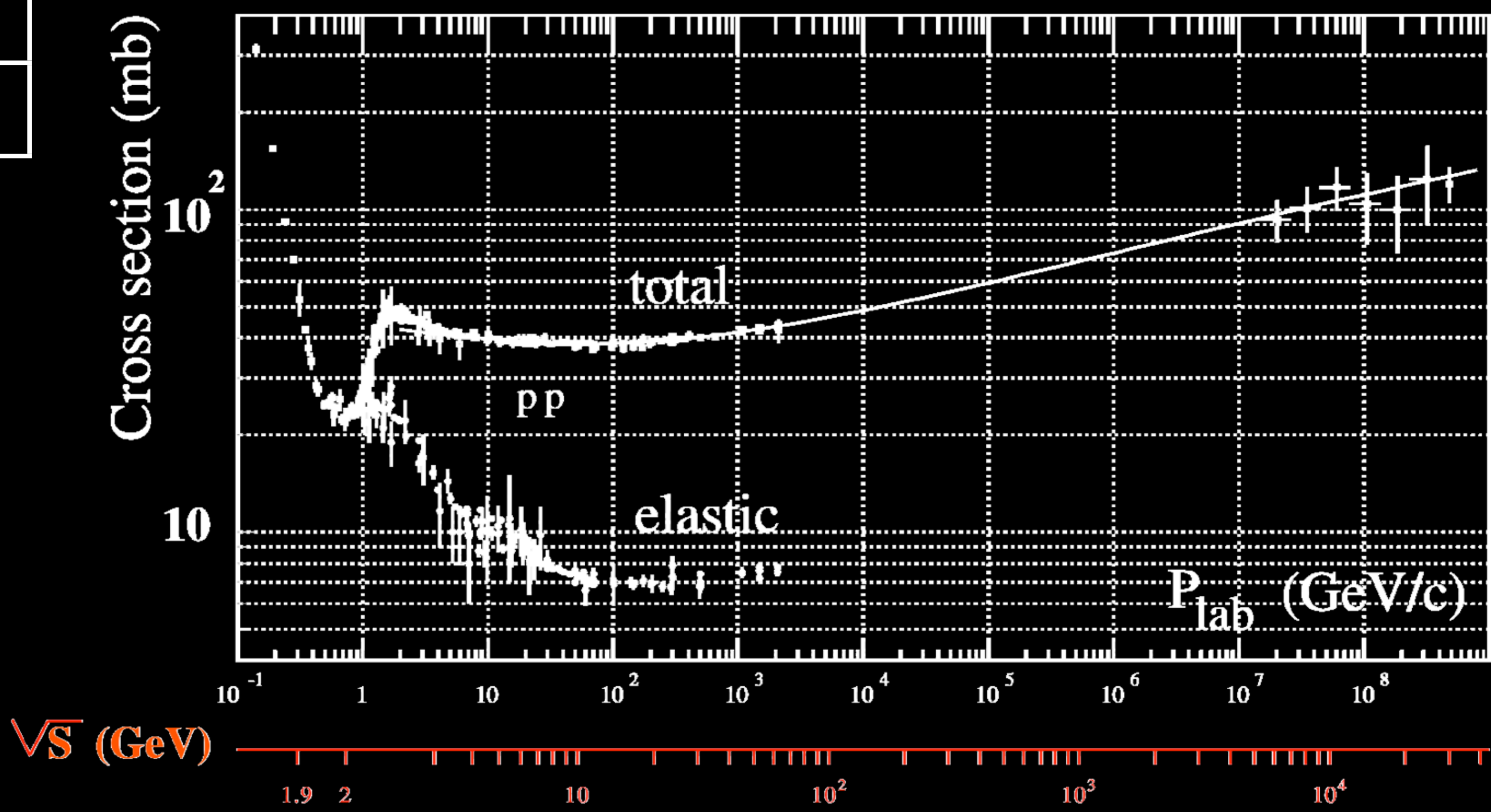
$$\rho(r) = \frac{\rho_0 \left(1 + wr^2 / R^2\right)}{1 + e^{(r-R)/a}}$$

Nucleus	A	R	a
Au	197	6.38	0.535
Pb	208	6.68	0.546

$\sqrt{S}$ (GeV)	$\sigma_{in,pp}$ (mb)
20	32
200	42
2700	~64

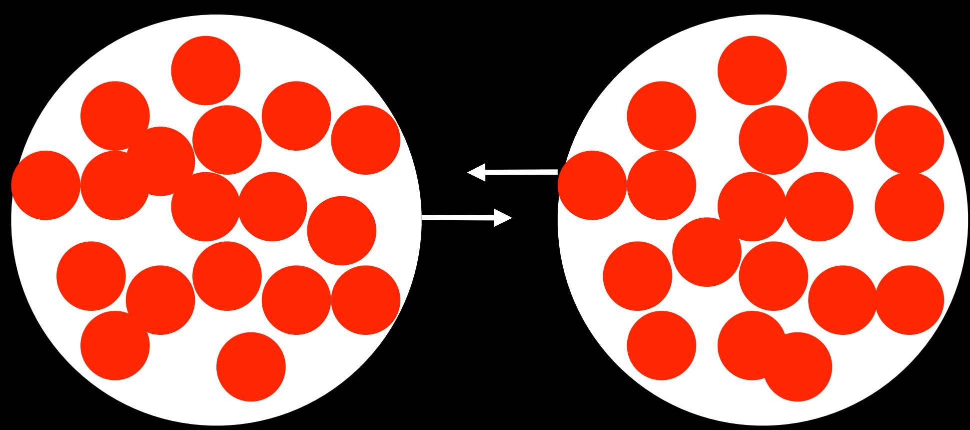


- ✓ nucleons travel on straight lines, no deflection after NN collision
- ✓ NN collision cross section from measured inelastic cross section in p+p
- ✓ NN cross section remains constant independent of how many collisions a nucleon suffered



Wounded nucleons and binary collisions

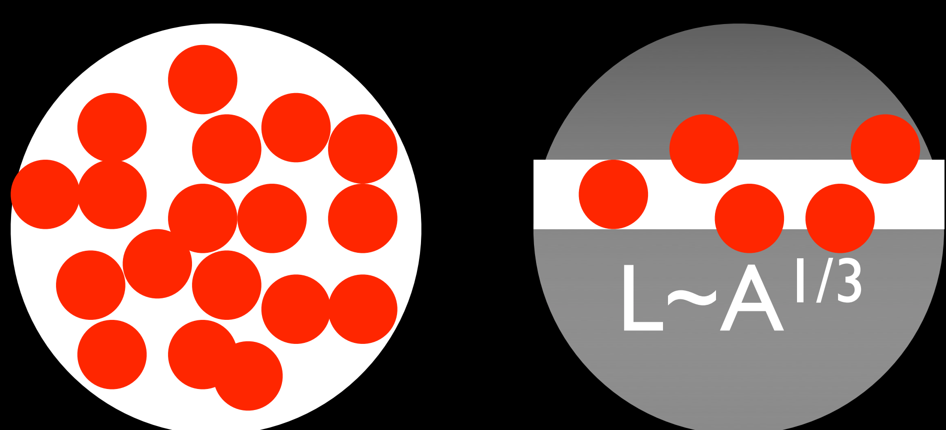
wounded nucleon scaling



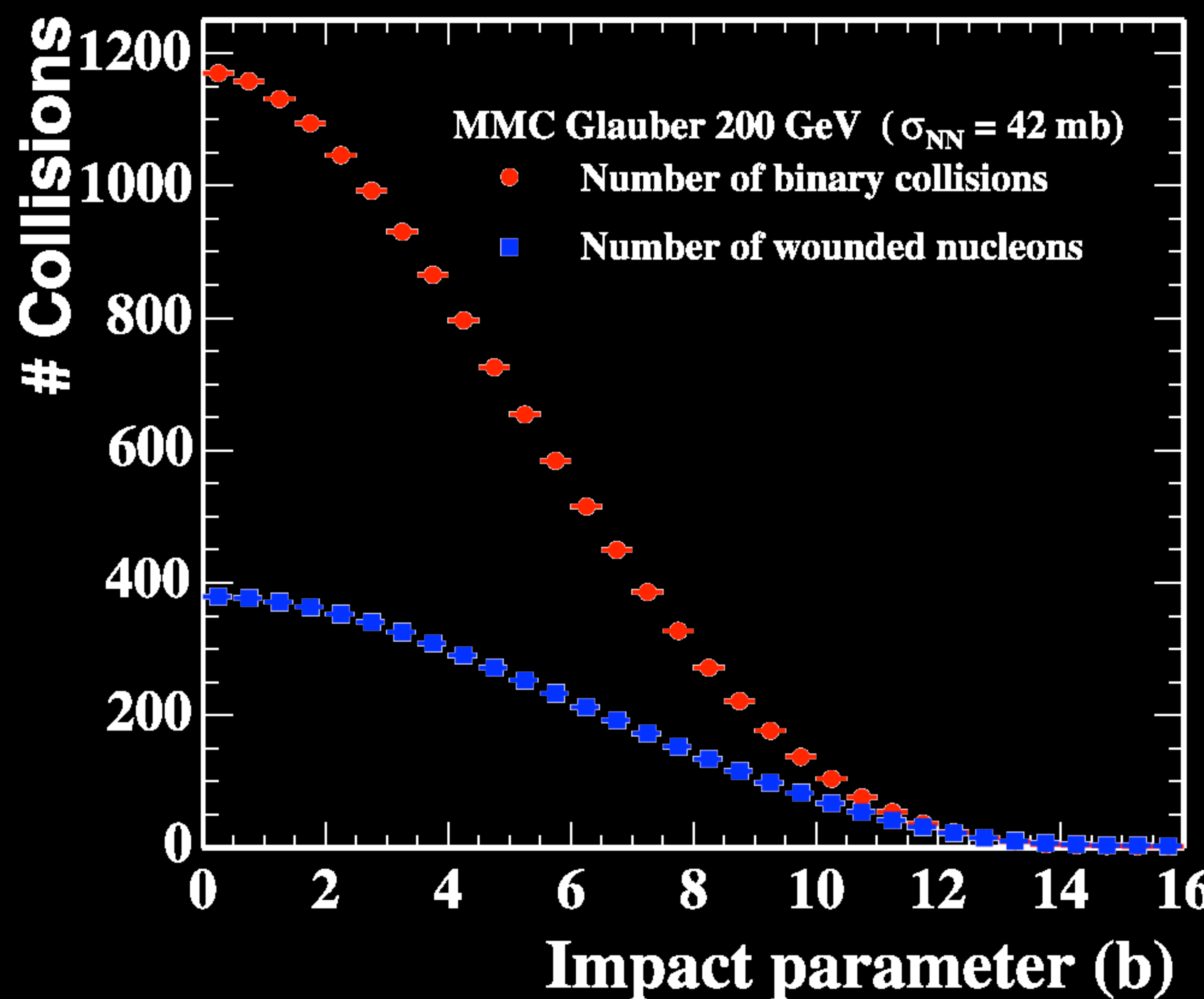
number of participating nucleons scales with volume ~ 2A

binary scaling

A

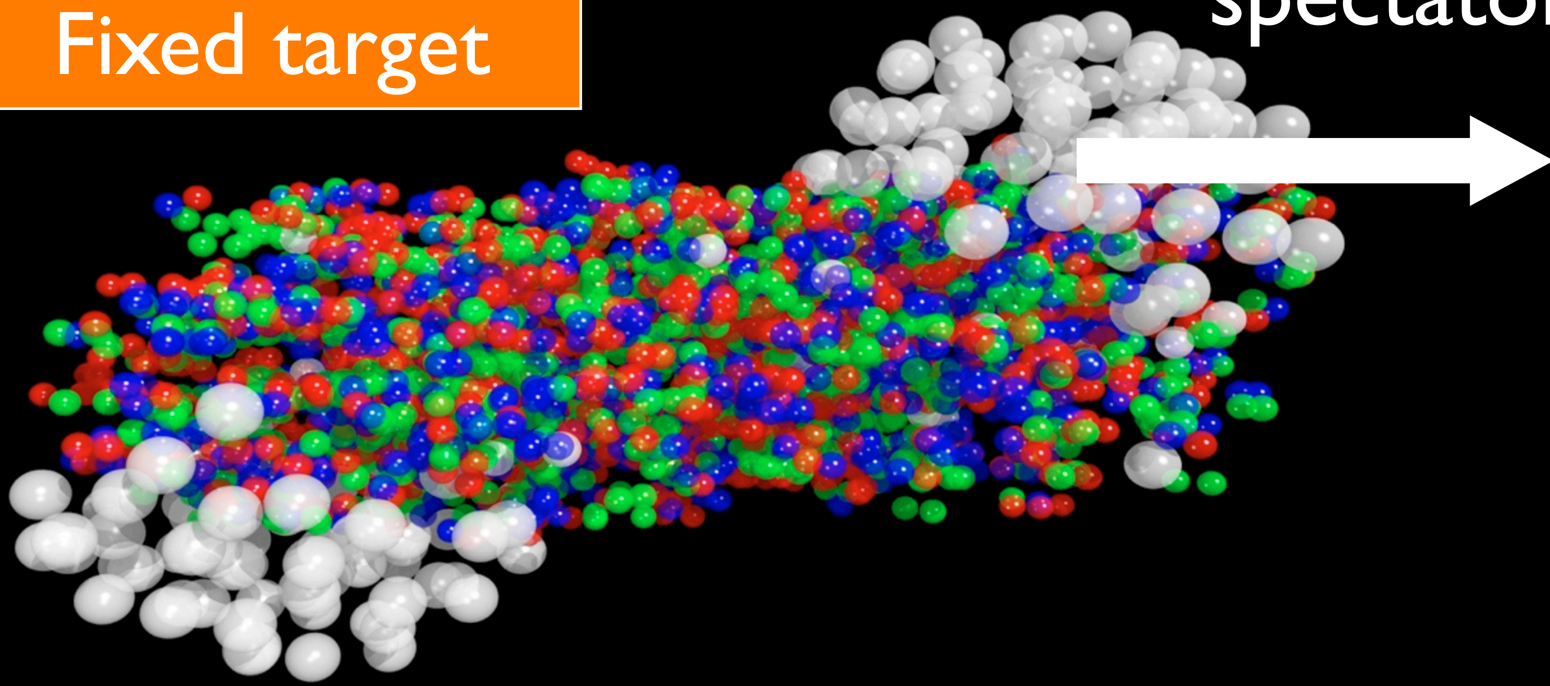


number of NN collisions, point like, scales with ~ $A^{4/3}$



Centrality determination (II)

Fixed target

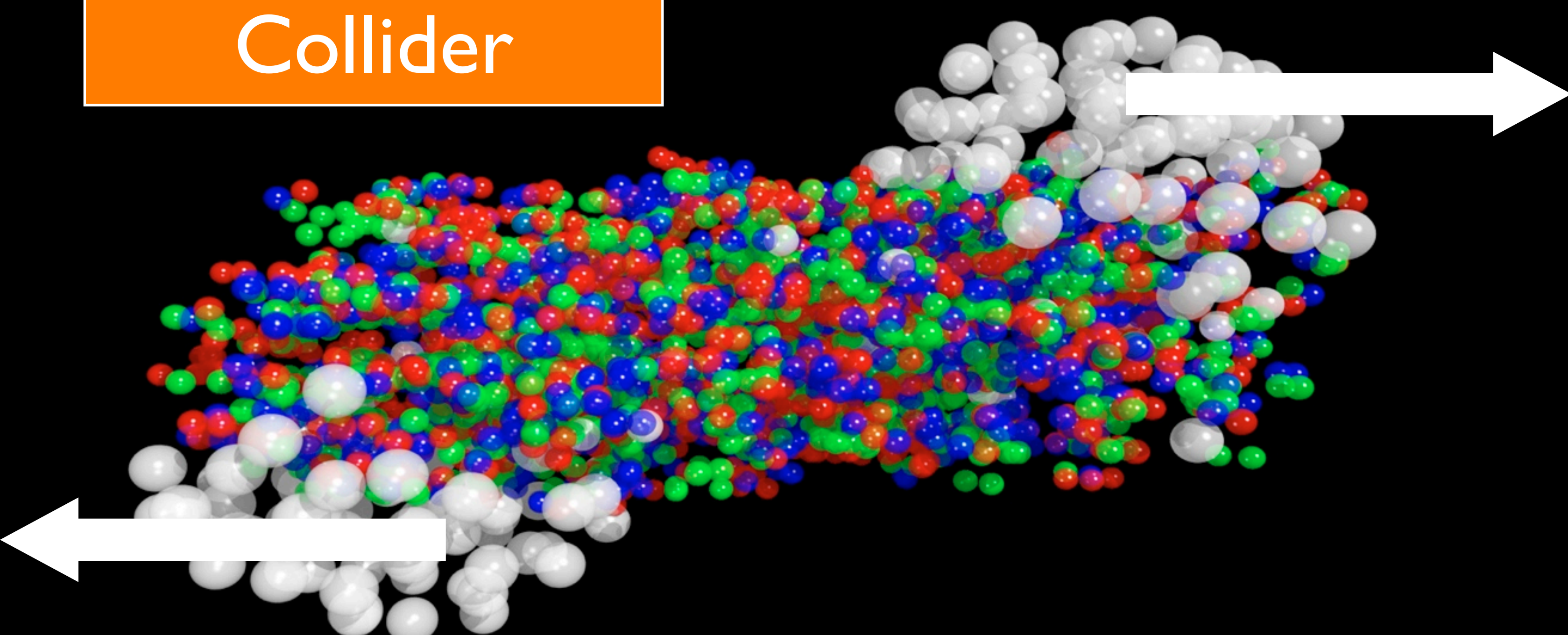


Zero-Degree-Calorimeter (ZDC) measures energy of all spectator nucleons

$$N_{\mathrm{spec}} \approx E_{\mathrm{ZDC}} / \left(E_{\mathrm{beam}} / A\right),$$

$$N_{part} \approx 2 \cdot \left(A - N_{\mathrm{spec}}\right)$$

Collider

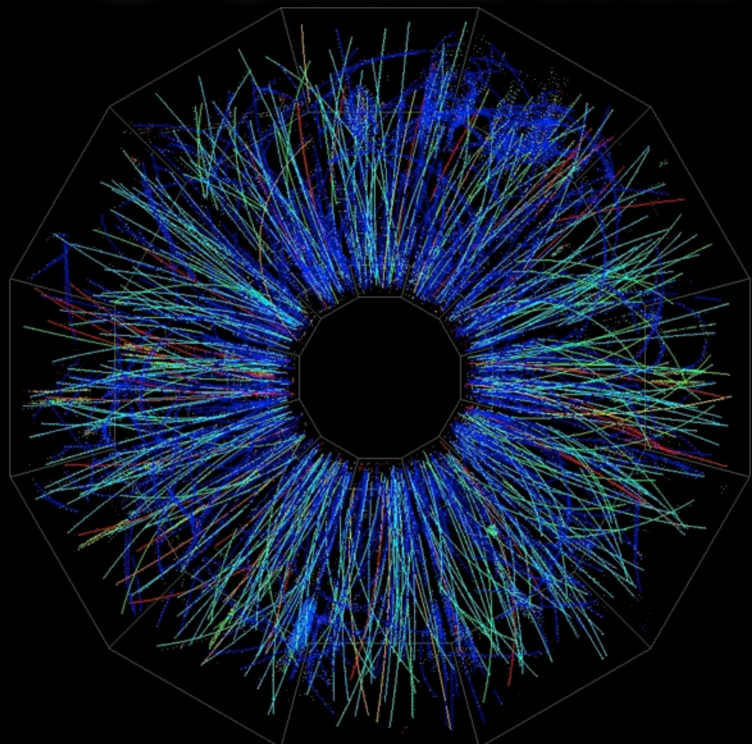


Zero-Degree-Calorimeter (ZDC) measures energy of all <u>unbound</u> spectator nucleons

➡ charged fragments (p, d, and heavier) are deflected by accelerator magnets

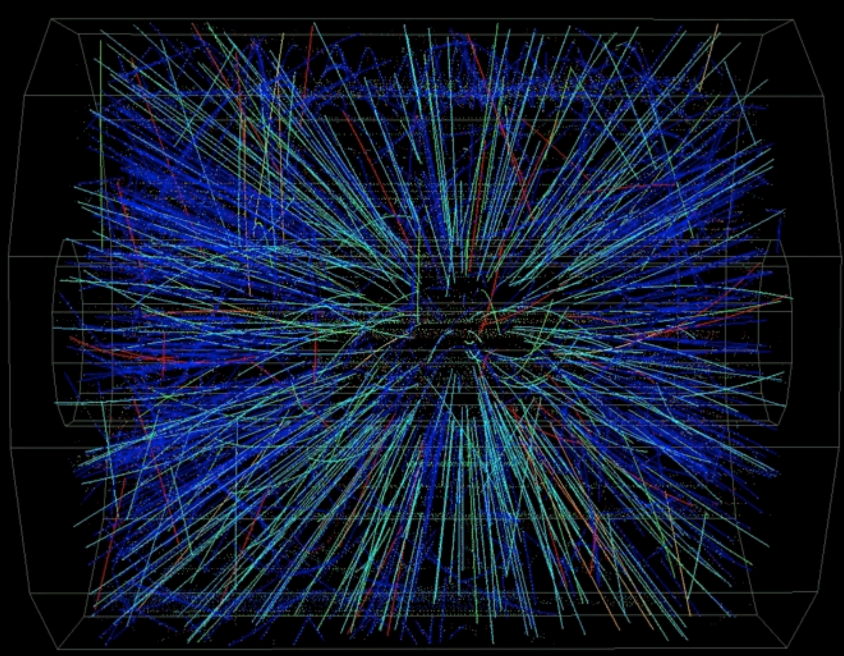
➡ E_{ZDC} small for very central and very peripheral collisions, ambiguous

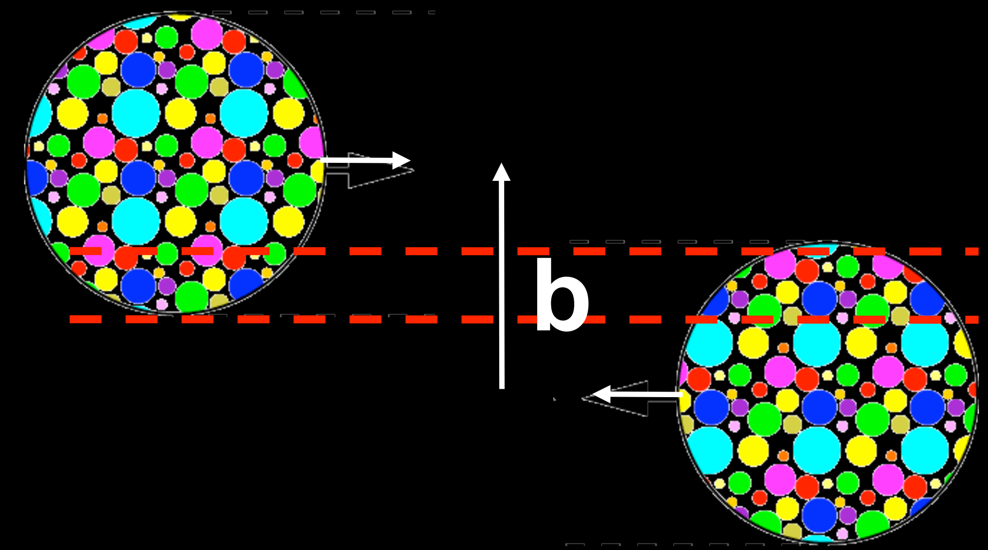
Centrality determination (III)



Peripheral Event

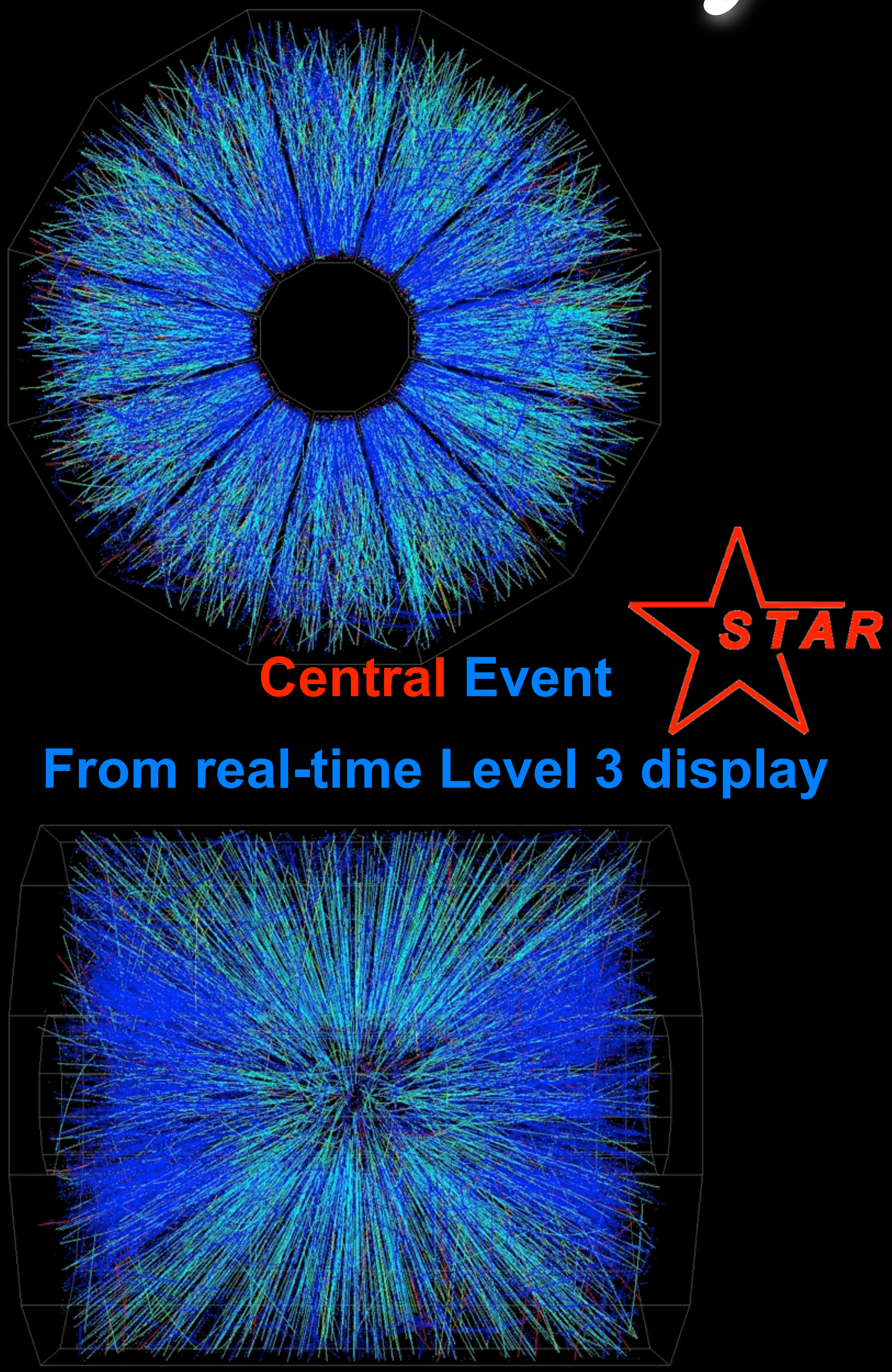
From real-time Level 3 display

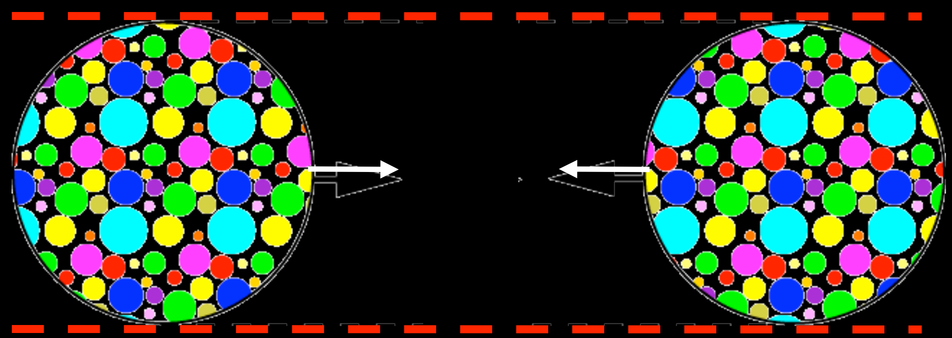




- ✓ peripheral collisions, largest fraction cross section
- ✓ many spectators
- ✓ “few” particles produced

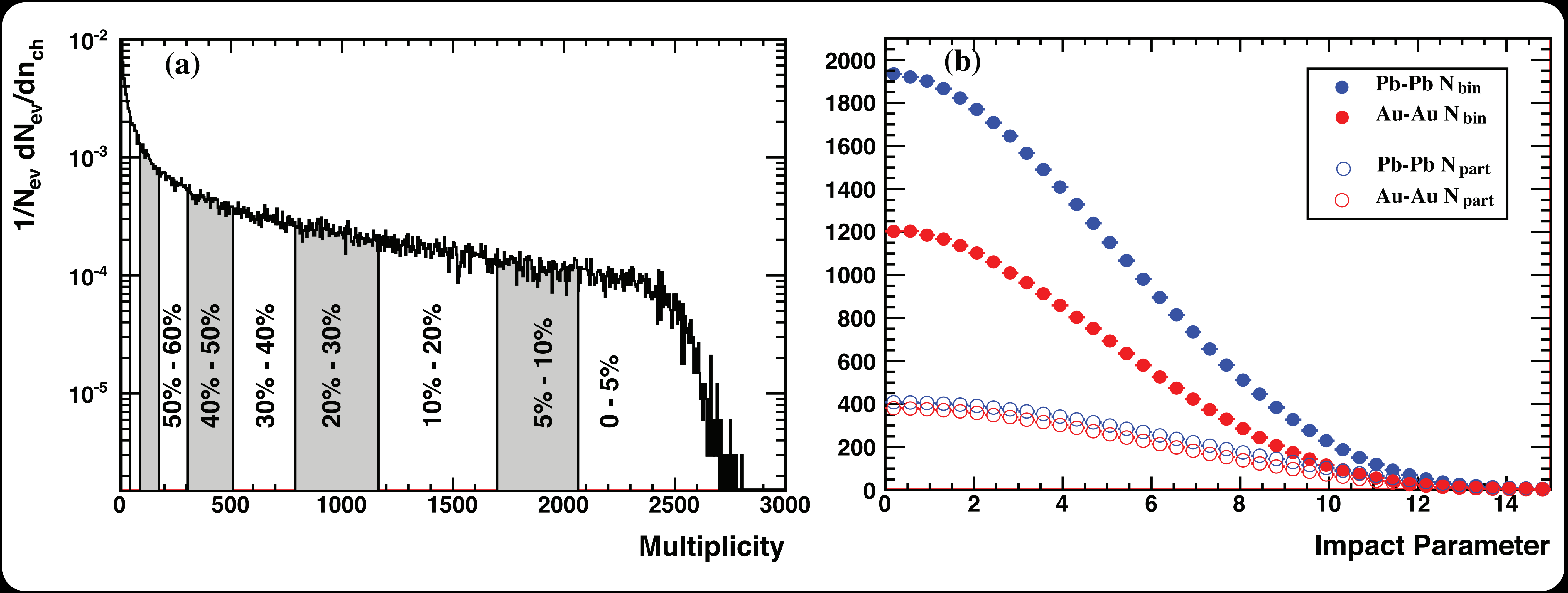
Centrality determination (IV)





- ✓ impact parameter **b** = 0
- ✓ central collisions, small cross section
- ✓ no spectators
- ✓ many particles produced

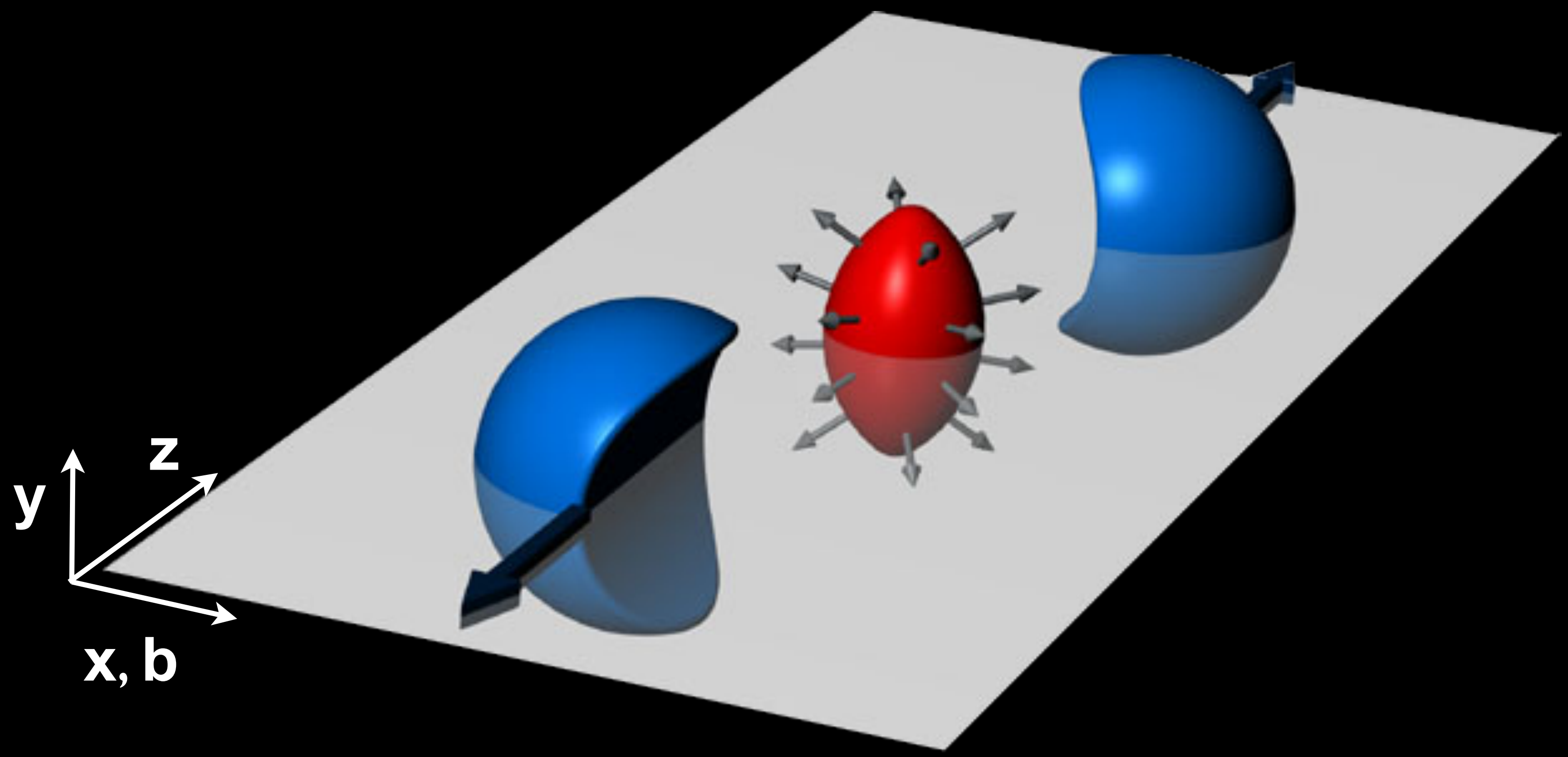
Centrality determination (ALICE)

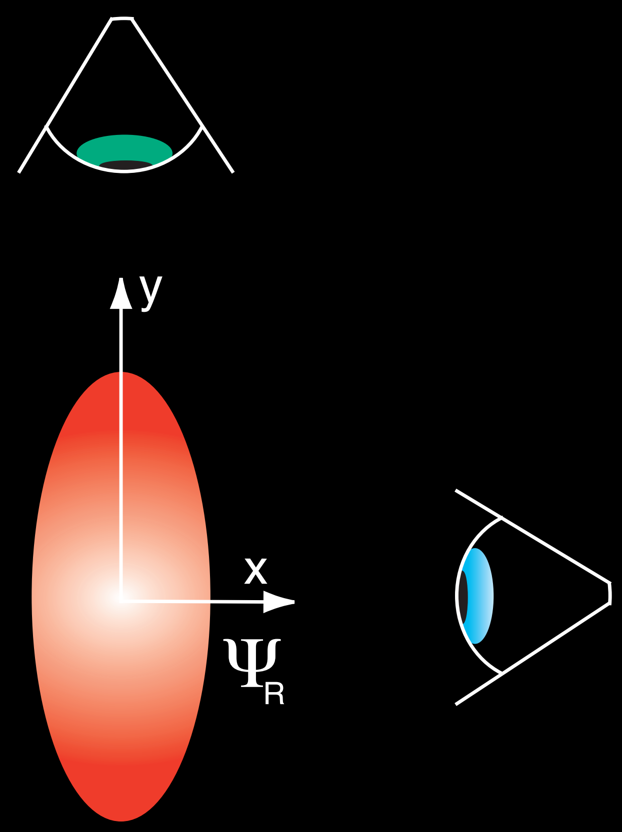


- ✓ Determines the magnitude of the impact parameter

$\%\sigma_{tot}$	$\langle N_{part} \rangle$	$\langle b \rangle$
0-5	386	2.48
20-30	177	7.85
60-70	25	12.66

The Reaction Plane

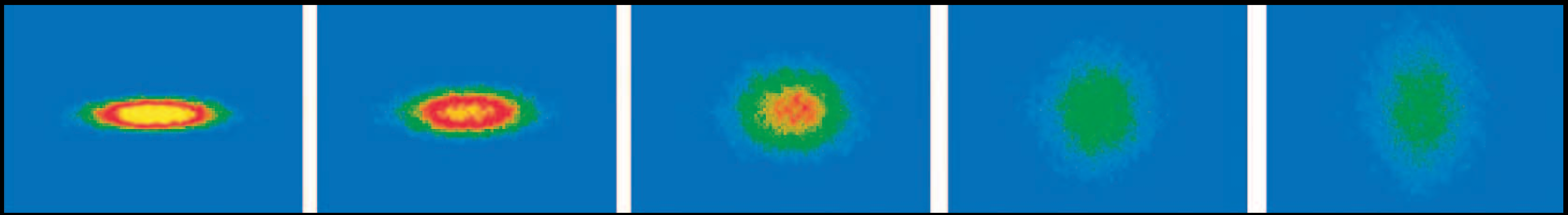




$$E\frac{d^3N}{d^3p} = \frac{d^3N}{p_t dp_t dy d(\phi - \Psi_R)}$$

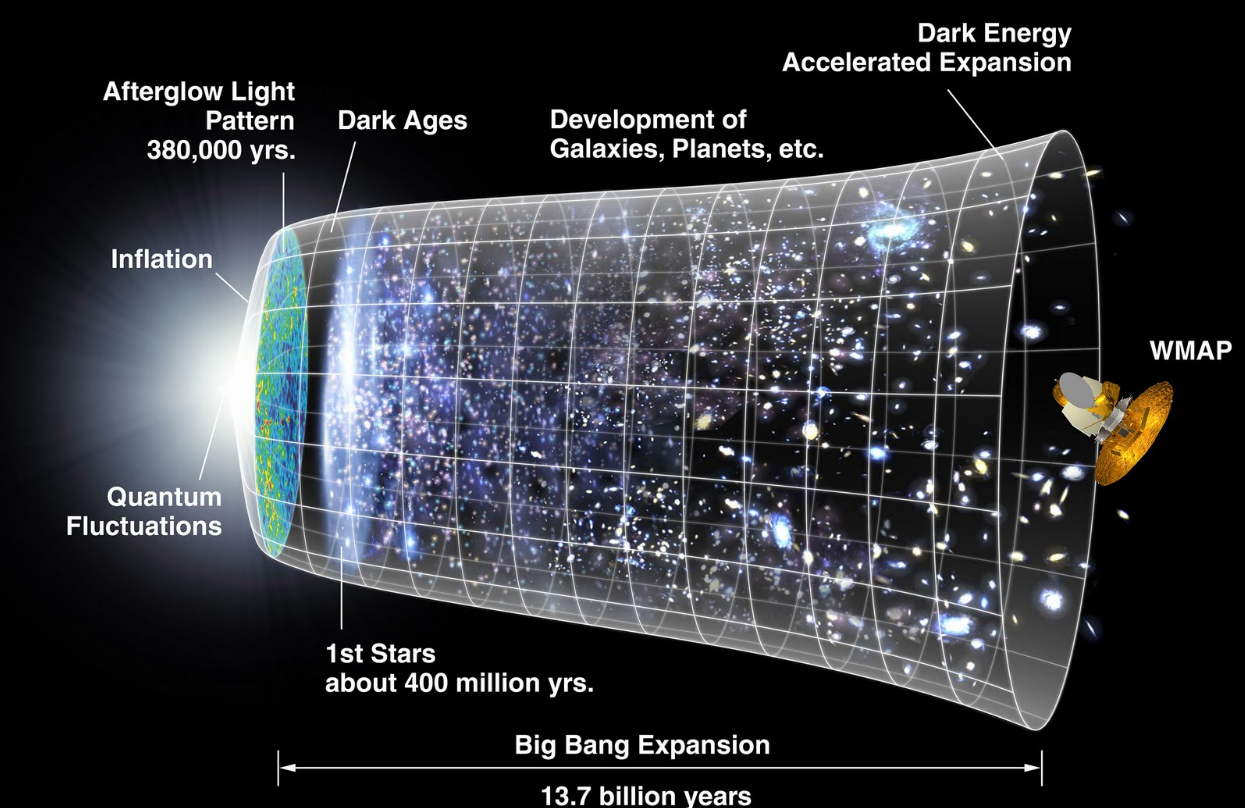
determine the angle of the reaction plane Ψ_R

How do we study these collisions?

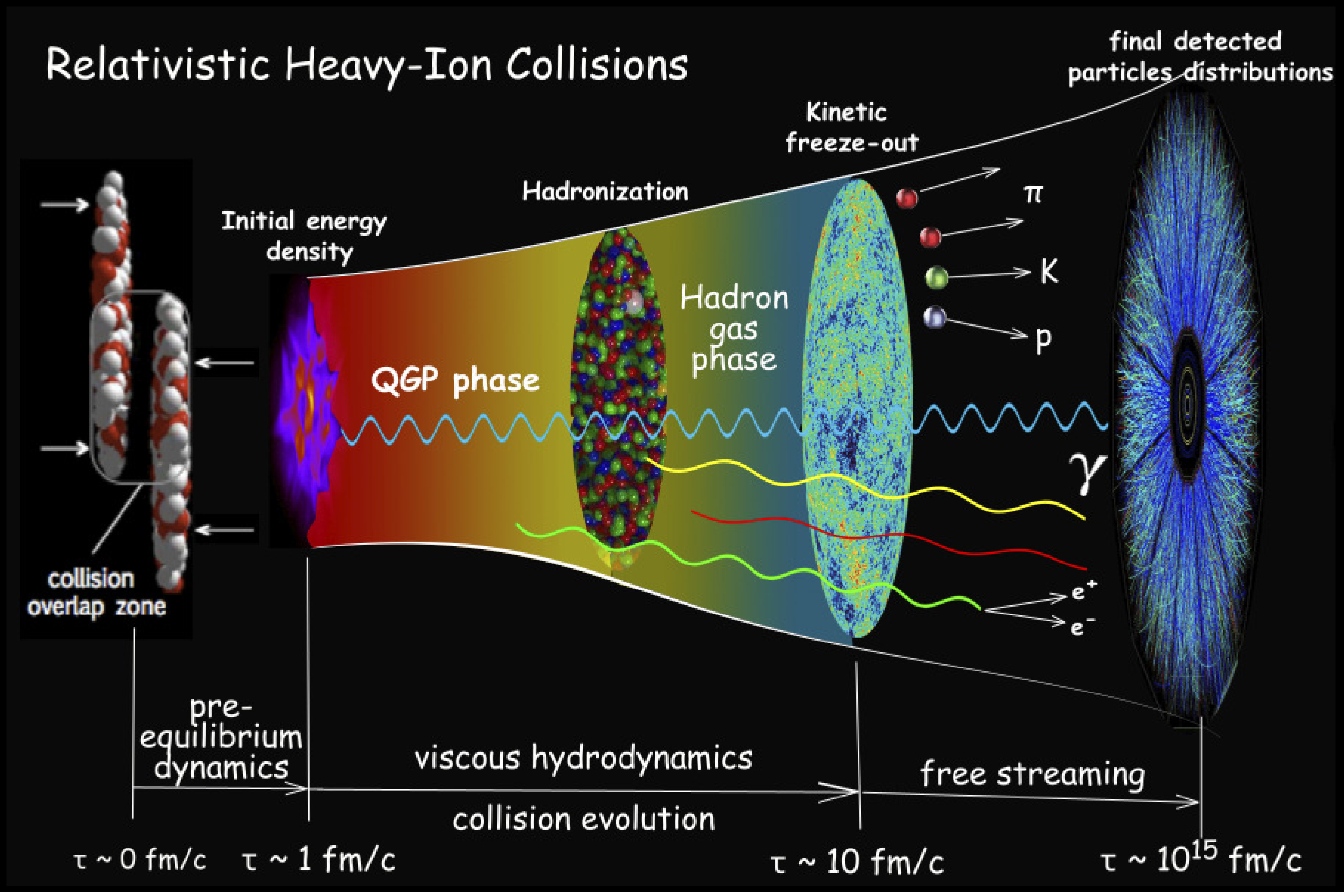


C. Cao et al, Science 331, 58 (2011)

ultracold strongly interacting Fermi gas



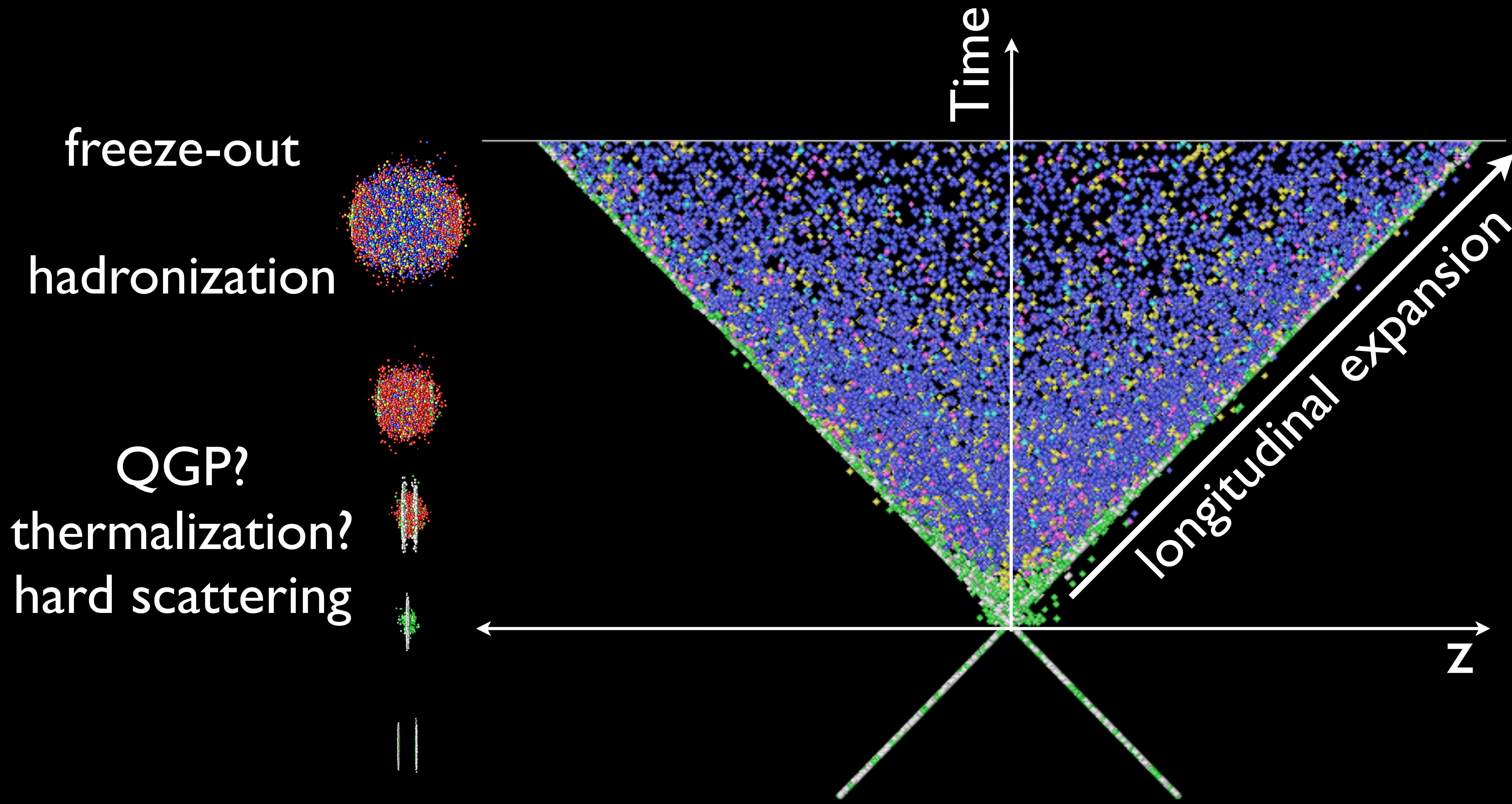
our Universe



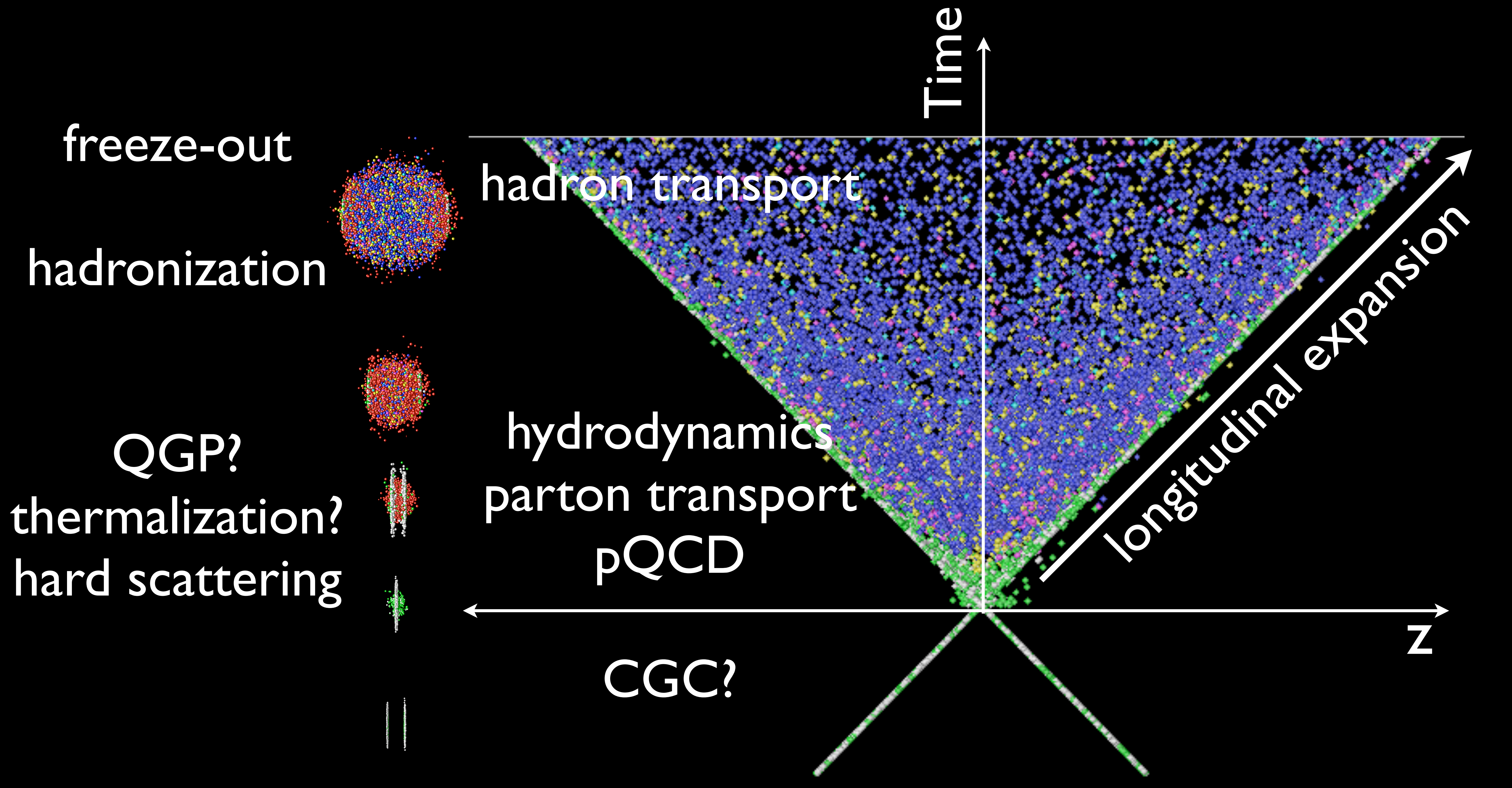
a heavy-ion collision

- What are the properties of the expanding hot and dense QCD matter?

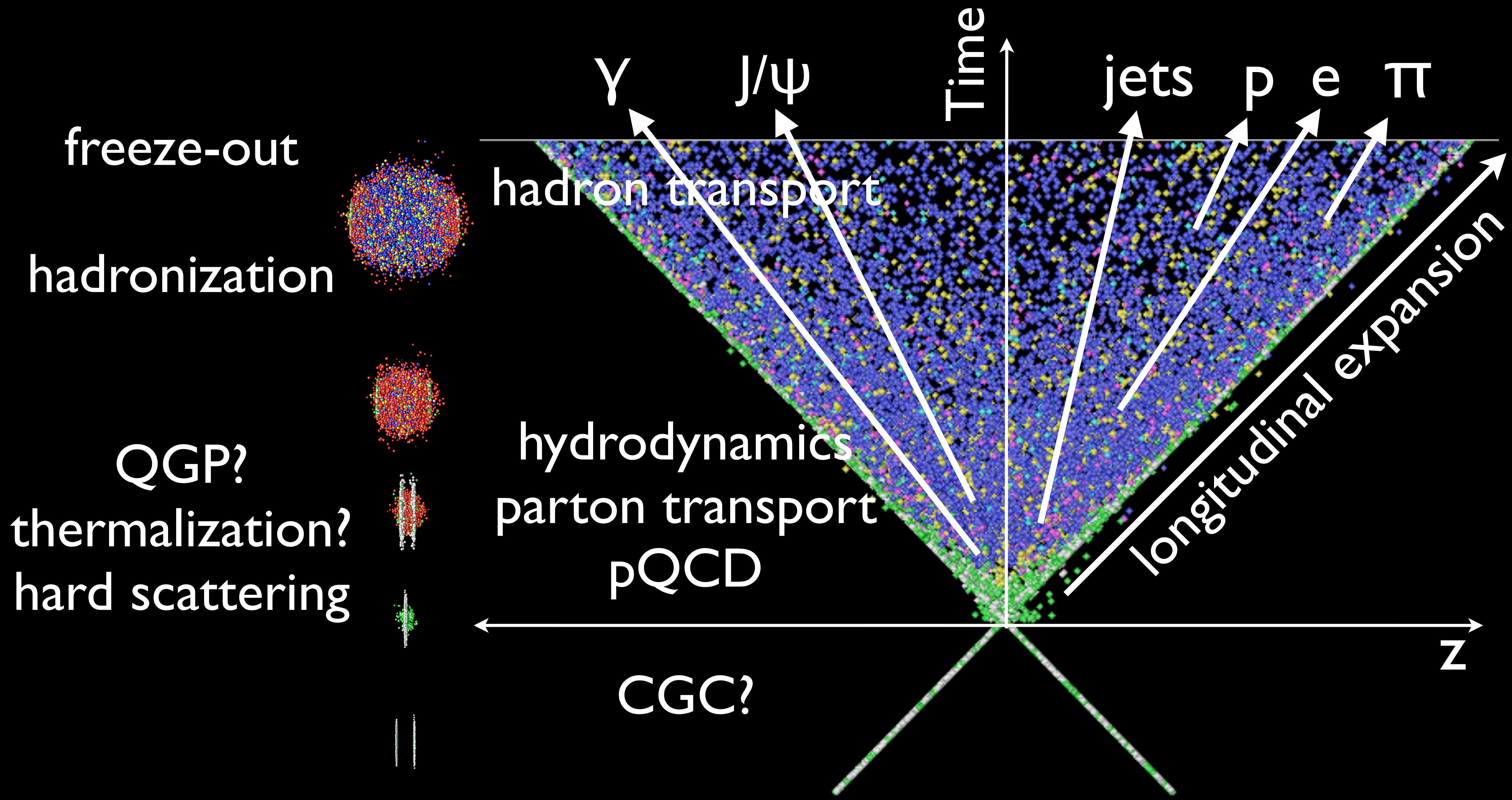
Time Evolution



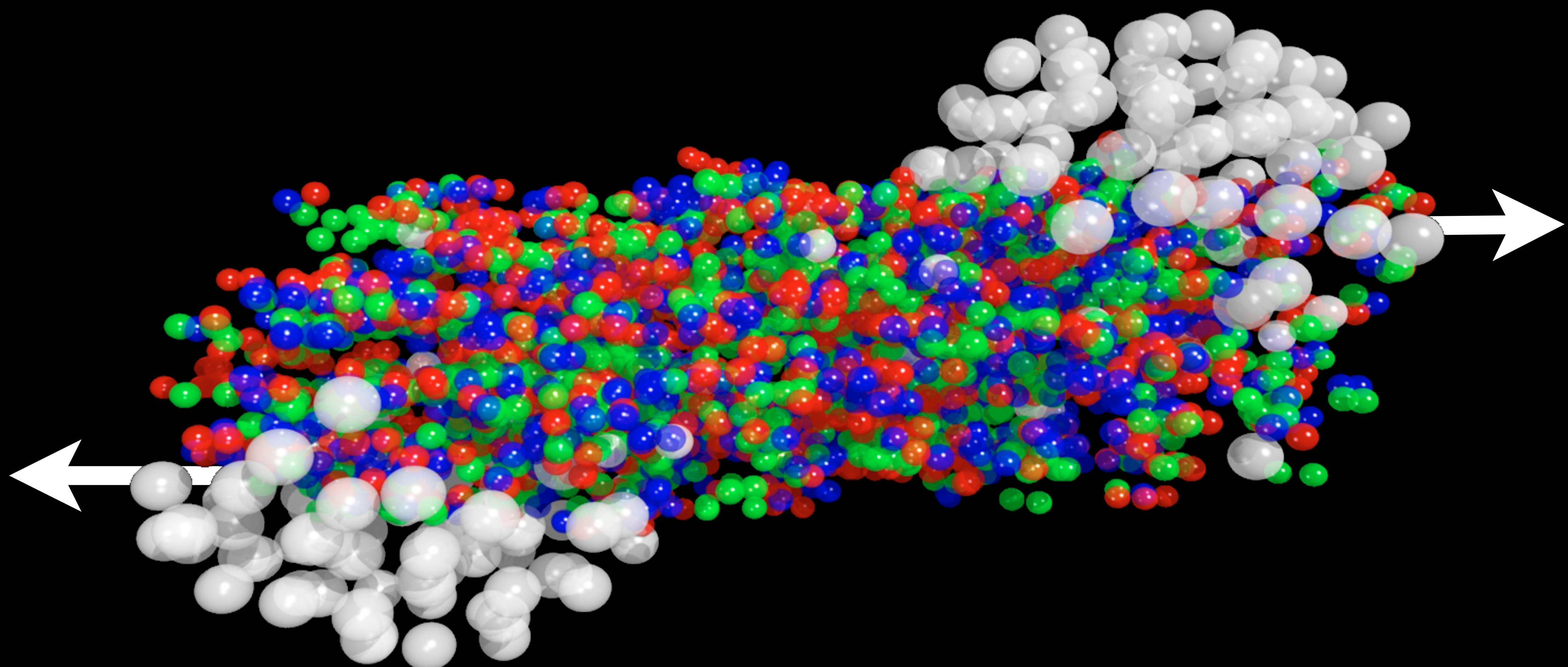
Time Evolution



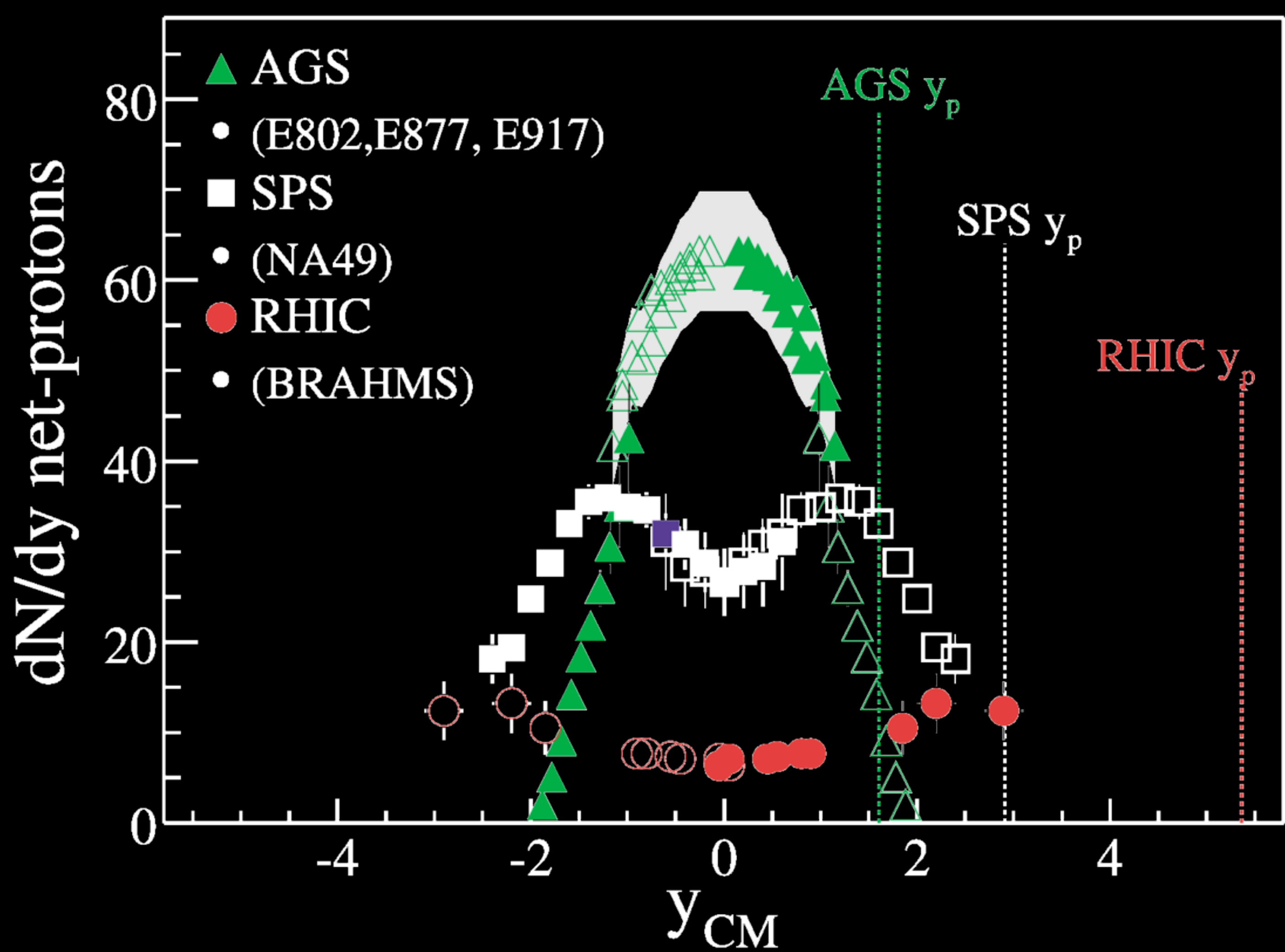
different observables

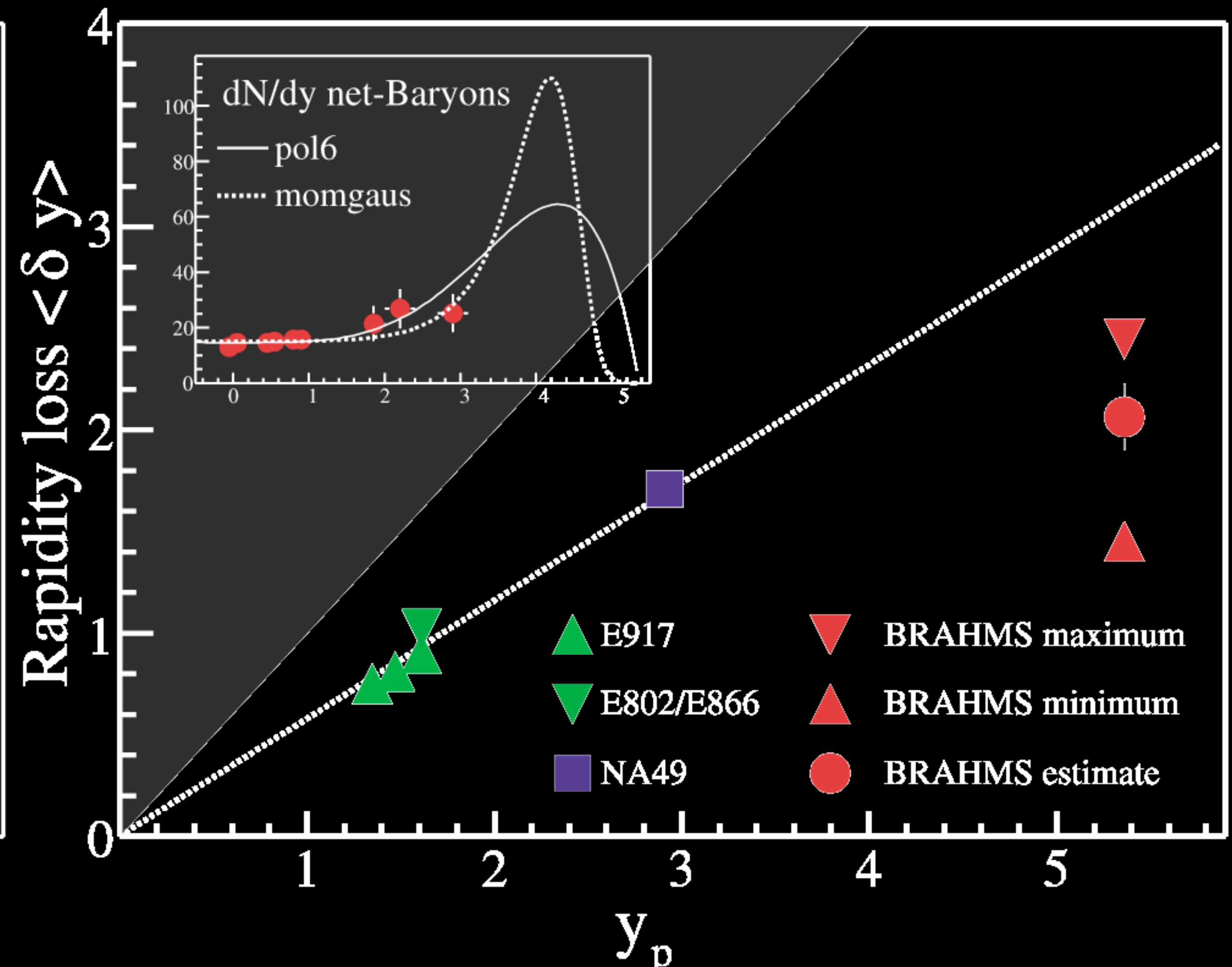


Stopping and Energy Density



Available Energy: Baryon-stopping

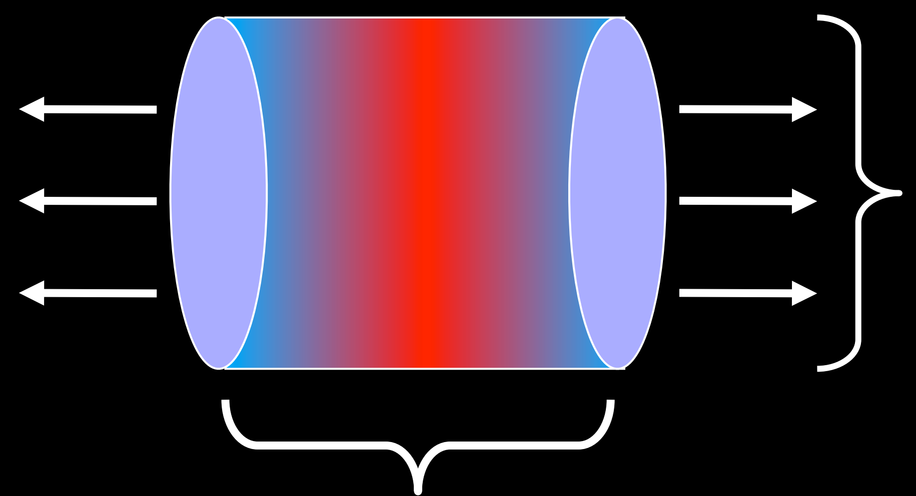


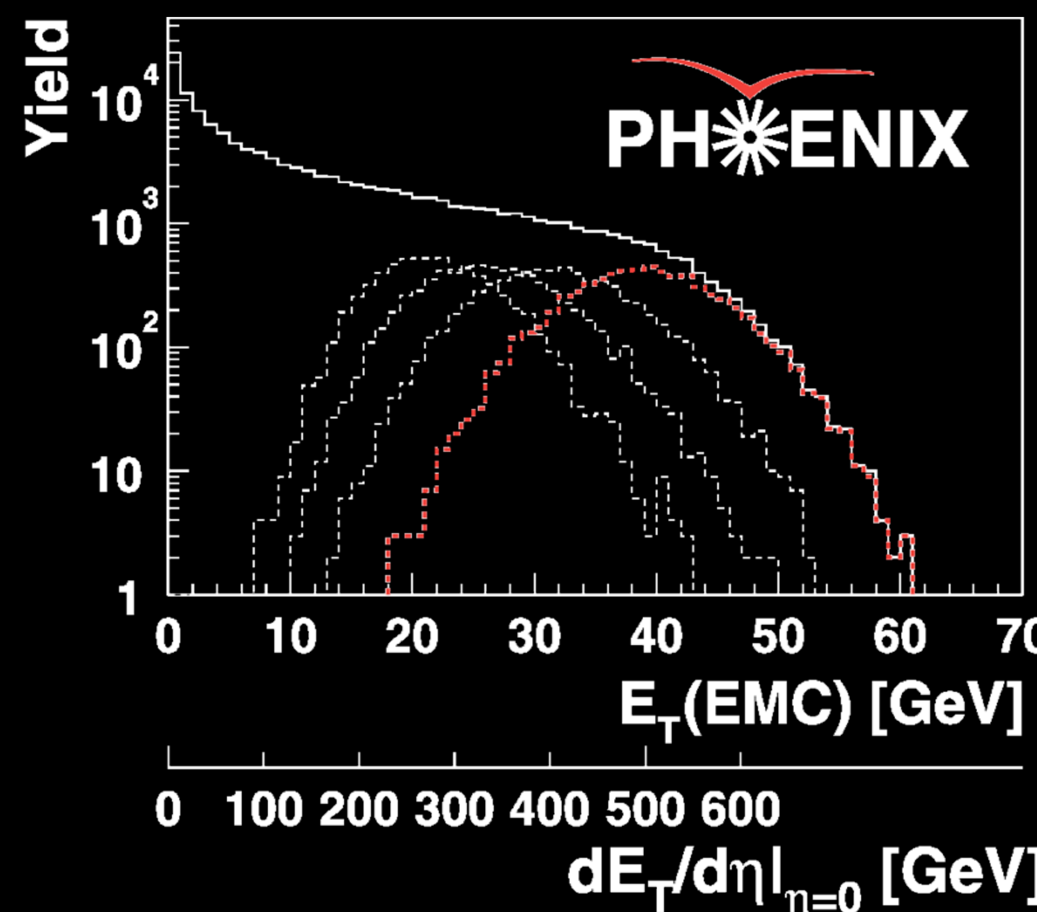


- In pp collisions 50% of beam energy available for particle production
- In AA collisions 70-80% of beam energy available for particle production (in accordance with expectations from pA)

Transverse Energy and Energy Density

Bjorken energy density estimate





$$\varepsilon_{\rm Bj} = \frac{1}{\pi R^2}\frac{1}{\tau}\frac{\rm dE_T}{\rm dy}$$

$$\frac{\rm dE_T}{\rm dy} = 503 \pm 2 \;\; \rm GeV$$

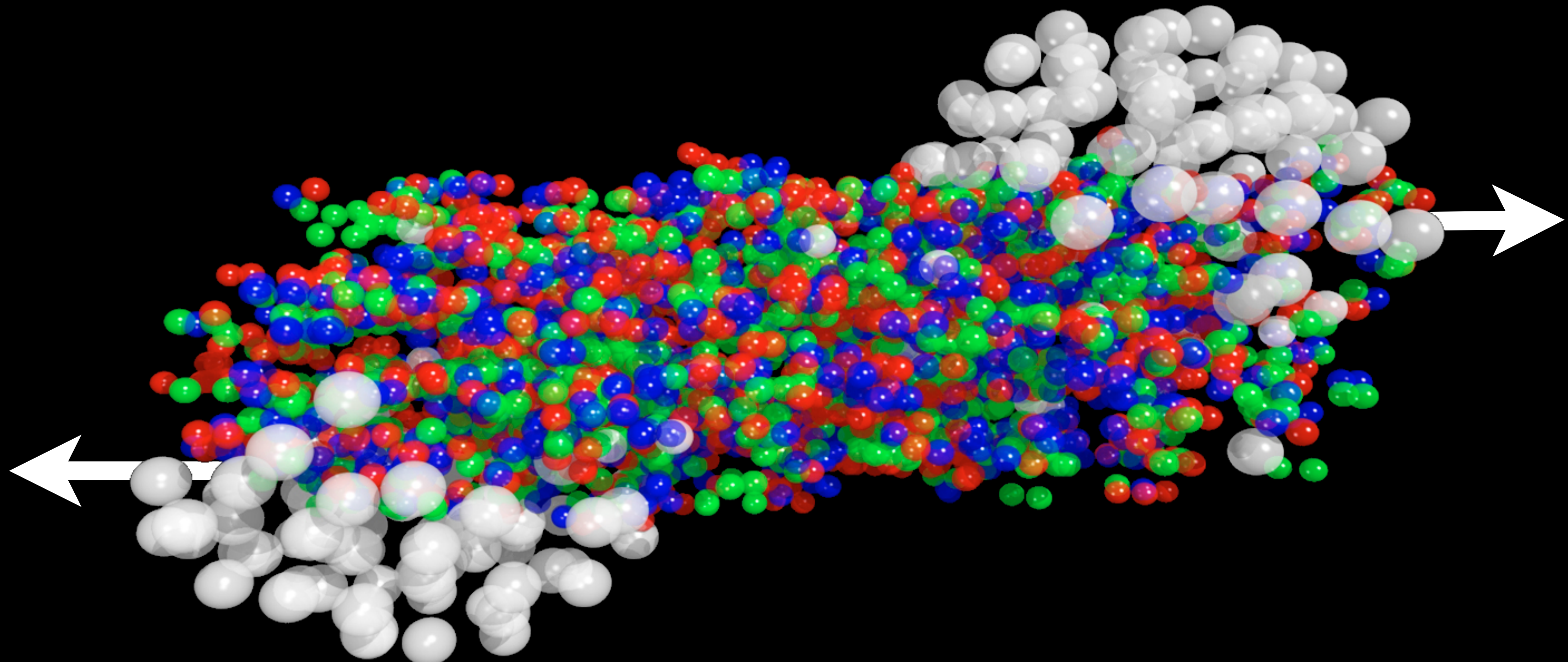
$$\varepsilon_{\rm Bj} = 4.6 \;\; \rm GeV/fm$$

Much larger than the critical energy density!

Do we produce a QGP?

are there smoking guns?

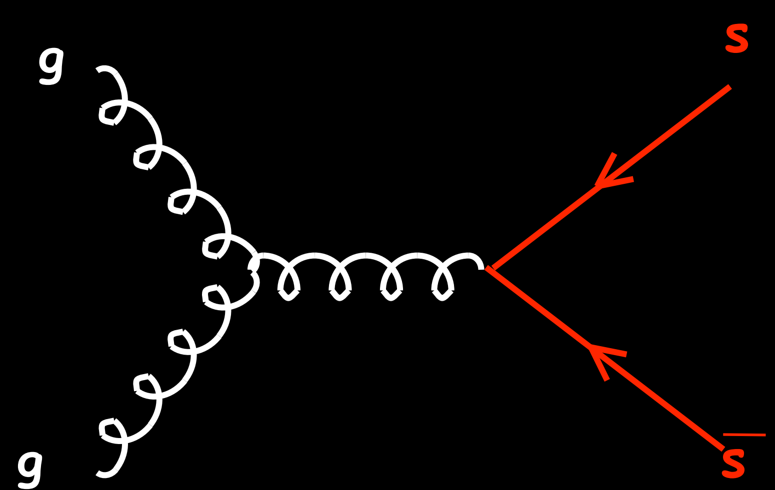
Strangeness Enhancement



Strangeness Enhancement

- QGP signature proposed by Rafelski and Muller, 1982
- The masses of deconfined quarks are expected to be about 350 MeV lower compared to confined
- m_s(constituent) ~ 500 MeV → m_s(current) ~ 150 MeV
- T_c ~ 170 MeV strange quark should be a sensitive probe

Strangeness Production in a QGP



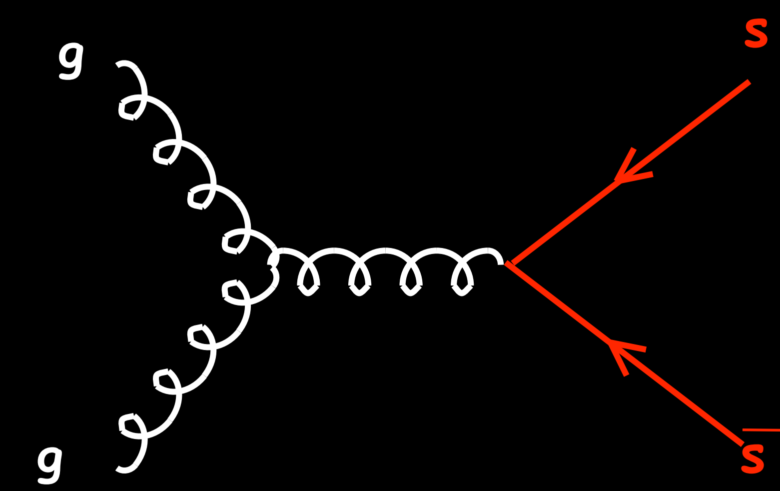
$$T \approx m_s = 150 \text{ MeV}$$

$$N(s) \propto \exp\left(-\frac{m_s}{T}\right)$$

- Copious strangeness production by gluon fusion:
- In a system which is baryon rich (i.e. an access of quarks over anti-quarks), the enhancement can be further enhanced due to Pauli blocking of light quark production

Strangeness abundances in a QGP

- The QGP strangeness abundance is enhanced
- The strange quarks recombine into hadrons (when the QGP cools down and hadronizes)
- The abundance of strange hadrons should also be enhanced
- This enhancement should be larger for particles of higher strangeness content



E(Ω^-) >	E(Ξ^-) >	E(Λ)
(sss)	(ssd)	(sud)

Strangeness abundances in a hadron gas

- In a relatively long lived strongly interacting hadronic system strangeness can also be enhanced
- These hadronic processes are relatively fast and easy for kaons and Λ, but progressively harder for particles of higher strangeness
- The production of multi-strange baryons is expected to be sensitive to deconfinement

$$\pi + \pi \circledR \; K + \bar{K}$$

$$\pi + N \rightarrow \Lambda + K$$

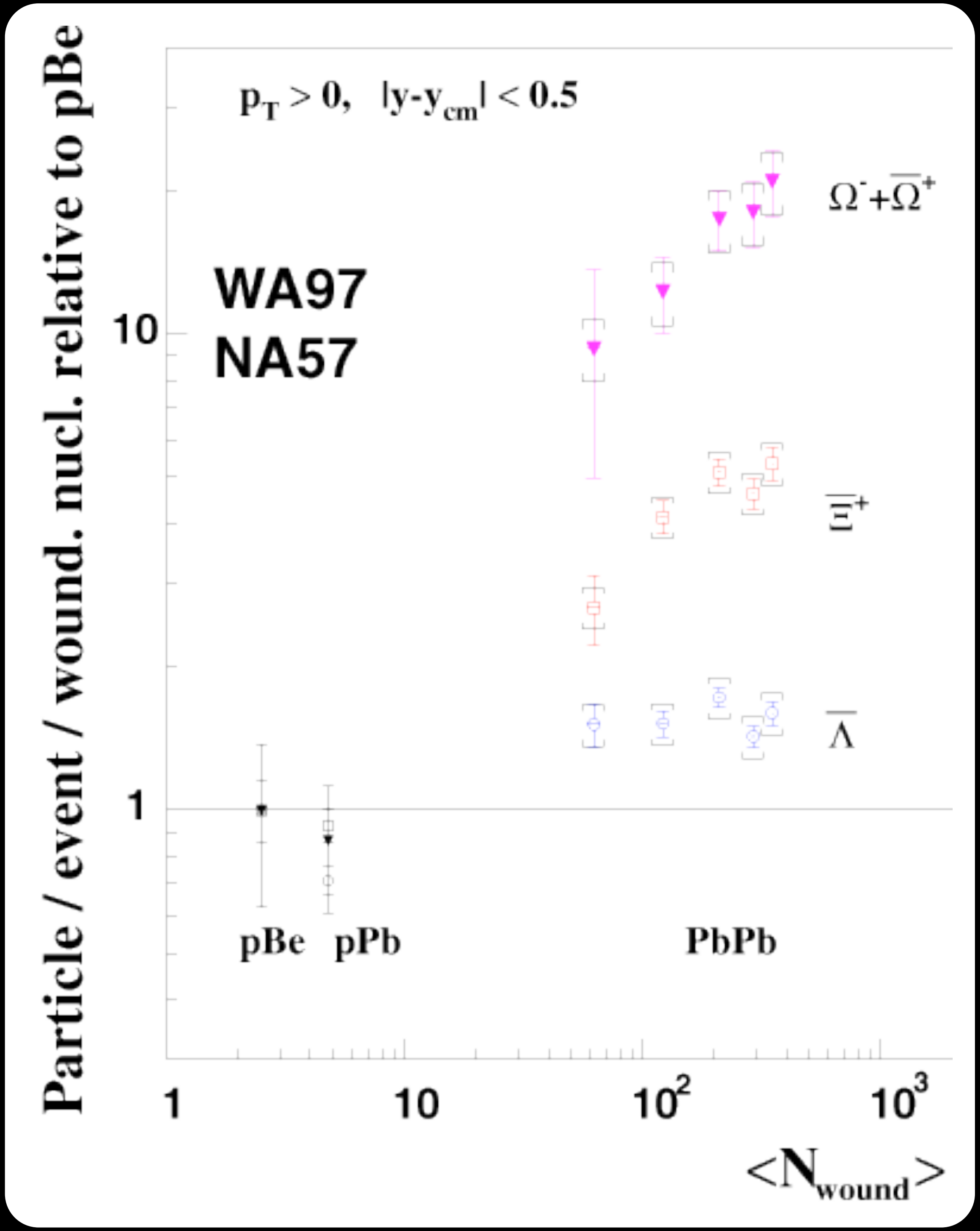
E(Ω^-) <	E(Ξ^-) <	E(Λ)
(sss)	(ssd)	(sud)

only 2→2 processes considered!!

Strangeness measurement at the SPS

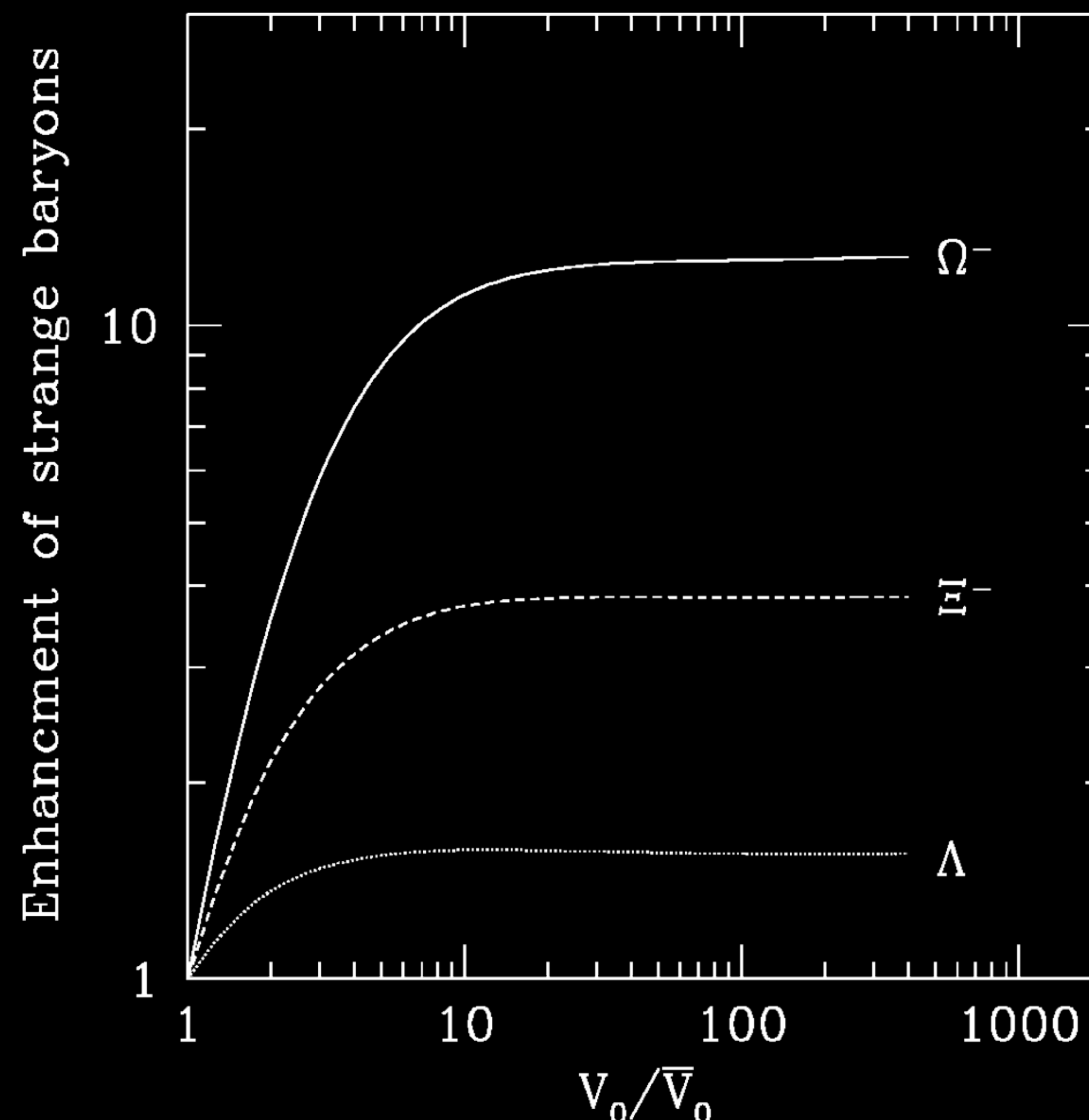
$$E_{\Omega^-} = \frac{\left(N_{\Omega^-} / \langle N_{\text{wounded}} \rangle\right)_{Pb+Pb}}{\left(N_{\Omega^-} / \langle N_{\text{wounded}} \rangle\right)_{p+Be}}$$

- Enhancement: yield per participant relative to yield per participant in p-Be
- The Ω yield is more than a factor 20 enhanced
- Relative order follows QGP prediction



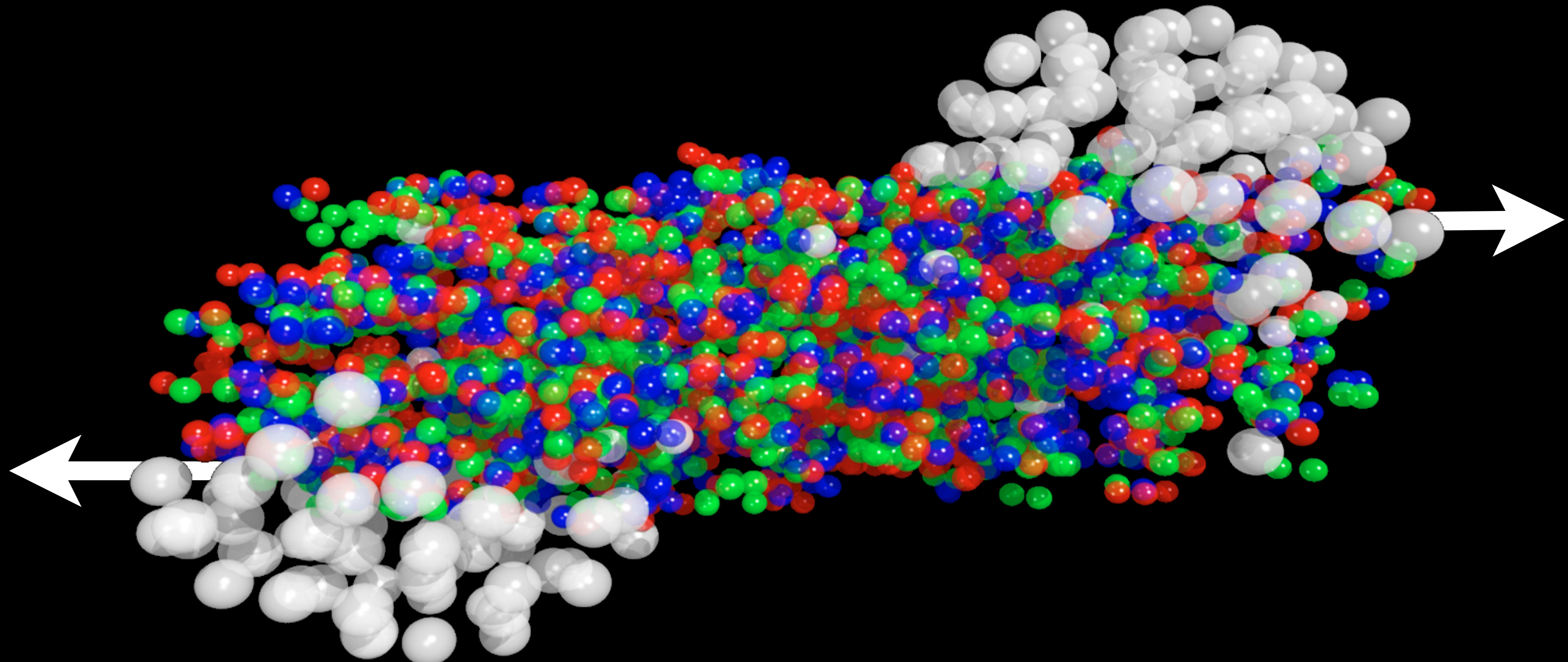
Canonical Suppression of Strangeness

- Successful description of strangeness production in heavy ion collisions with a thermal model using a grand canonical ensemble
- For small systems exact strangeness conservation becomes important, canonical ensemble, reduces available phase space



Strangeness enhancement is not necessarily a smoking gun

Charmonium Suppression



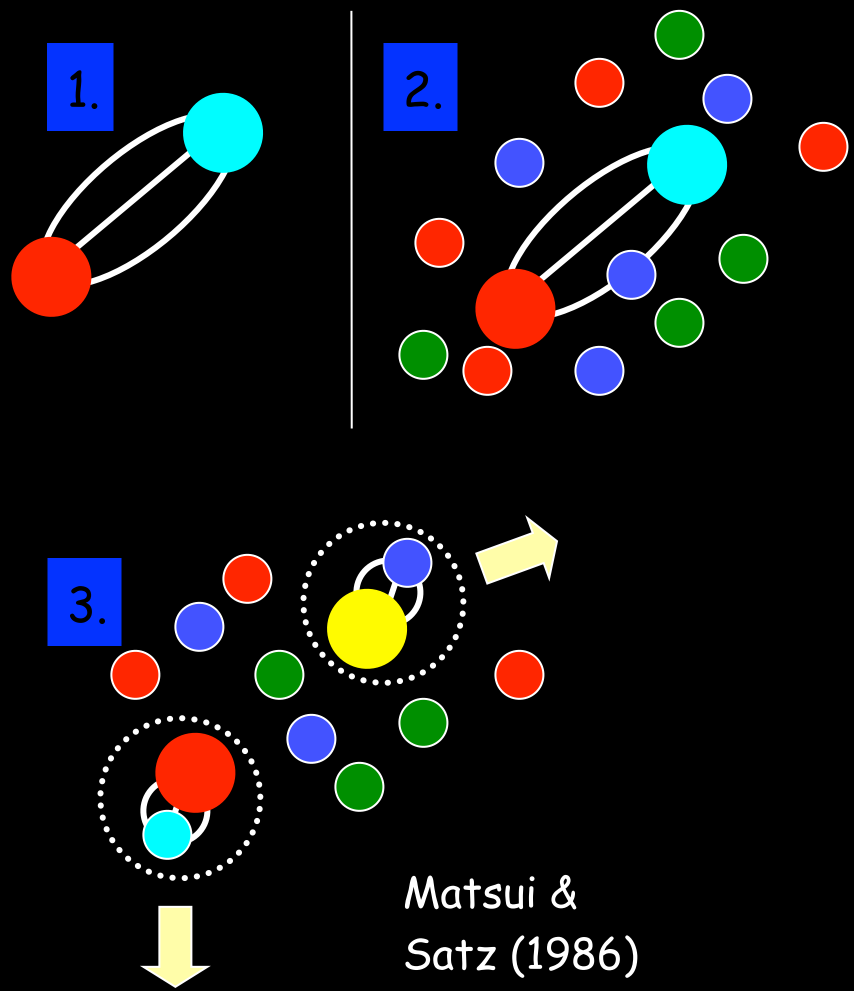
Charmonium Suppression

- a QGP signature predicted by Matsui and Satz, 1986
- In the plasma phase the interaction potential is expected to be screened beyond the Debye length λ_D (analogous to e.m. Debye screening)
- Charmonium (cc_{bar}) and bottonium (bb_{bar}) states with $r > \lambda_D$ will not bind; their production will be suppressed

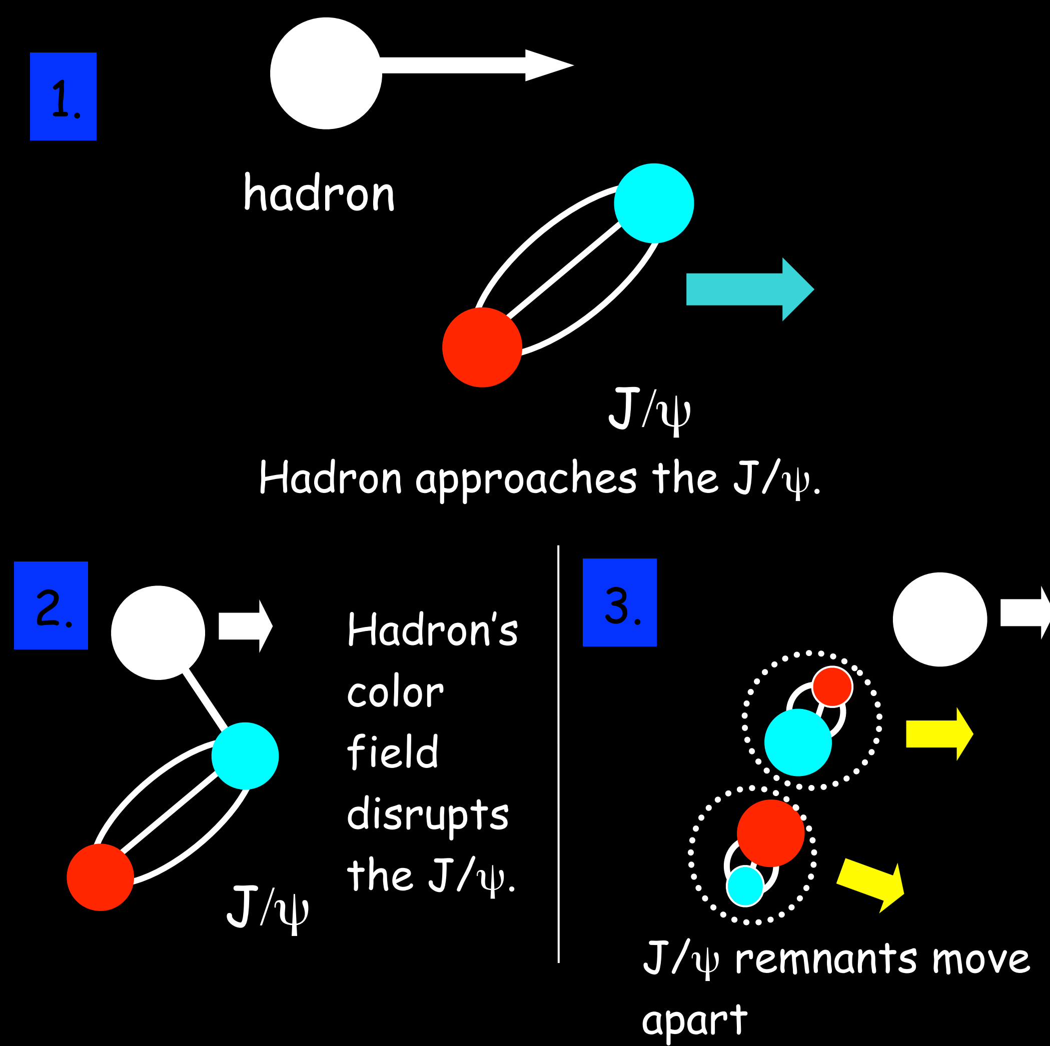
Charmonium Suppression

- λ_D depends on temperature, thus which states are suppressed depends on temperature
- Charmonium suppression key signature of deconfinement
- cc_{bar} and bb_{bar} bound states are particularly sensitive probes because the probability of combining an uncorrelated pair at the hadronization stage is small
- In fact, at the SPS the only chance of producing a cc_{bar} bound state is shortly after the pair is produced. Debye screening destroys this correlations

Sources of Suppression



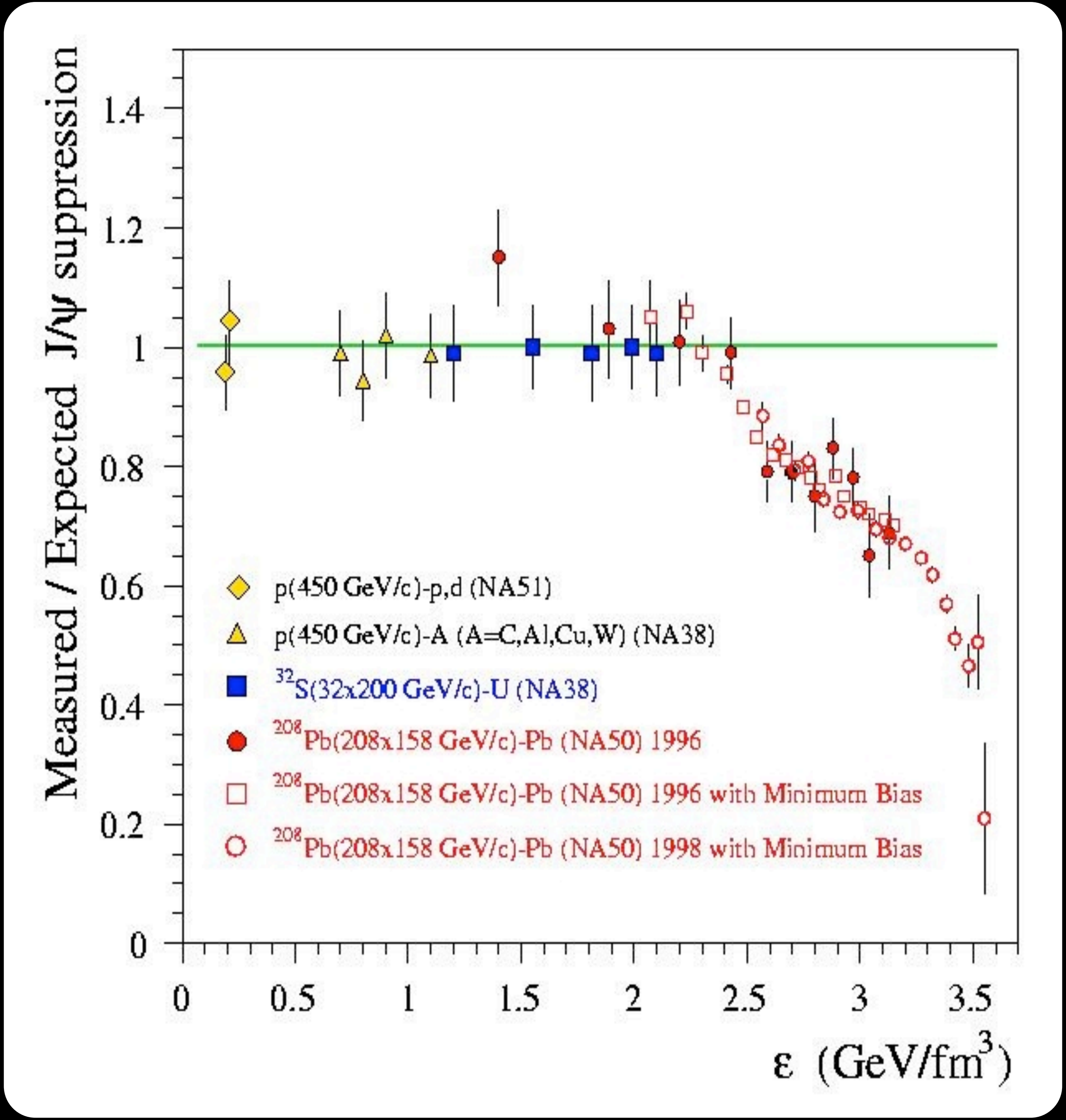
Debye screening of the J/ψ



Co-movers suppressing the J/ψ

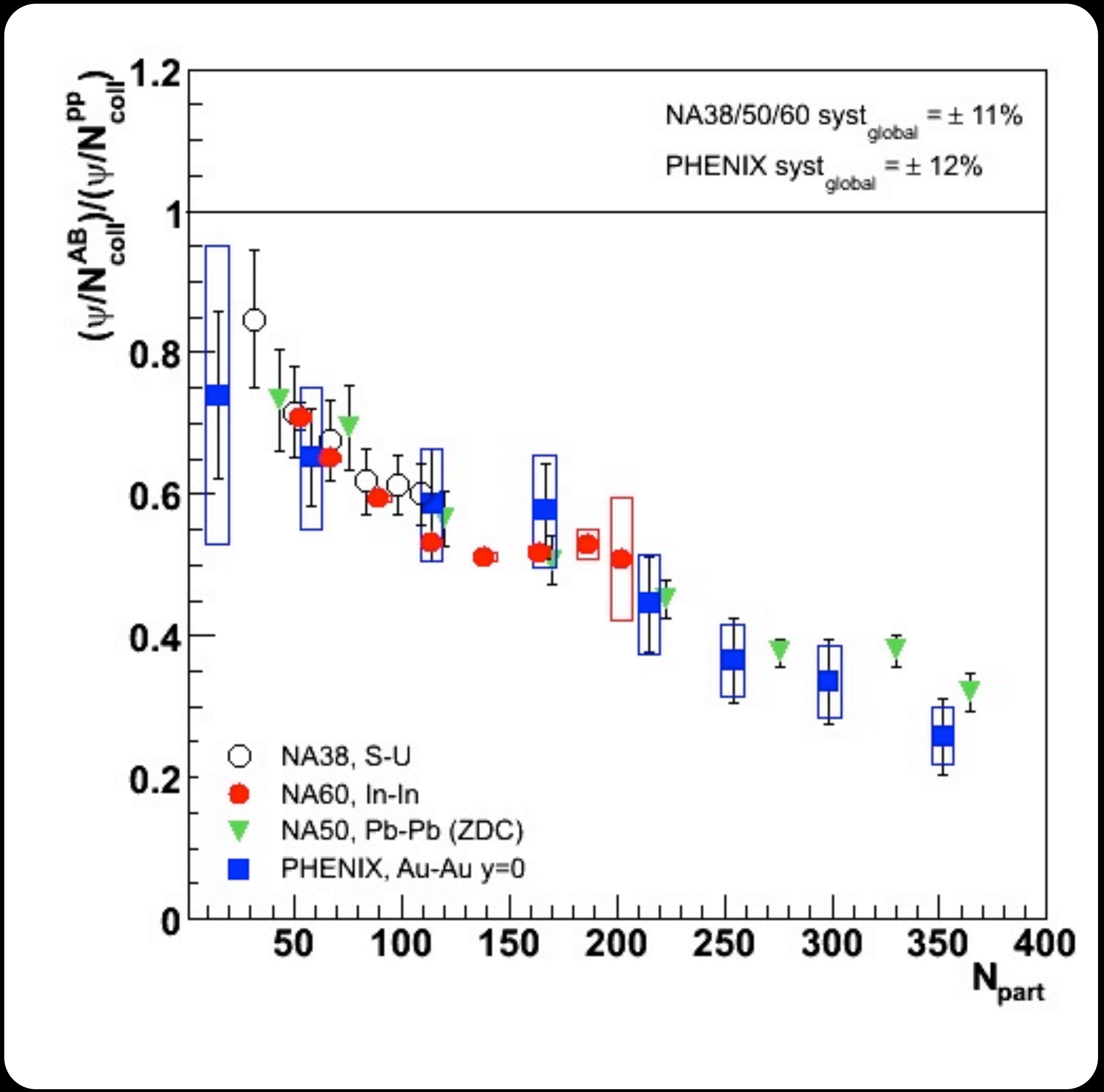
The J/Ψ measurement at the SPS

- Measured/expected J/Ψ suppression versus estimated energy density
  - Anomalous suppression sets in at ε~ 2.3 GeV/fm^3
  - Double step was interpreted as successive melting of the χ_C and of the J/Ψ



The J/Ψ measurement at RHIC

- Suppression pattern almost the same as at the SPS???
- J/Ψ production at RHIC is more complicated due to possible contributions from coalescence



See lectures PBM

End Lecture 1

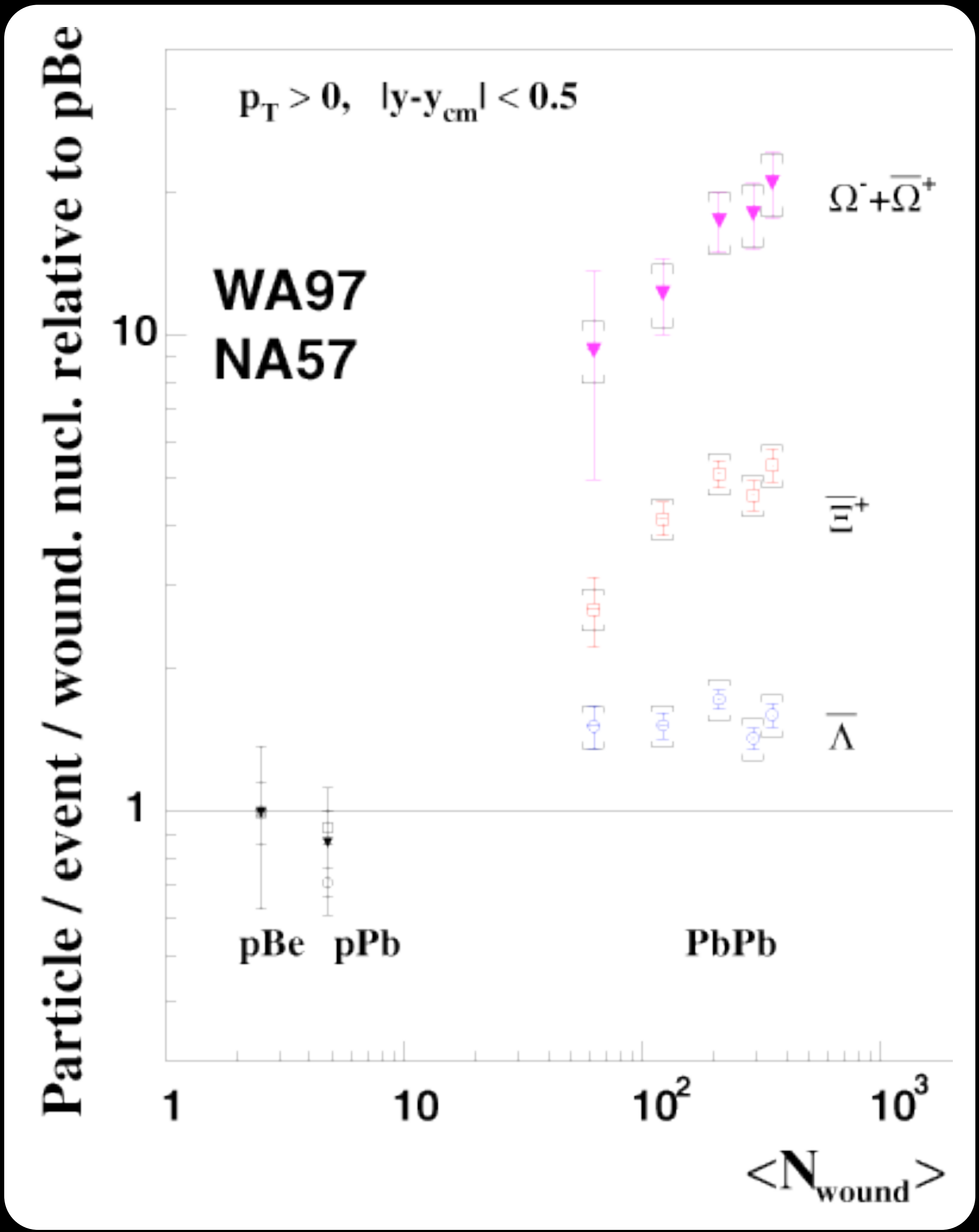
Content (lecture 1-3)

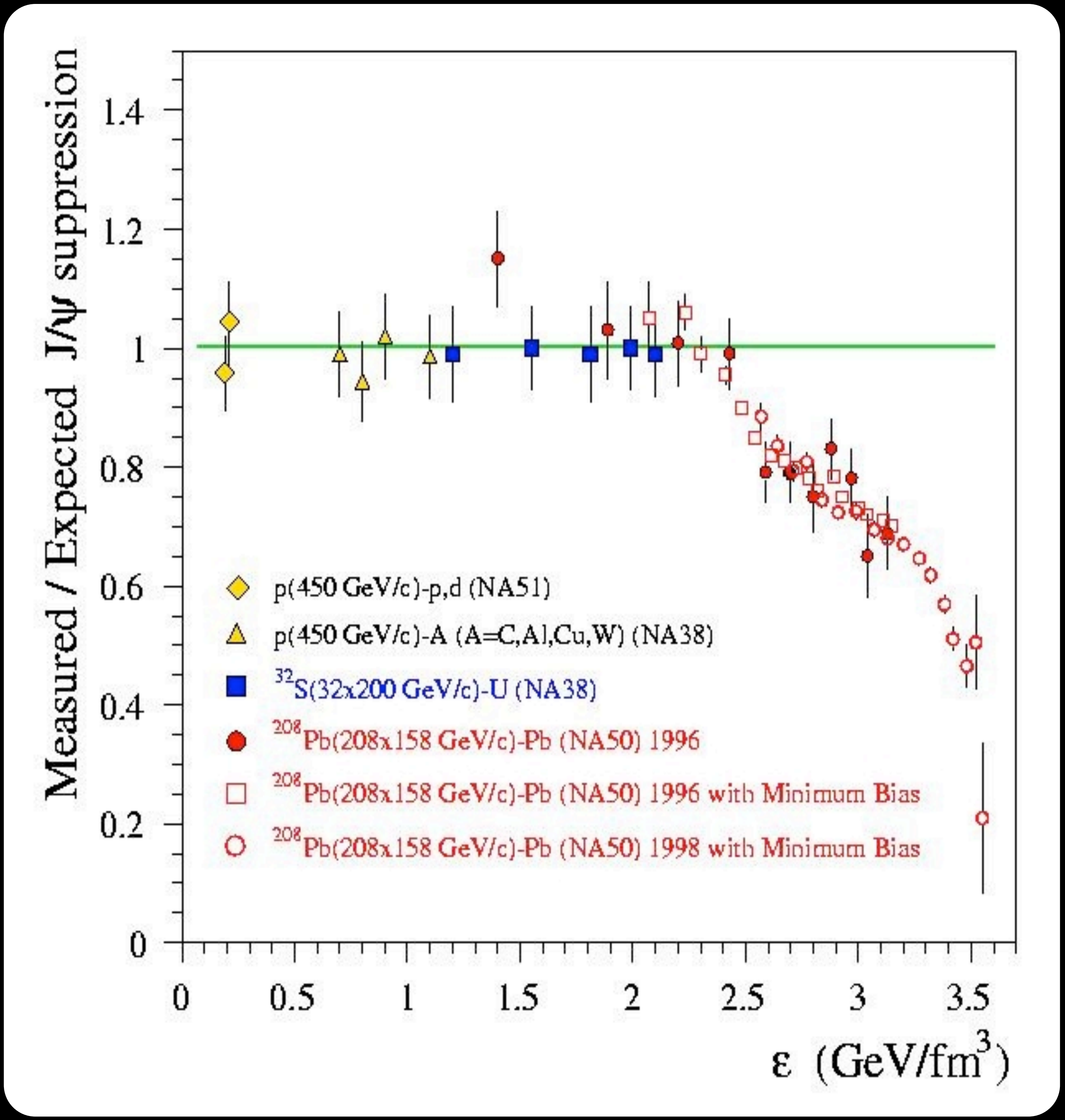
1) Heavy Ion Collisions

2) Thermal Model and Collective Flow

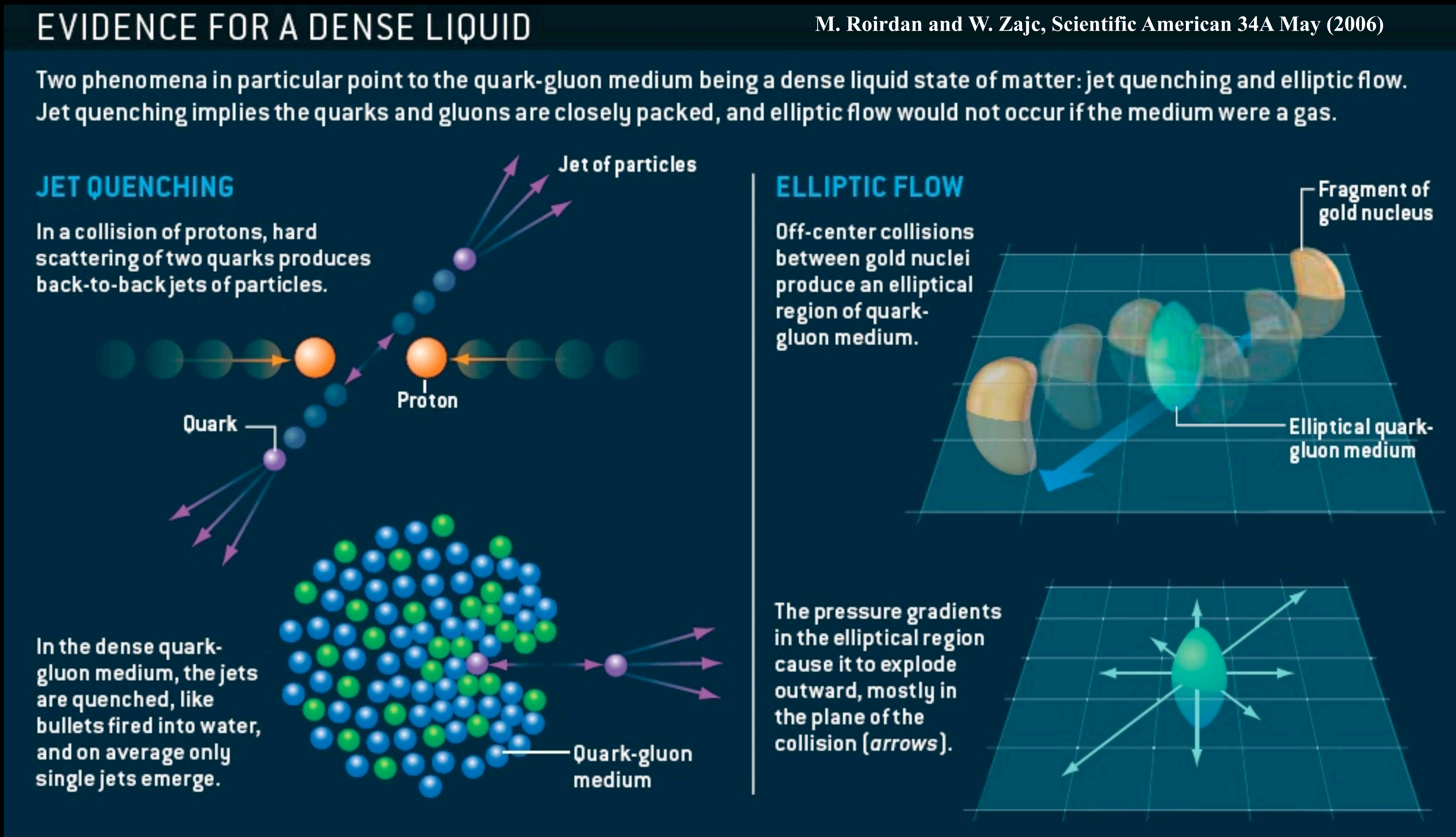
3) Current results

some highlights at SPS



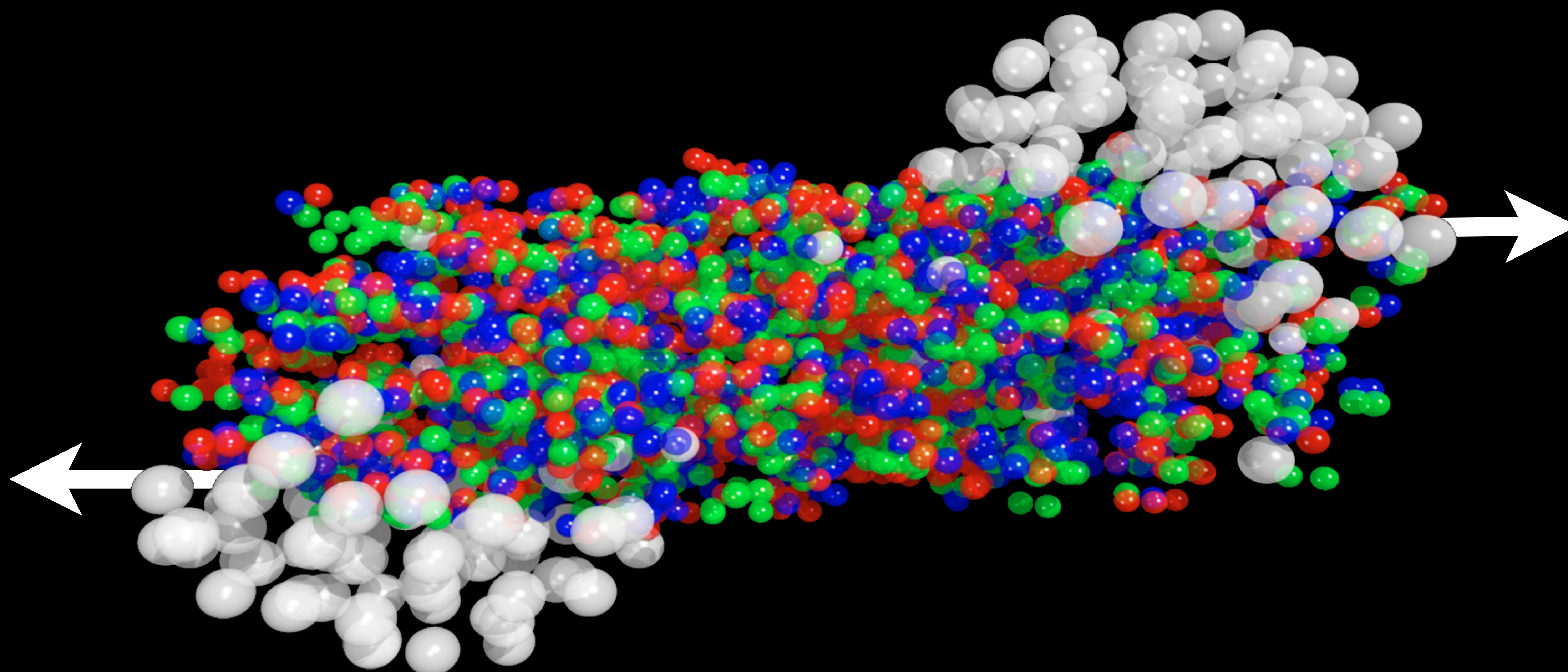


highlights at RHIC

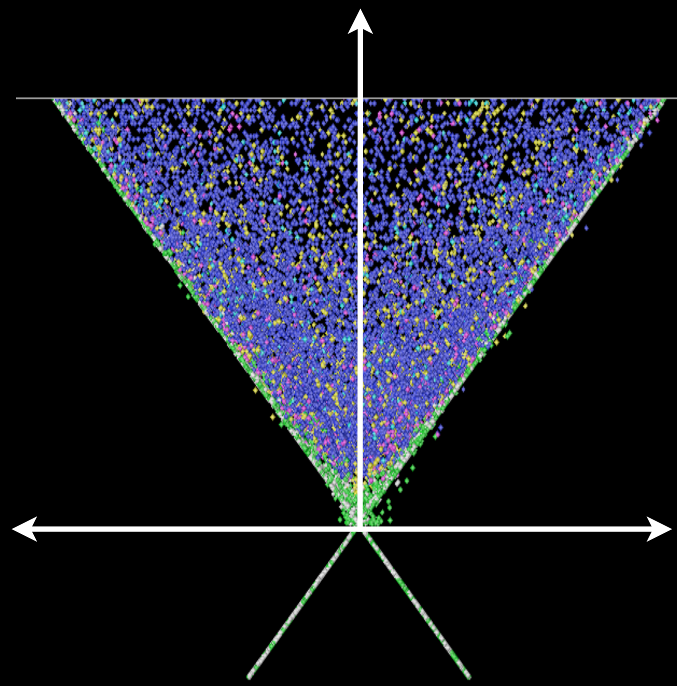


M. Roirdan and W. Zajc, Scientific American 34A May (2006)

Particle Yields



Particle Yields



- Chemical freeze-out
  - “sharp” end of inelastic collisions
  - particle yields are fixed
  - entropy of the system stays constant
- Chemical temperature
  - if system is in local equilibrium the chemical temperature is the temperature of the system at hadronization

Ensembles of Statistical Mechanics

- Microcanonical Ensemble
    - describes and isolated system
- Canonical Ensemble
    - describes a system in contact with a heat bath
    - T is constant, Energy can be exchanged
- Grand Canonical Ensemble
    - describes a system in contact with a heat and particle bath
    - T is constant, Energy can be exchanged, number of particles can change

- For these systems we can define
    - Ps: probability of observing the System in energy state Es
    - Z: the partition function which describes how the probability is distributed among the states
    - S: entropy of the system S = k log(Z), Boltzmann law

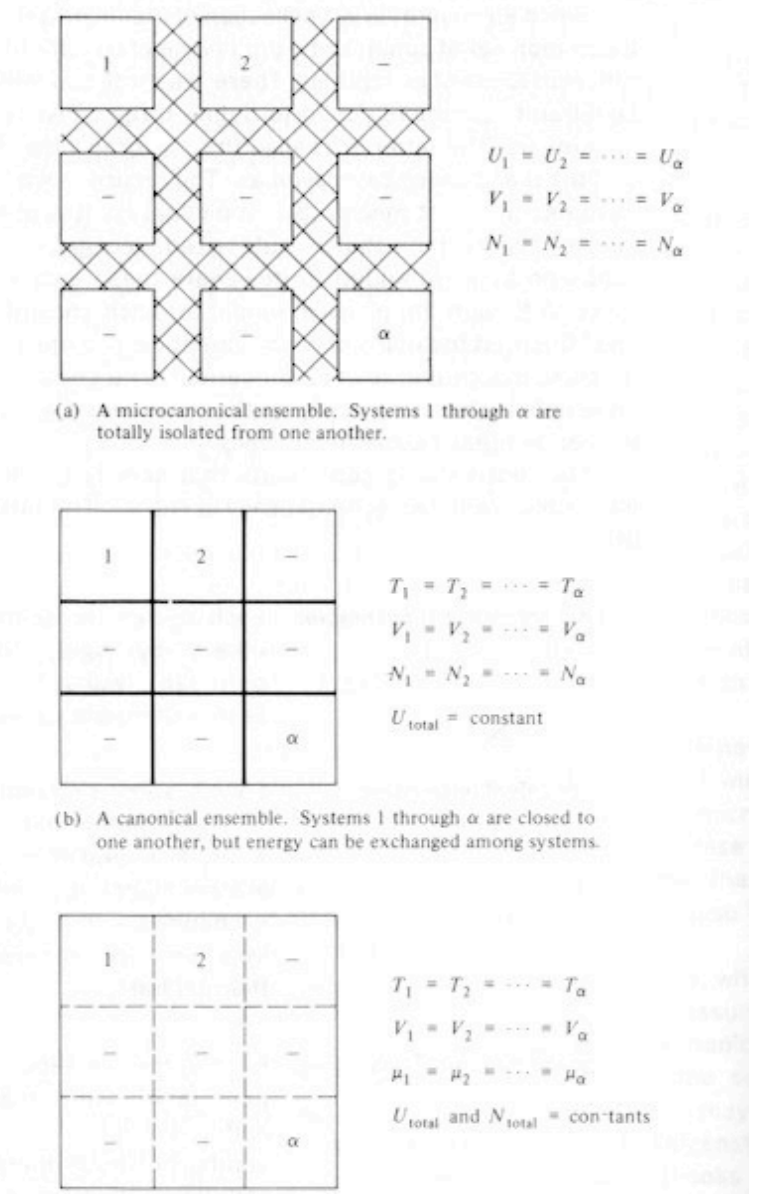


Fig. 8.1 Schematic representation of three kinds of ensembles.

(a) A microcanonical ensemble. Systems 1 through α are totally isolated from one another.

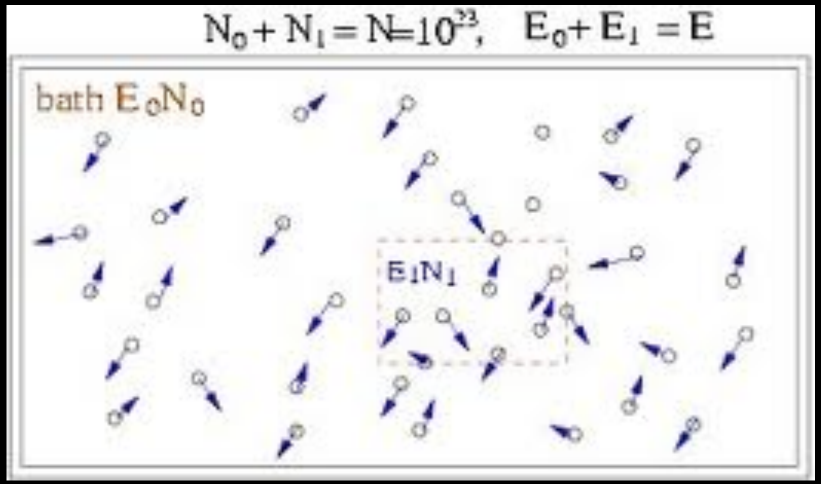
(b) A canonical ensemble. Systems 1 through α are closed to one another, but energy can be exchanged among systems.

(c) A grand canonical ensemble. Energy and mass can be exchanged among systems.

We use a Grand Canonical Ensemble

Canonical Ensemble

www.physics.udel.edu/~glyde/PHYS813/Lectures/chapter_6.pdf



$$P_s = \frac{1}{Z} e^{-\beta E_s}$$

probability to see a state with energy Es
β= 1/(kT)

$$Z = \sum_s e^{-\beta E_s}$$

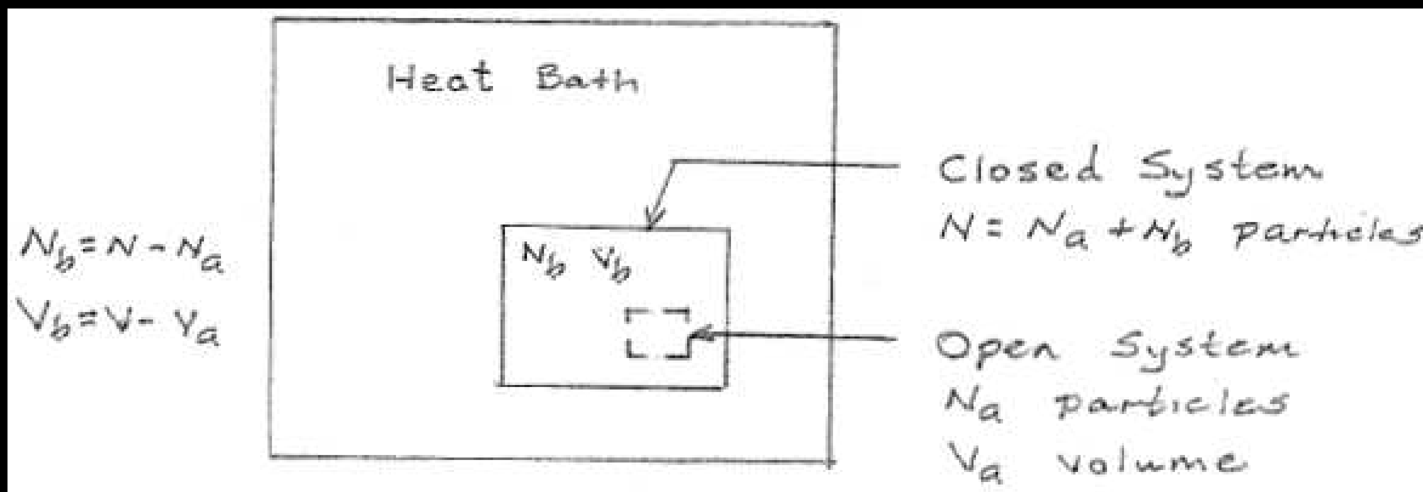
Z = Canonical partition function

F = U - TS Helmholtz free energy, maximal energy that can be converted into work

$$F(T, V, N) = -kT \log Z(T, V, N) \rightarrow Z = e^{\beta F}$$

$$\rightarrow P_s = e^{\beta(F - E_s)} \quad \text{and} \quad \sum_s P_s = 1$$

Grand Canonical Ensemble



www.physics.udel.edu/~glyde/PHYS813/Lectures/chapter_6.pdf

We build the GCE from the CE

$$P_s = Z^{-1} e^{-\beta E_s} = e^{\beta(F - E_s)}$$

$$E_s = E_{sa}(N_a) + E_{sb}(N_b)$$

$$F(T, V, N) = F_a(T, V_a, N_a) + F_b(T, V_b, N_b)$$

$$P_s = e^{-\beta E_{s_a}(N_a)} \times e^{-\beta E_{s_b}(N_b)} \times e^{\beta F(T,V,N)}$$

now lets look at the probablity of observing a subsystem N_a with energy $E_{sa}(N_a)$

$$P_{s_a}(N_a) = e^{-\beta E_{s_a}(N_a)} \times e^{\beta(F - F_b)} \quad \text{with} \quad F_b = -kT \log \sum_{s_b} e^{-\beta E_{s_b}(N_b)}$$

Grand Canonical Ensemble

$$P_{s_a}(N_a) = e^{-\beta E_{s_a}(N_a)} \times e^{\beta(F - F_b)} \quad \text{with} \quad F_b = -kT \log \sum_{s_b} e^{-\beta E_{s_b}(N_b)}$$

$$\text{since} \quad N_a << N \quad \text{and} \quad V_a << V$$

$$F_a = F - F_b = \left(\frac{\partial F}{\partial N}\right) N_a + \left(\frac{\partial F}{\partial V}\right) V_a = \mu N_a - p_a V_a$$

$$P_{s_a}(N_a) = e^{-\beta p_a V_a} \; e^{-\beta(E_{s_a}(N_a) - \mu N_a)}$$

$$P_{s_a}(N_a) \propto e^{-\beta(E_{s_a}(N_a) - \mu N_a)}$$

The probability depends on the number of particles N_a and on the chemical potential μ

Grand Canonical Ensemble

$$P_{s_a}(N_a) \propto e^{-\beta(E_{s_a}(N_a)-\mu N_a)}$$

$$F_a = \mu N_a - p_a V_a$$

$$N_a = \frac{dF_a}{d\mu} = \frac{dT \ln Z_a}{d\mu}$$

now more general
(without subsript a):

$$P_s(N) \propto e^{-\beta(E_s(N)-\mu N)}$$

$$Z = \sum_{s,N} \propto e^{-\beta(E_s(N)-\mu N)}$$

in general given he total number of states N

$$N = \sum_i n_i \quad E = \sum_i n_i E_i$$

we can write the total particion functions as:

$$Z(T,V,\mu) = \sum_{n_1,n_2,\dots} e^{-\beta \sum (E_i-\mu)n_i}$$

$$= \sum_{n_1,n_2,\dots} \prod_i e^{-\beta(E_i-\mu)n_i}$$ sum in exponential goes to product

$$= \prod_i \left[\sum_{n_i} e^{-\beta(E_i-\mu)n_i} \right]$$ each exponential only depends on one of the n_i

Example

$$Z(T,V,\mu) = \sum_{n_1,n_2,\dots} e^{-\beta\sum(E_i-\mu)n_i}$$

$$= \sum_{n_1,n_2,\dots}\prod_i e^{-\beta(E_i-\mu)n_i} \qquad \text{A}$$

$$= \prod_i\left[\sum_{n_i} e^{-\beta(E_i-\mu)n_i}\right] \qquad \text{B}$$

$$n_1 = 1,2 \quad \text{and} \quad n_2 = 1,2$$

$$\sum_{n_1,n_2} e^{-\beta[(E_1-\mu)n_1+(E_2-\mu)n_2]} =$$

$$e^{-\beta[(E_1-\mu)+2(E_2-\mu)]} + e^{-\beta[2(E_1-\mu)+(E_2-\mu)]}$$

$$+e^{-\beta[(E_1-\mu)+(E_2-\mu)]} + e^{-\beta[2(E_1-\mu)+2(E_2-\mu)]}$$

A

$$\Pi_i(e^{-\beta(E_i-\mu)} + e^{-2\beta(E_i-\mu)}) =$$

$$(e^{-\beta(E_1-\mu)} + e^{-2\beta(E_1-\mu)})\cdot$$

$$(e^{-\beta(E_2-\mu)} + e^{-2\beta(E_2-\mu)})$$

B

A = B

$$Z = \prod_i z_i$$

partition function of whole system factorizes in products of partition functions of single particle states

Grand Canonical Ensemble

For Fermions (n_i = 0,1) and Bosons (n_i = 0,1,2,3,...)
we get respectively:

$$Z_F = \prod_i (1 + e^{-\beta(E_i - \mu)}), \quad Z_B = \prod_i \frac{1}{(1 - e^{-\beta(E_i - \mu)})}$$

ln Z transforms the product into a sum, The sum runs over the number of states and if we include the phase space density

$$\frac{d^3p}{(2\pi\hbar^3)}$$

we get:

$$\ln Z(T, V, \mu) = \pm gV \int \frac{d^3p}{(2\pi\hbar)^3} \ln(1 \pm e^{-\beta(E_p - \mu)})$$

Grand Canonical Ensemble

$$\ln Z(T,V,\mu) = \pm gV \int \frac{d^3p}{(2\pi\hbar)^3} \ln(1 \pm e^{-\beta(E_p-\mu)})$$

With this we can define the usual way pressure, N and energy:

$$P = \frac{\partial T \ln Z}{\partial V}, \quad N = \frac{\partial T \ln Z}{\partial \mu}$$

$$E = V \int \frac{d^3p}{(2\pi\hbar)^3} \frac{E_p}{1 \pm e^{-\beta(E_p-\mu)}}$$

For an ideal gas $E_p = |p|$, $\mu=0$:

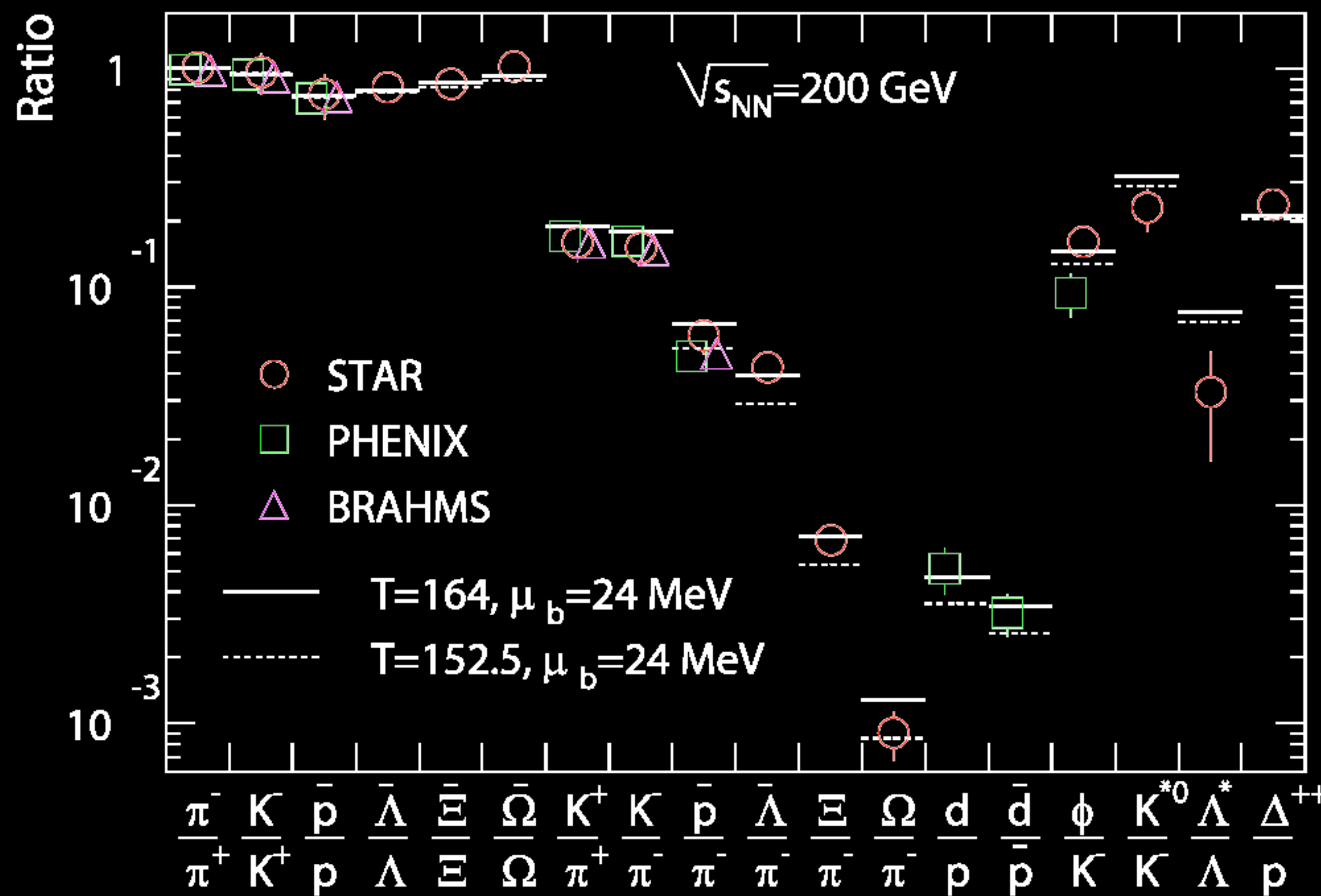
fermions

$$\epsilon = g_F \frac{7}{8}\frac{\pi^2}{90}T^4, \quad P = \frac{1}{3}\epsilon$$

bosons

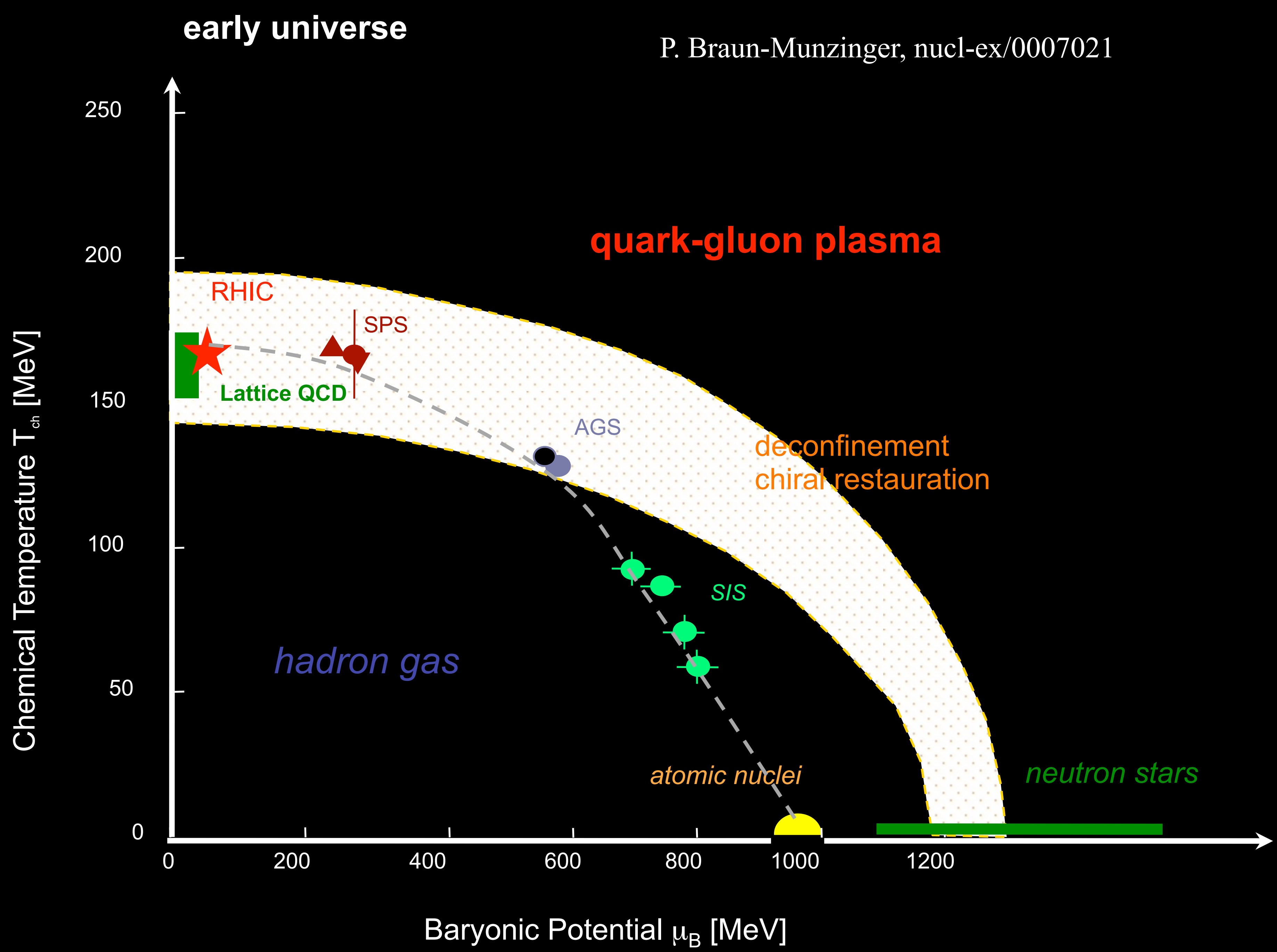
$$\epsilon = g_B \frac{\pi^2}{90}T^4, \quad P = \frac{1}{3}\epsilon$$

Identified Particle Yields

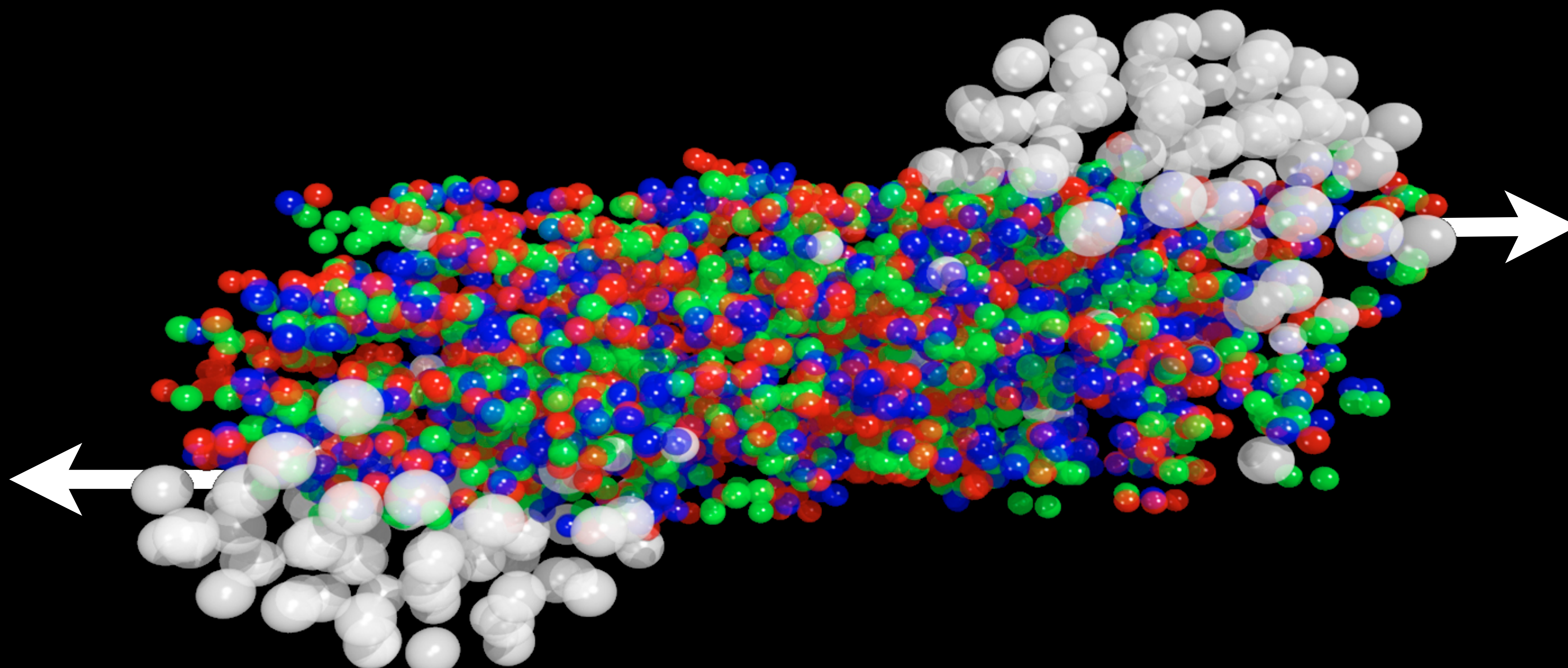


- the thermal model (T, μ, mass) fits very well
- Works rather well in $e^+ e^-$ and proton-proton collisions as well, except for strange particles

The Phase Diagram



Collective Flow

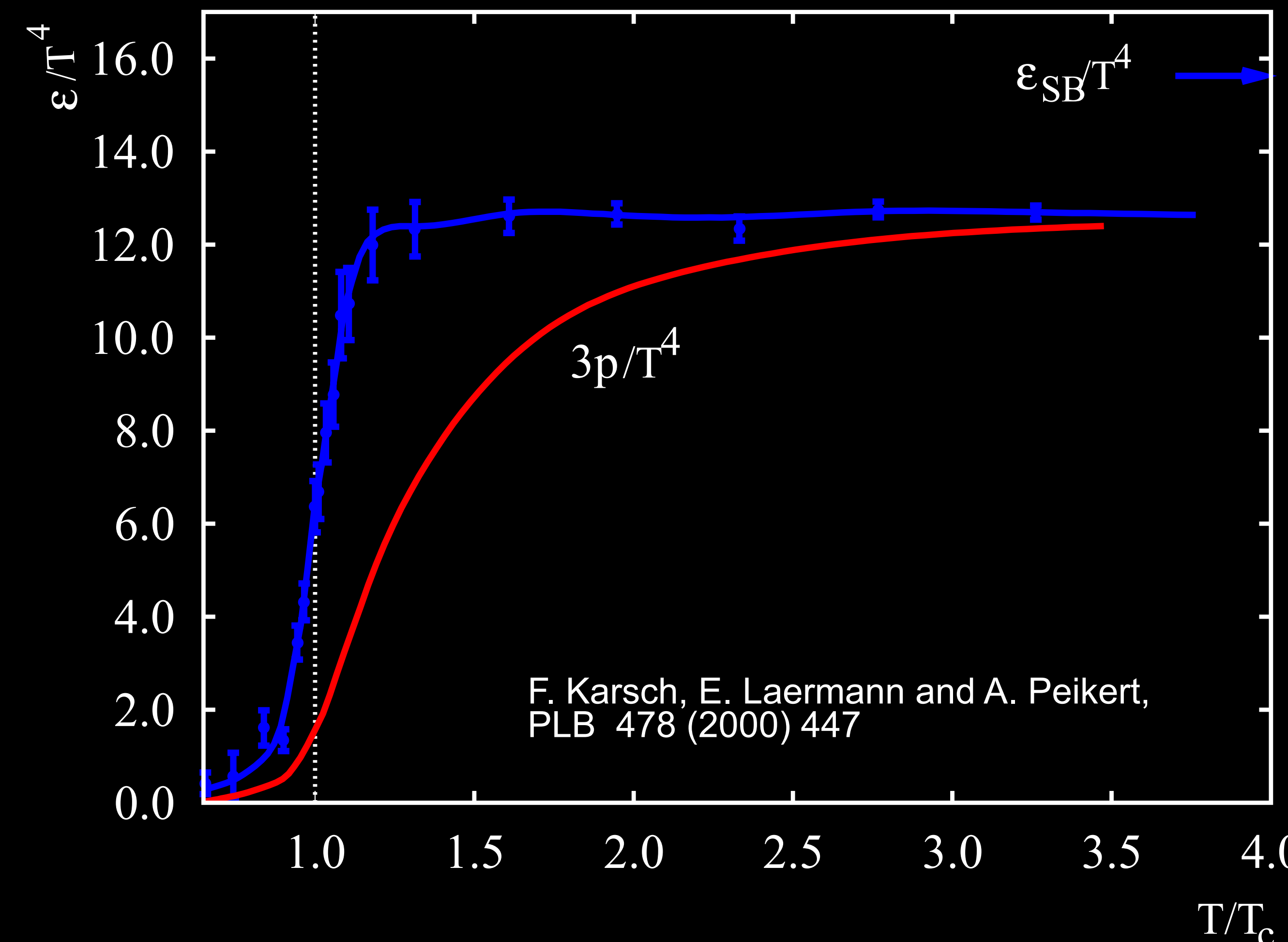


Collective Motion

$$c_s^2 = \frac{\partial P}{\partial e}$$

$$\dot{\mu}^{\mu} = \frac{\nabla^{\mu} P}{e + P} = \frac{c_s^2}{1 + c_s^2} \frac{\nabla^{\mu} e}{e}$$

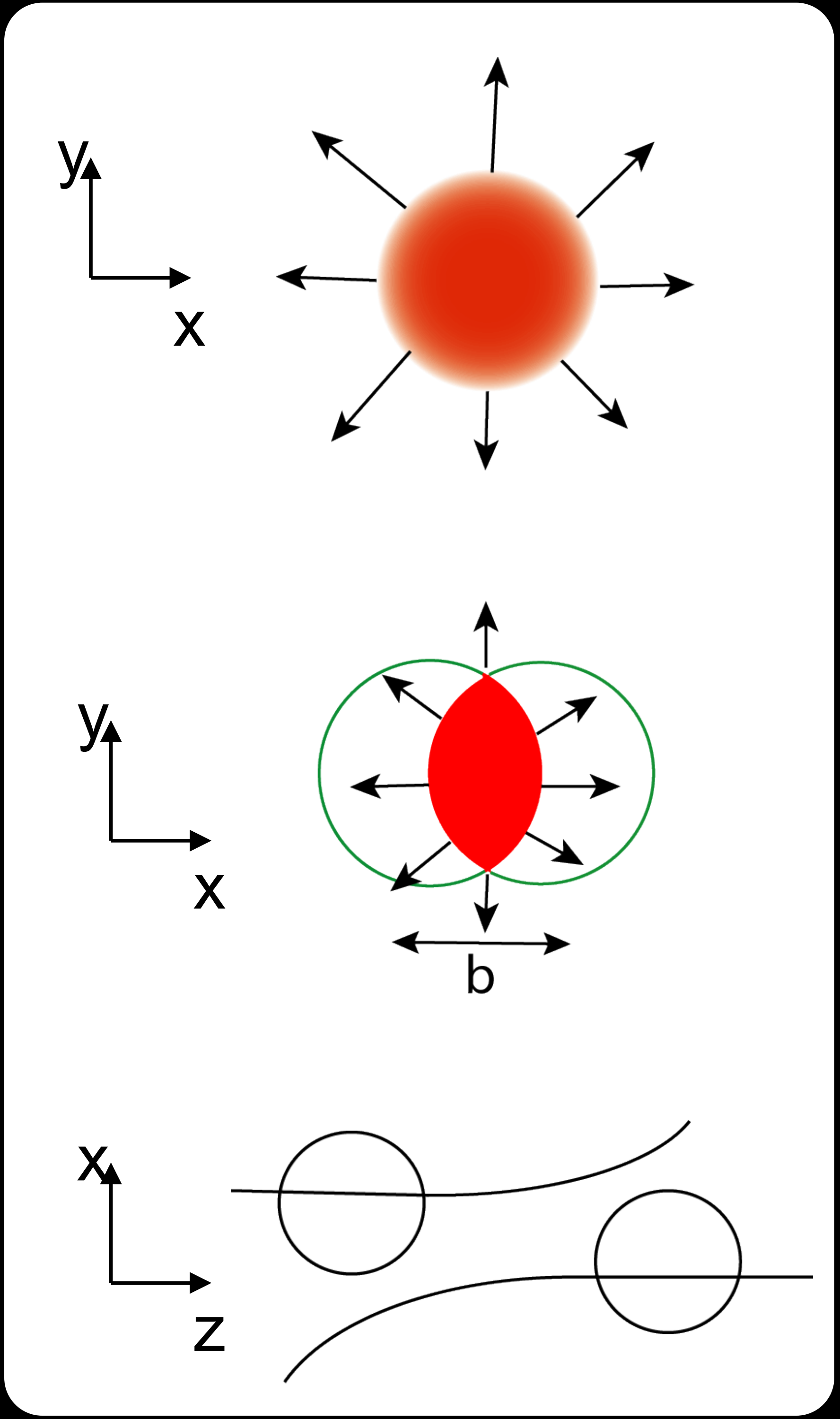
measurements of the collective expansion constrain c_s and thus the EoS



for a noninteracting gas of massless quarks

$$\begin{aligned} p &= \frac{1}{3}\epsilon = g\frac{\pi^2}{90}T^4 \\ g_{\mathrm{H}} &\approx 3 \quad g_{\mathrm{QGP}} \approx 37 \\ g &= 2_{\mathrm{spin}} \times 8_{\mathrm{gluons}} + \frac{7}{8} \times 2_{\mathrm{flavors}} \times 2_{q\bar{q}} \times 2_{\mathrm{spin}} \times 3_{\mathrm{color}} \end{aligned}$$

Collective Motion



only type of transverse flow in central collision (b=0) is radial flow Integrates pressure history over complete expansion phase

elliptic flow (v_2) , v_4 , v_6, … caused by anisotropic initial overlap region (b > 0) more weight towards early stage of expansion

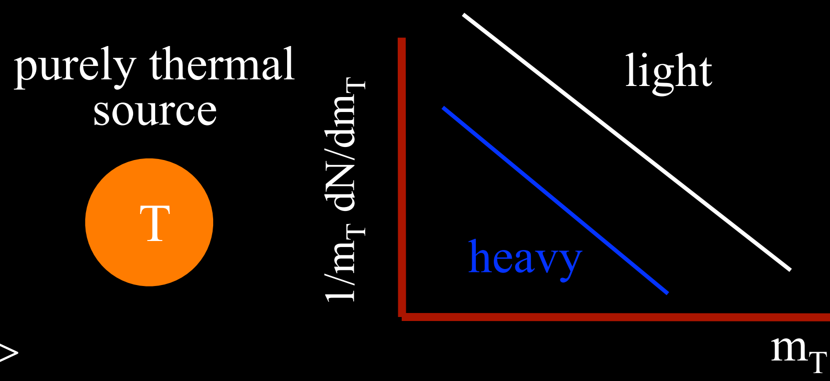
directed flow (v_1) , sensitive to earliest collision stage (b > 0), pre-equilibrium at forward rapidity, at midrapidity perhaps different origin

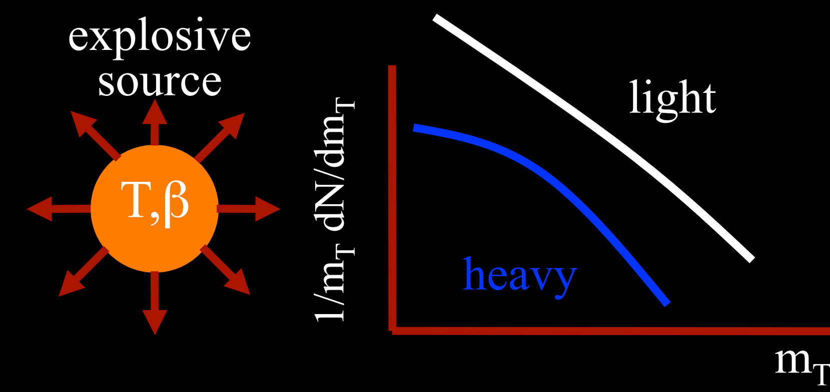
Collective Motion

$$m_T = \sqrt{(m^2 + p_t^2)}$$

$$\frac{dN}{m_T dm_T} \propto e^{-m_T/T}$$

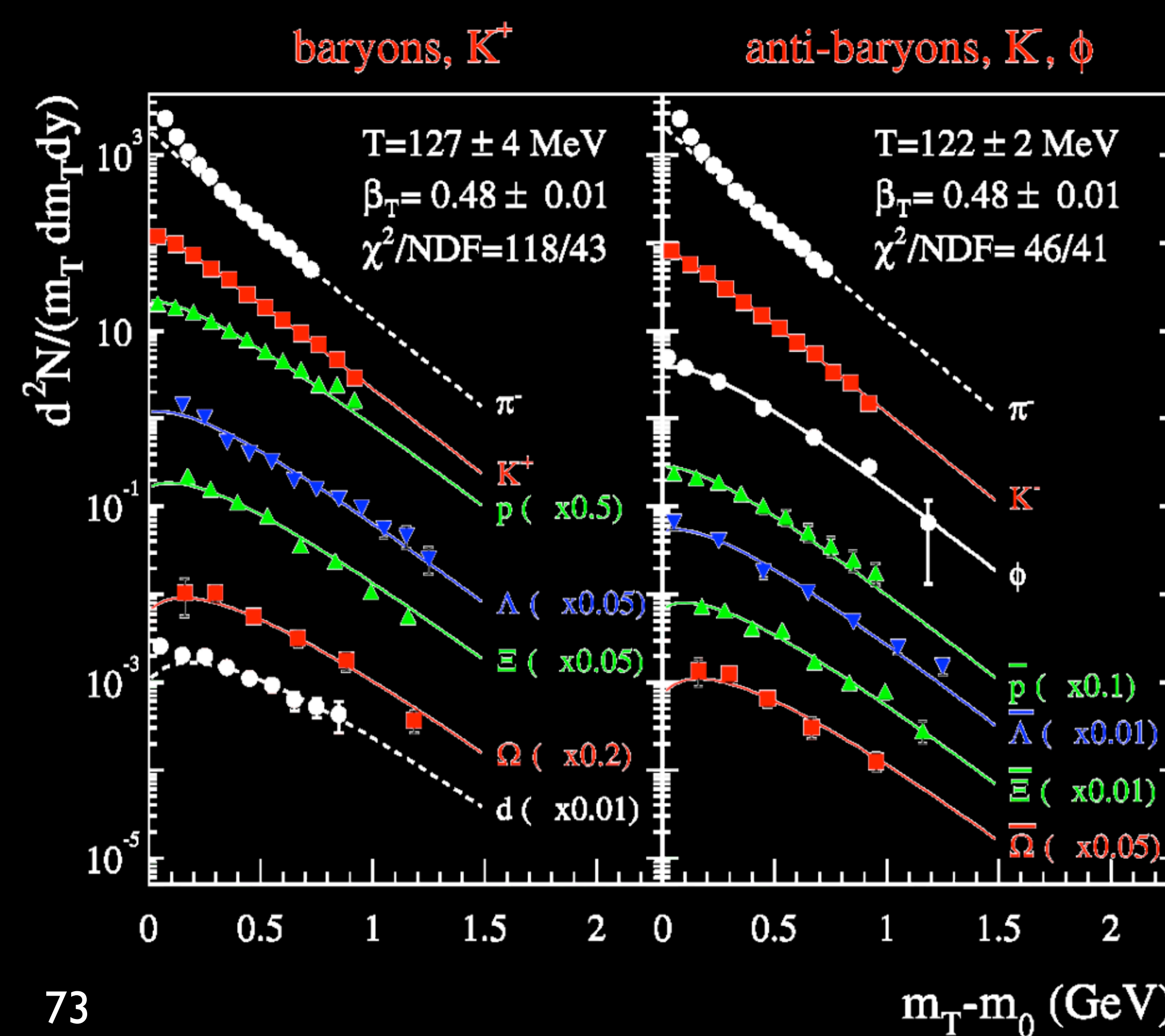
$$p_T \approx p_T^{\rm th} + m < v_T >$$



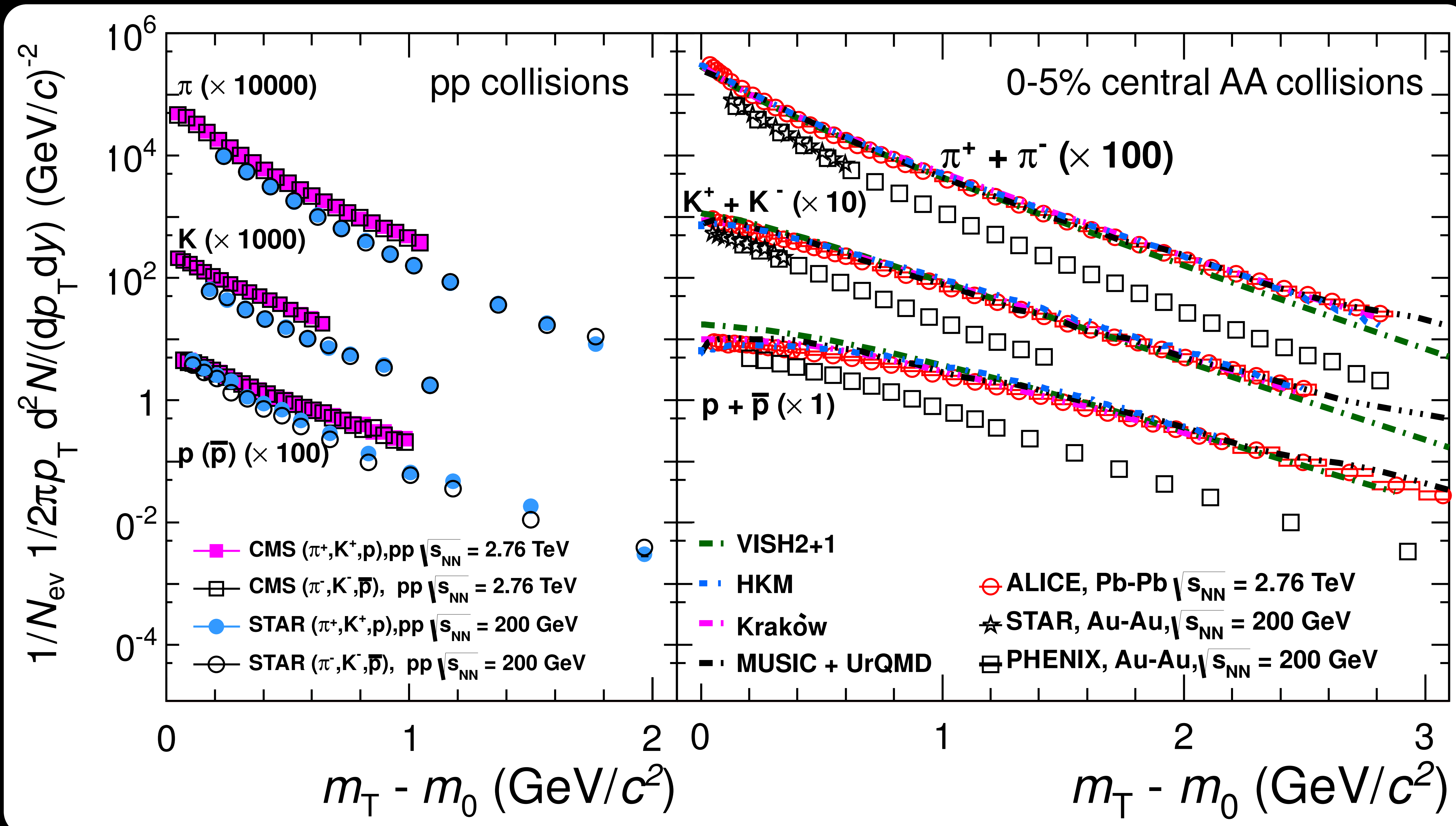


in p-p at low transverse momenta the particle yields are well described by thermal spectra (m_T scaling)

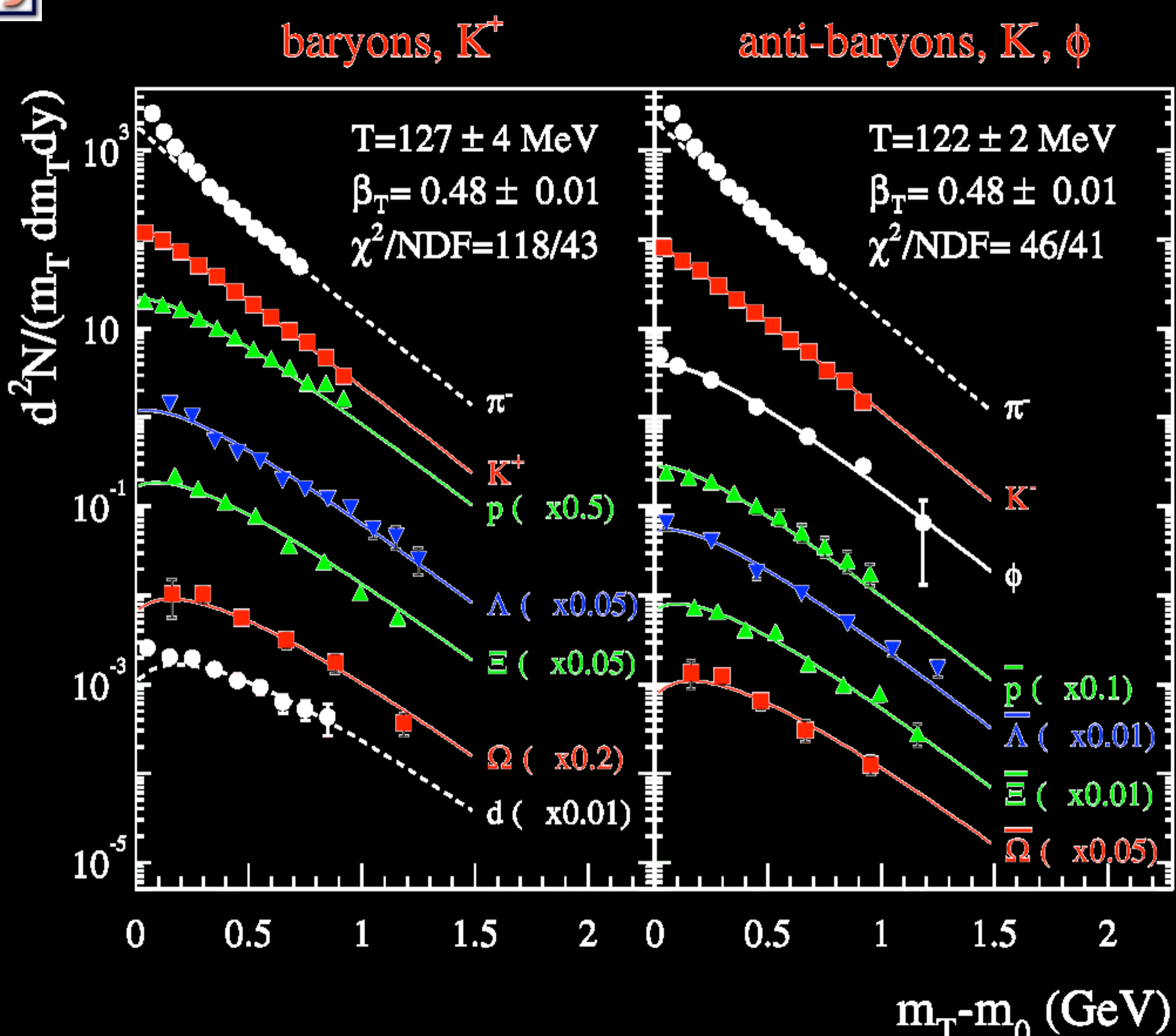
boosted thermal spectra give a very good description of the particle distributions measured in heavy-ion collisions



Collective Motion



Collective Motion



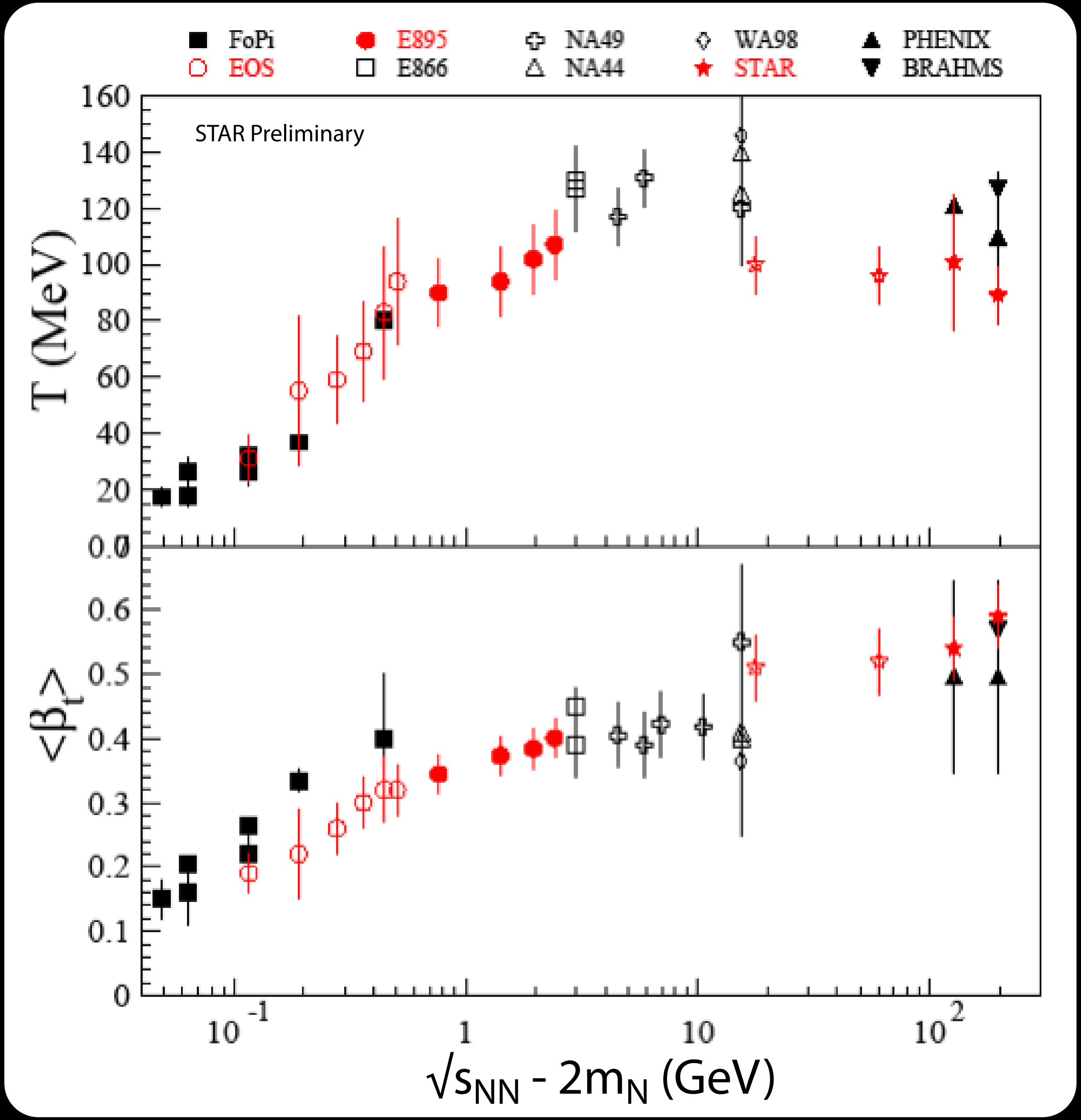
E. Schnedermann and U. Heinz, PRC 50, 1675 (1994)

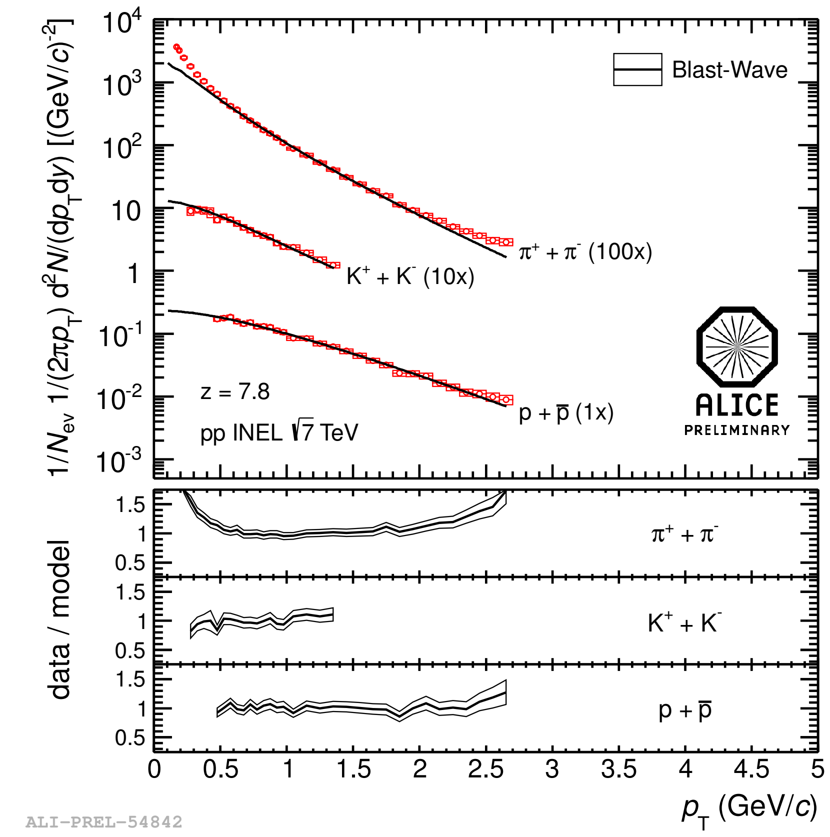
"Blastwave fit"

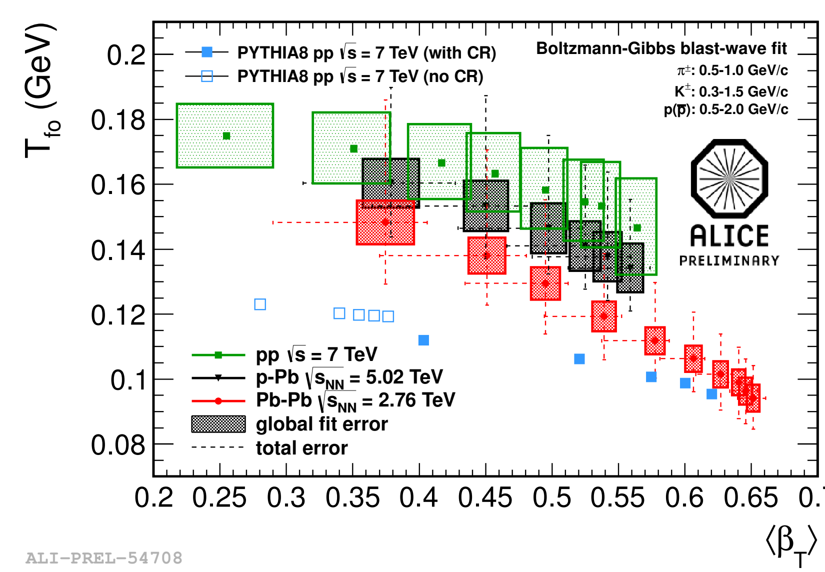
$$\frac{1}{m_T}\frac{dN}{dm_T} \propto m_T K_1\left(\frac{m_T \cosh \eta_T}{T_{\text{kin}}}\right) I_0\left(\frac{p_T \sinh \eta_T}{T_{\text{kin}}}\right)$$

$$\eta_T = \tanh^{-1}\beta_T$$

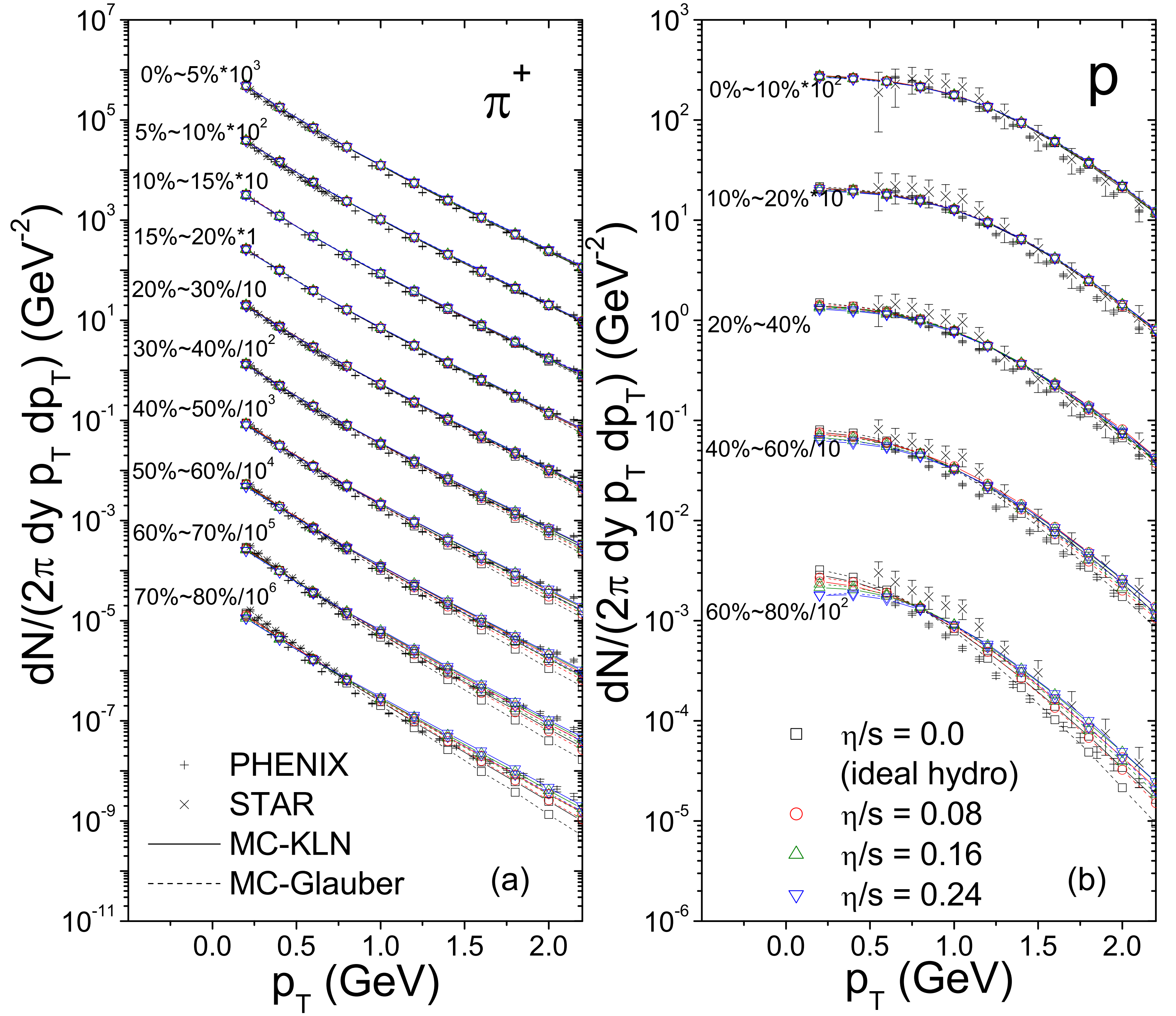
Collective Motion







Collective Motion

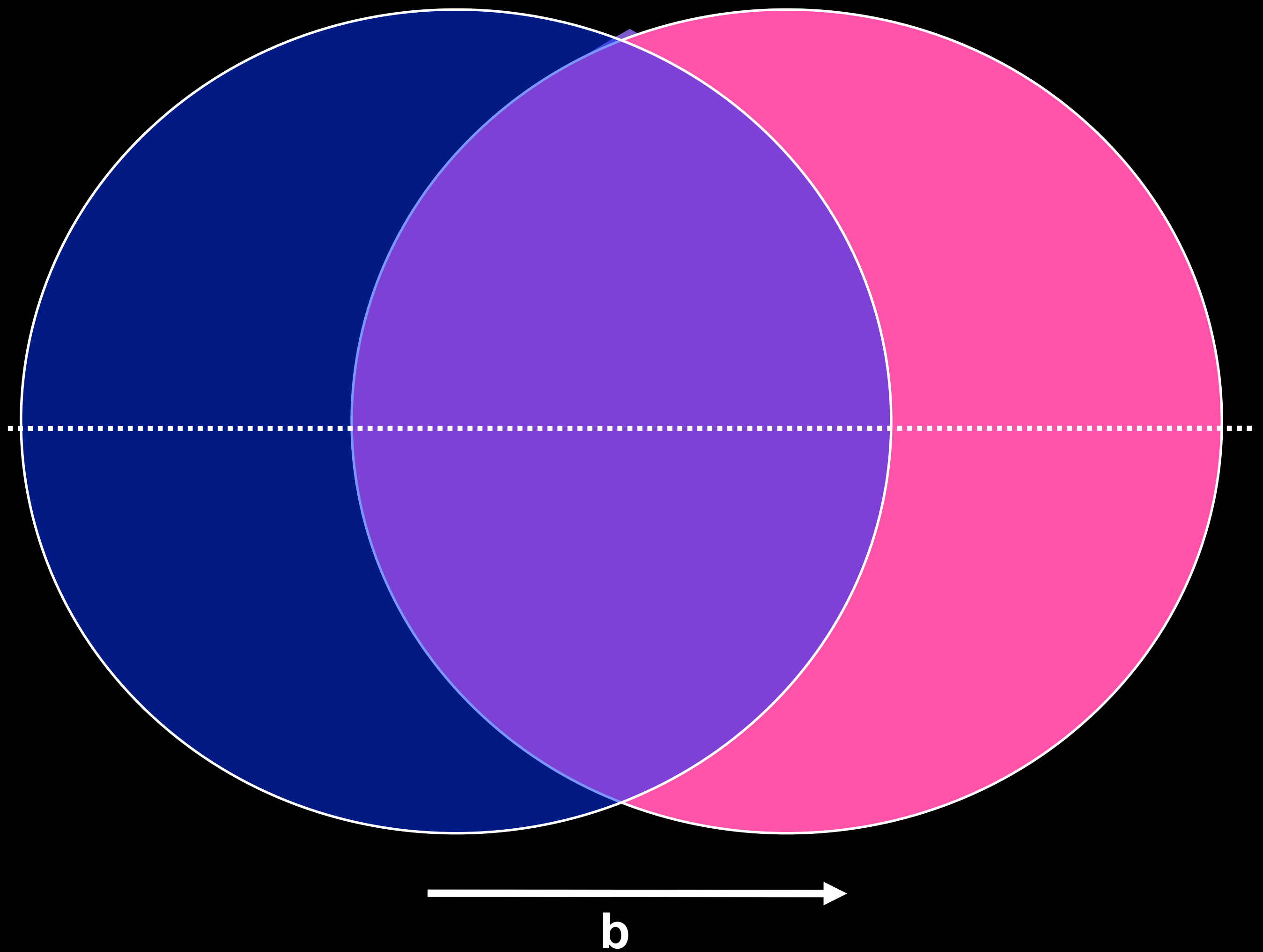


Viscous hydrodynamics does a good job explaining the observed soft particle spectra

Elliptic Flow

Ollitrault 1992

Animation: Mike Lisa

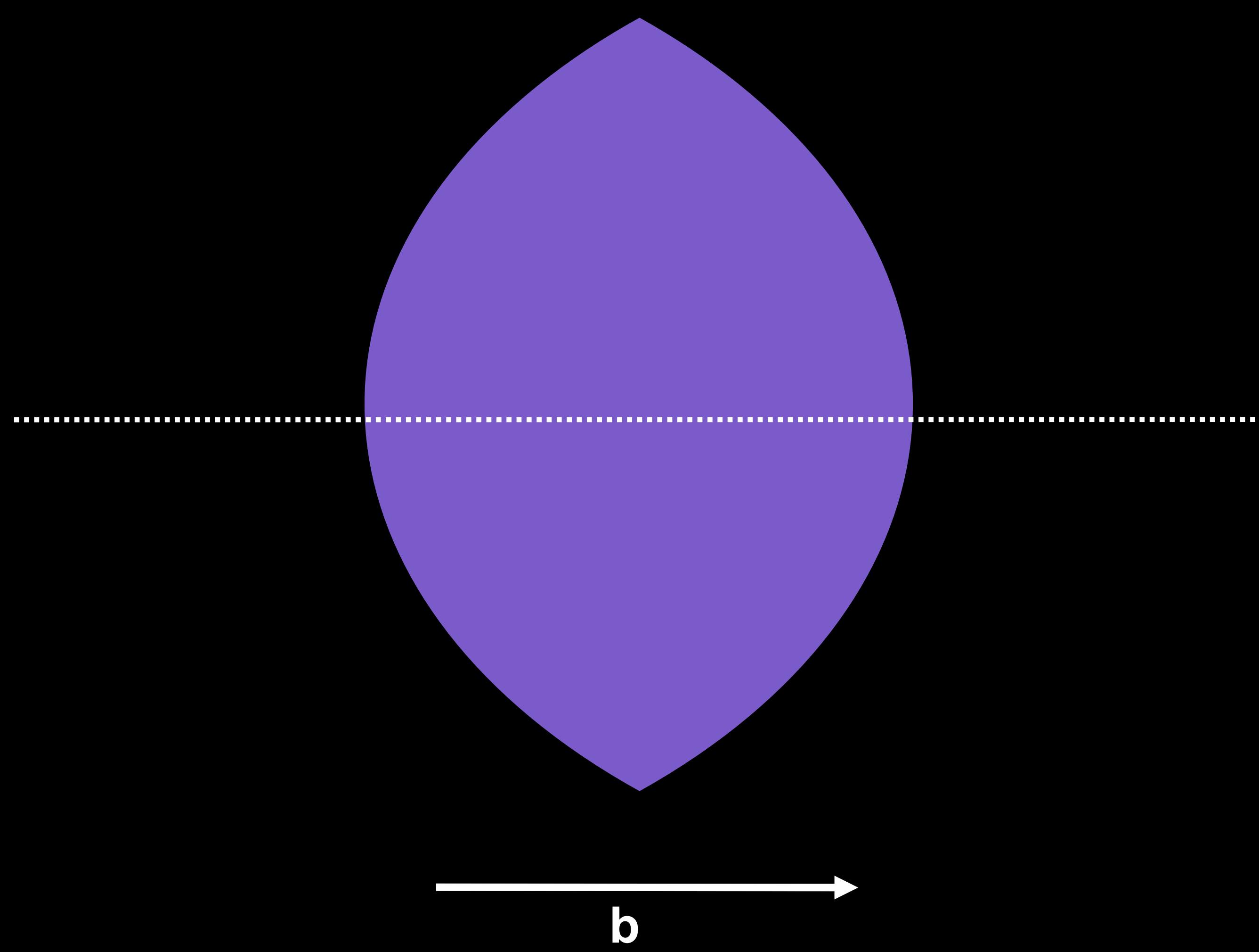


Elliptic Flow

Ollitrault 1992

Animation: Mike Lisa

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



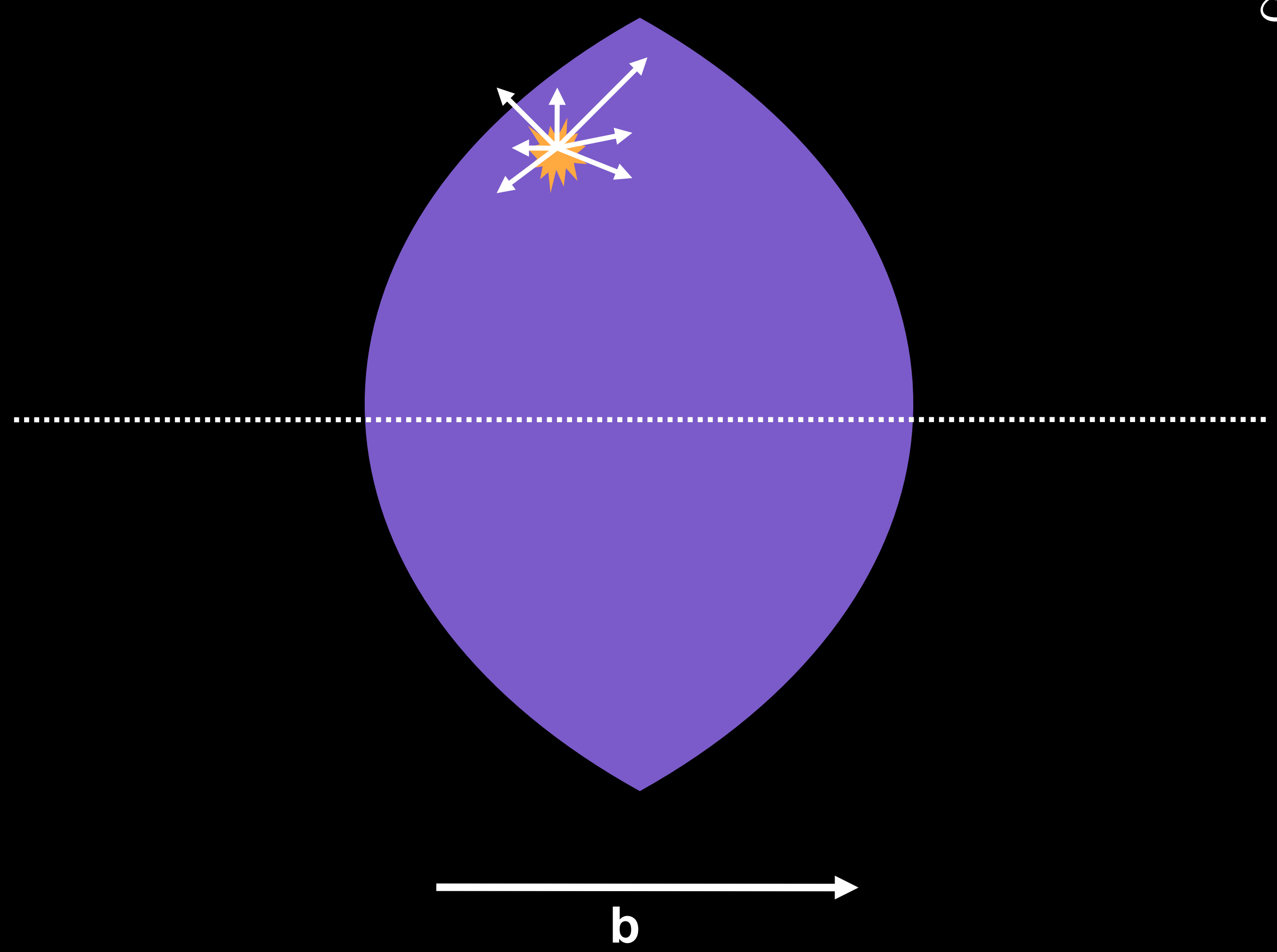
Elliptic Flow

Ollitrault 1992

1) superposition of independent p+p:

Animation: Mike Lisa

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



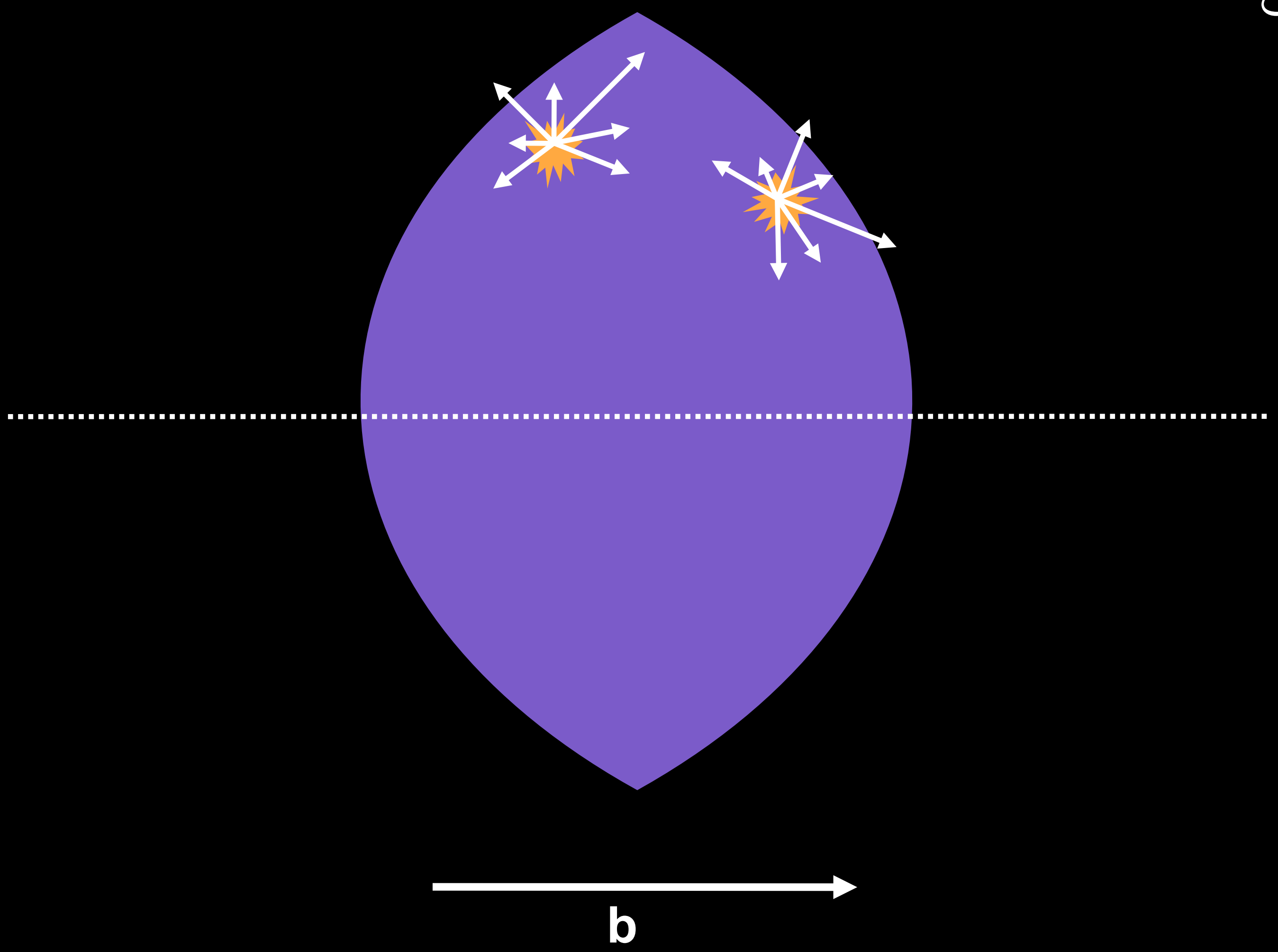
Elliptic Flow

Ollitrault 1992

1) superposition of independent p+p:

Animation: Mike Lisa

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



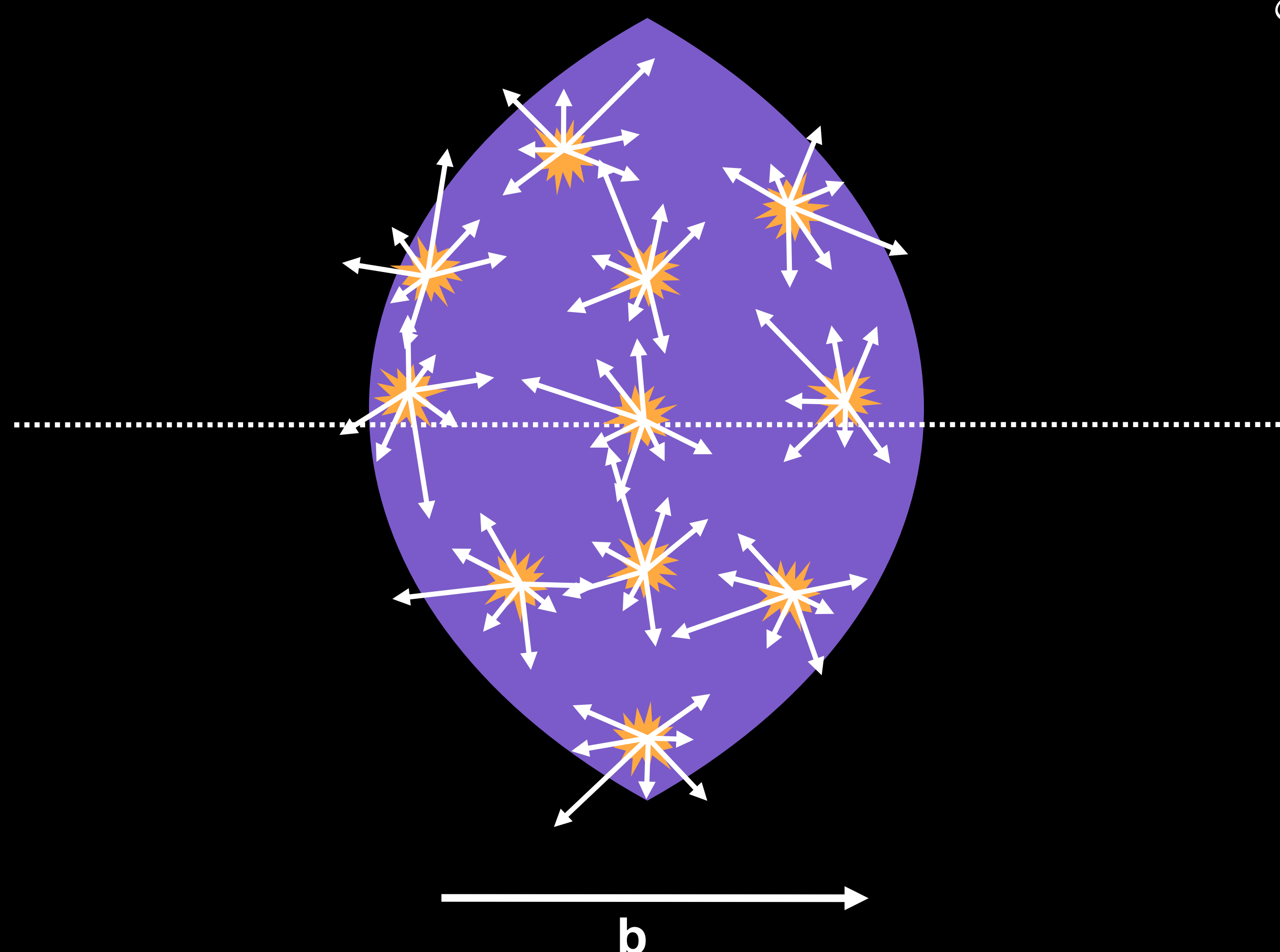
Elliptic Flow

Ollitrault 1992

Animation: Mike Lisa

1) superposition of independent p+p:

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



Elliptic Flow

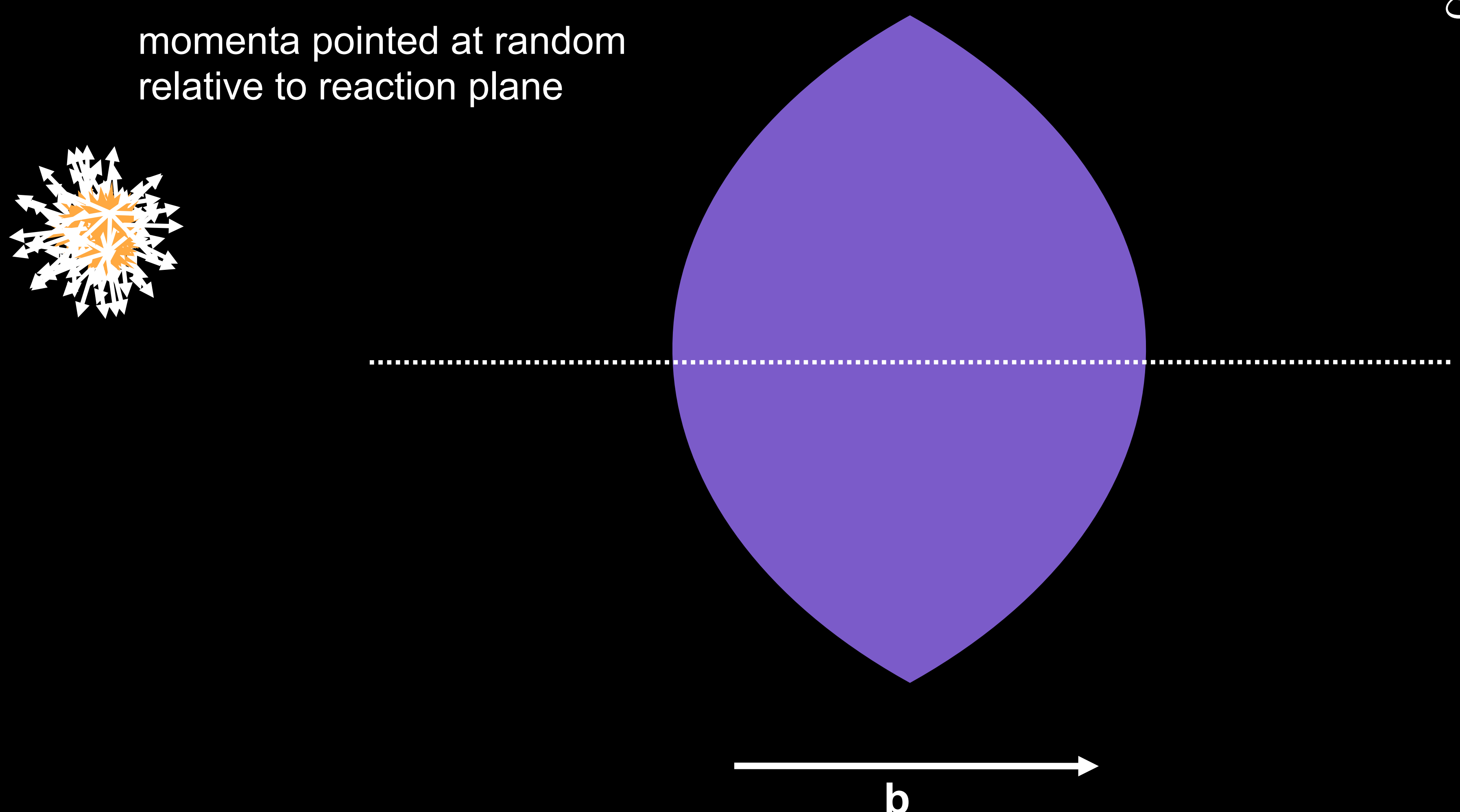
Ollitrault 1992

Animation: Mike Lisa

1) superposition of independent p+p:

momenta pointed at random
relative to reaction plane

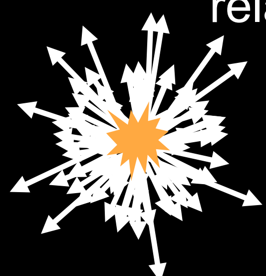
$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



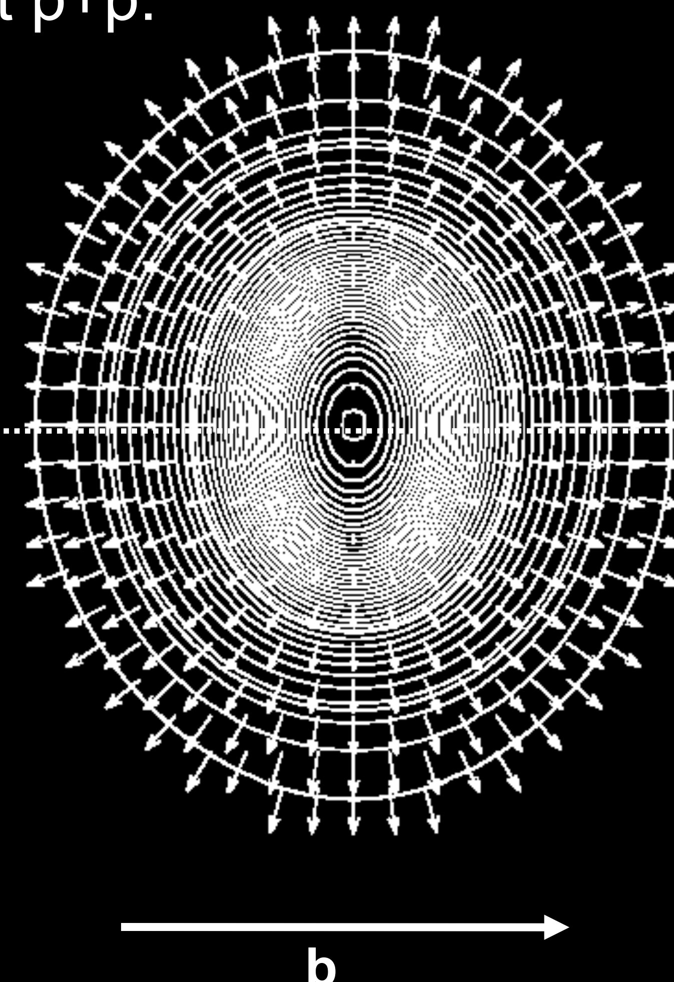
Elliptic Flow

1) superposition of independent p+p:

momenta pointed at random
relative to reaction plane



2) evolution as a **bulk system**



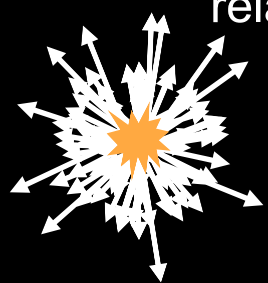
$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

Elliptic Flow

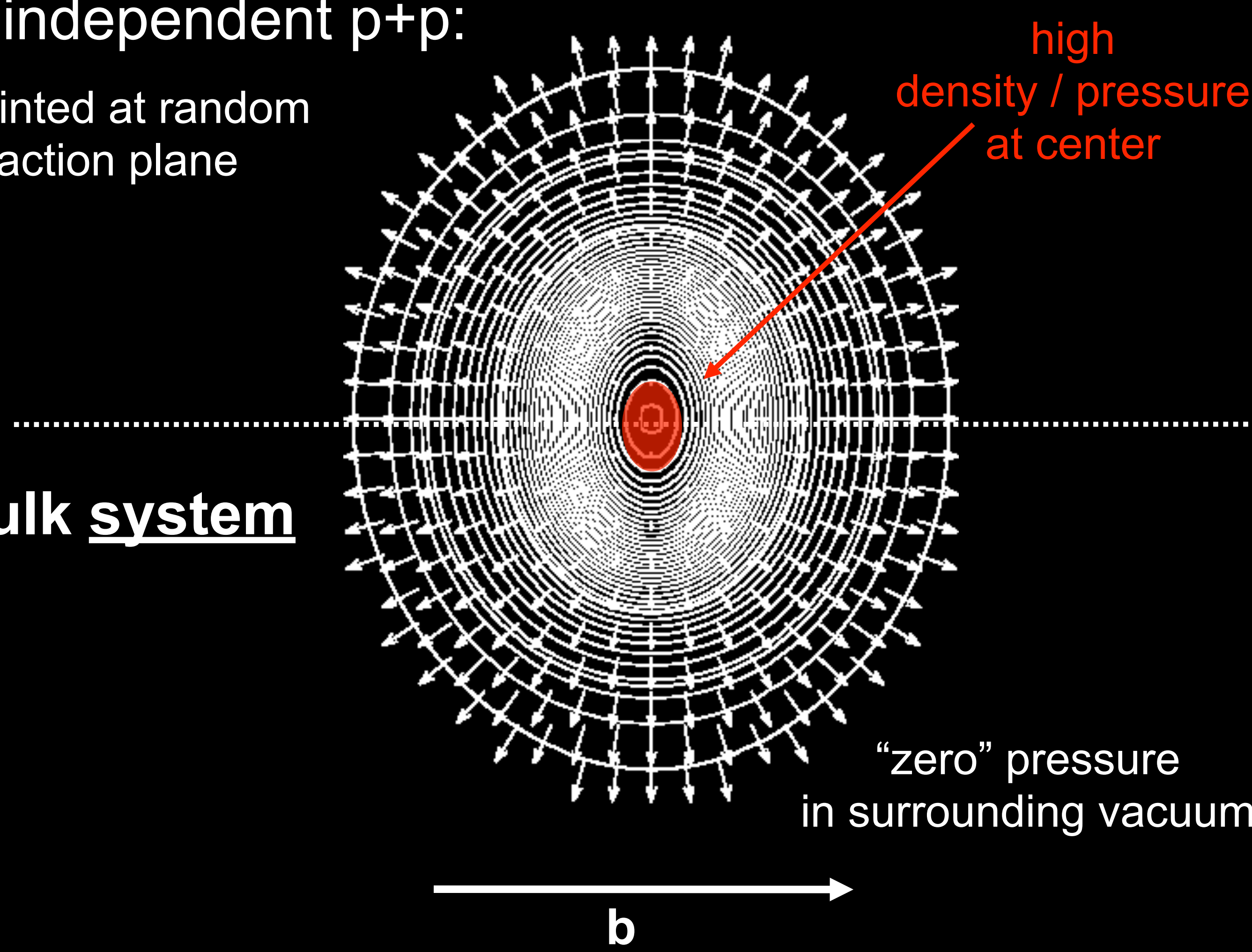
$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

1) superposition of independent p+p:

momenta pointed at random
relative to reaction plane



2) evolution as a **bulk <u>system</u>**

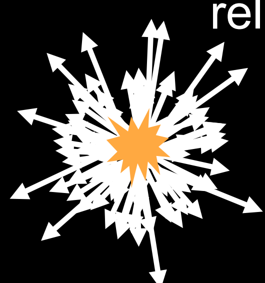


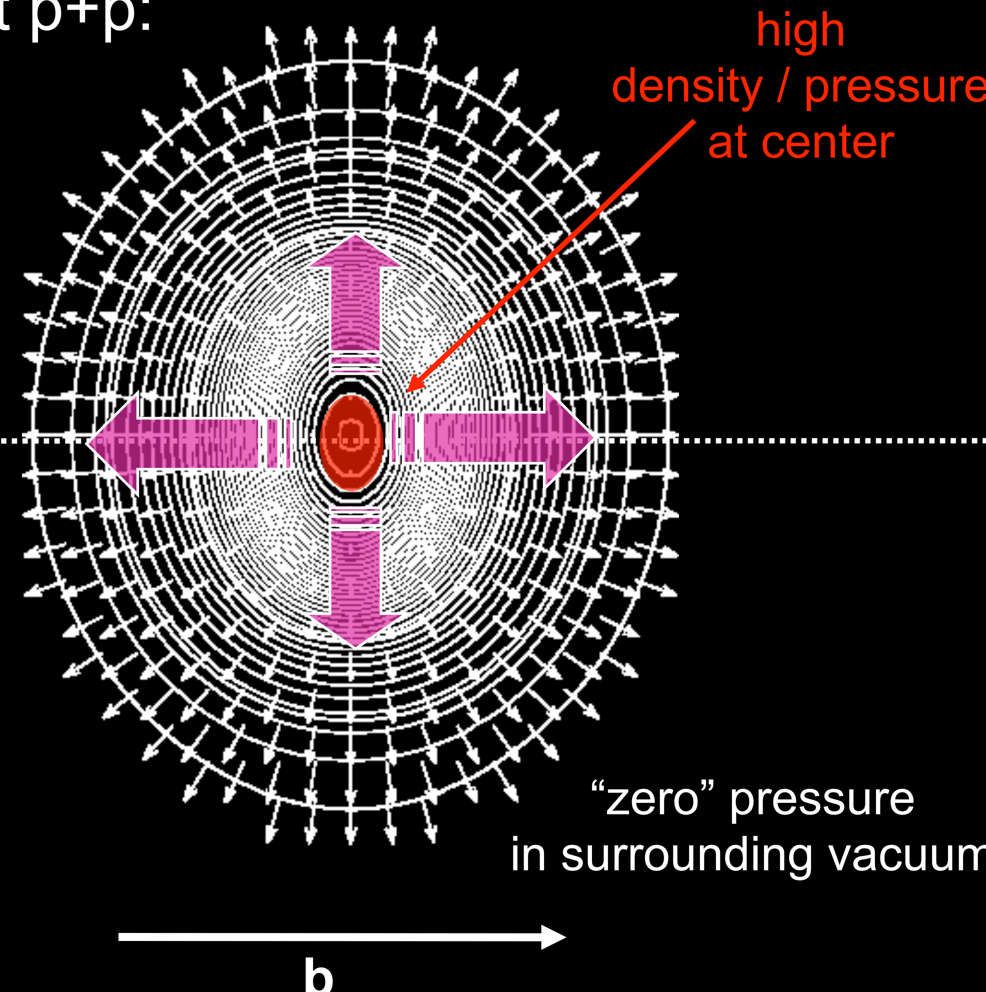
Elliptic Flow

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

1) superposition of independent p+p:

momenta pointed at random
relative to reaction plane





2) evolution as a **bulk** <u>**system**</u>

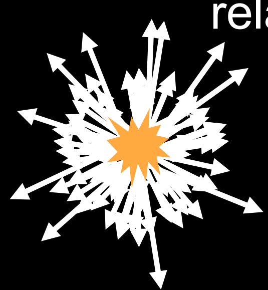
pressure gradients (larger in-plane)
push bulk “out” → “flow”

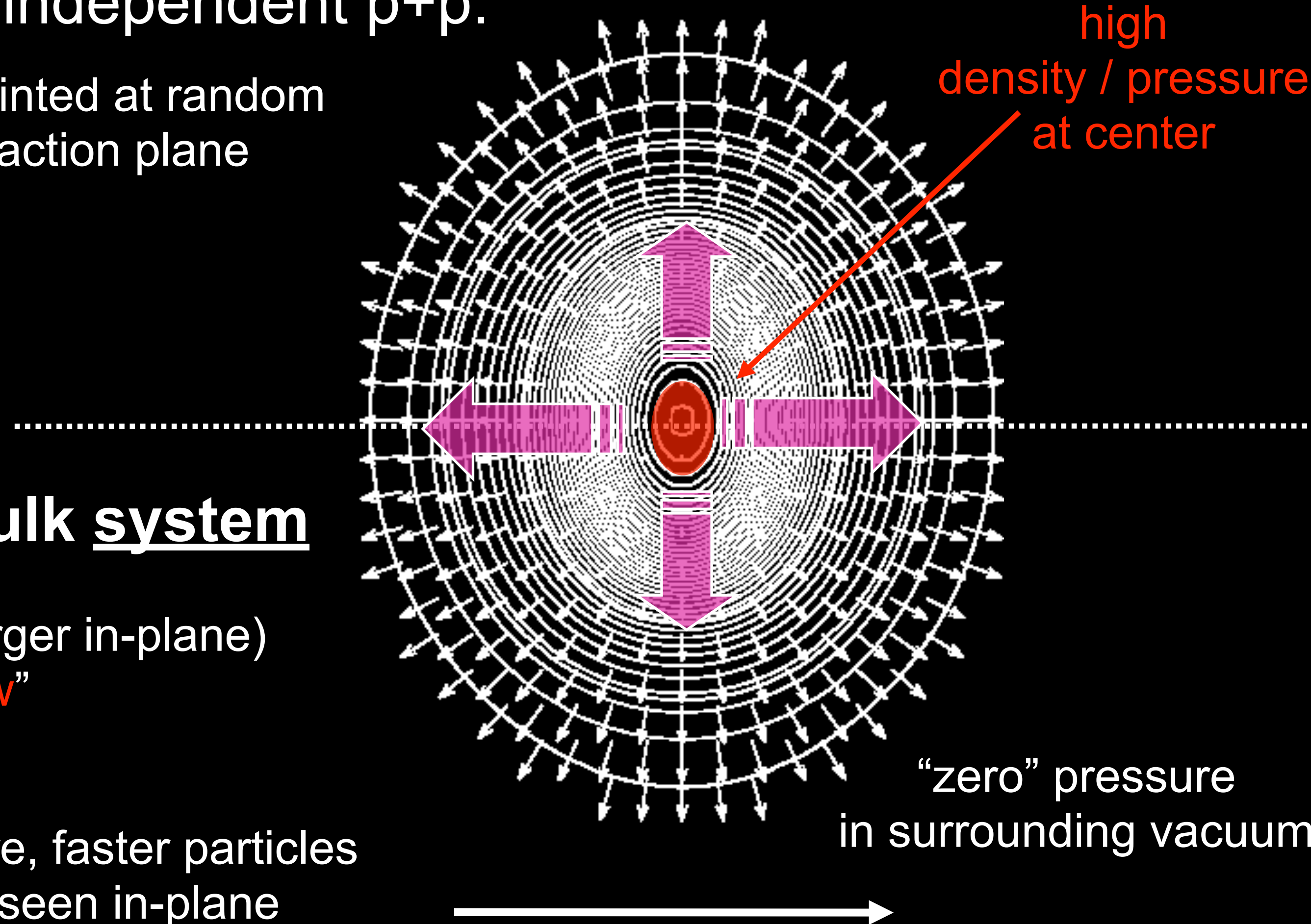
Elliptic Flow

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

1) superposition of independent p+p:

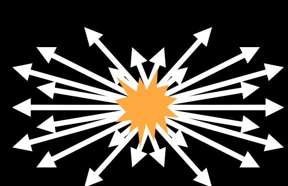
momenta pointed at random
relative to reaction plane





2) evolution as a **bulk system**

pressure gradients (larger in-plane)
push bulk "out" → "flow"

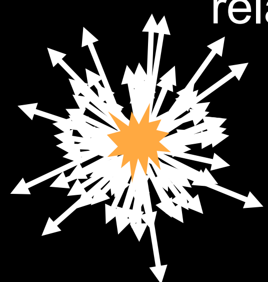


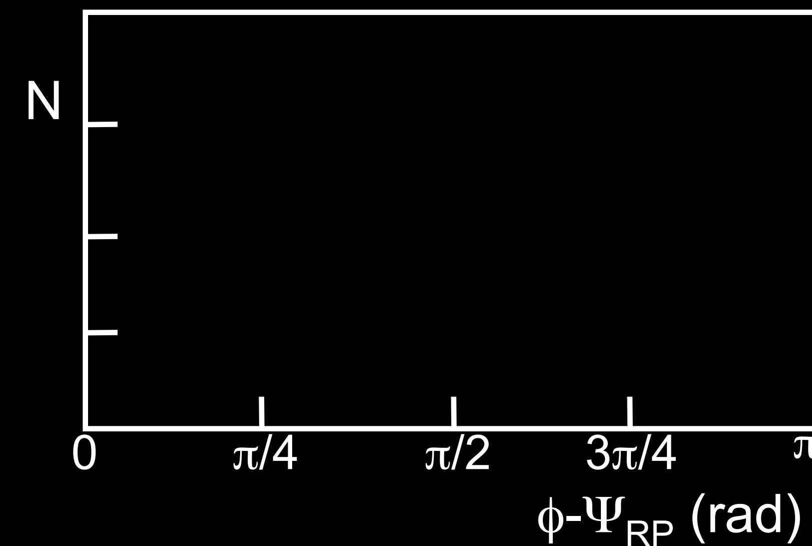
more, faster particles
seen in-plane

Elliptic Flow

1) superposition of independent p+p:

momenta pointed at random
relative to reaction plane

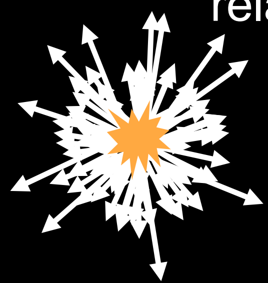


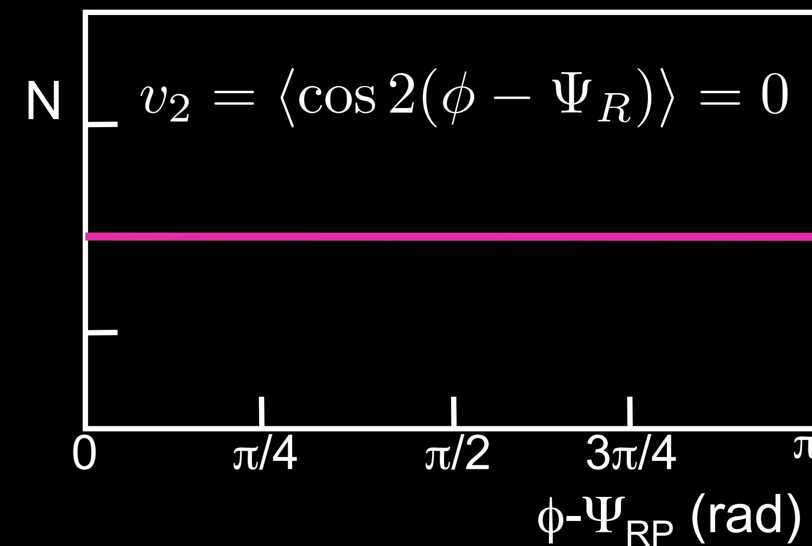


Elliptic Flow

1) superposition of independent p+p:

momenta pointed at random
relative to reaction plane

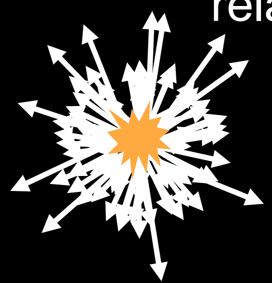


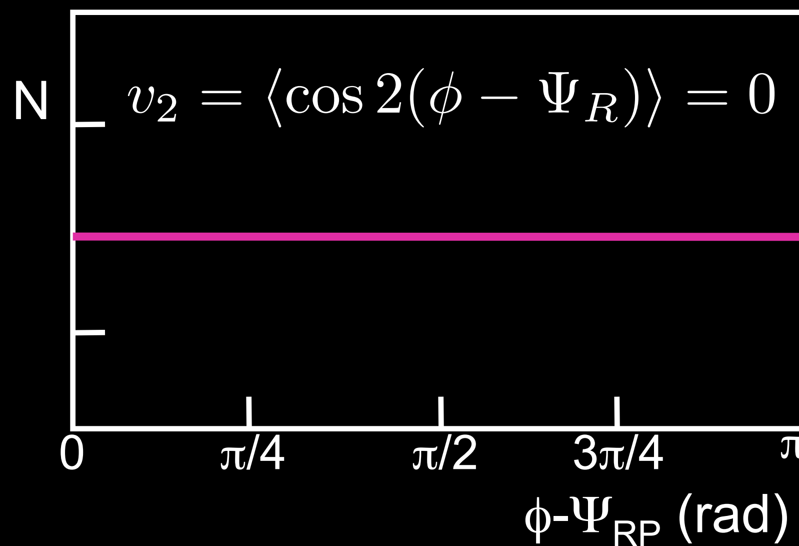


Elliptic Flow

1) superposition of independent p+p:

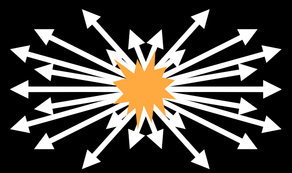
momenta pointed at random
relative to reaction plane





2) evolution as a **bulk system**

pressure gradients (larger in-plane)
push bulk "out" → "flow"

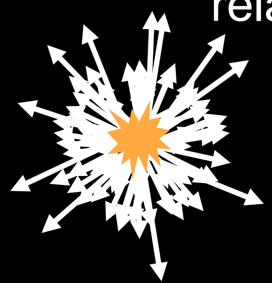


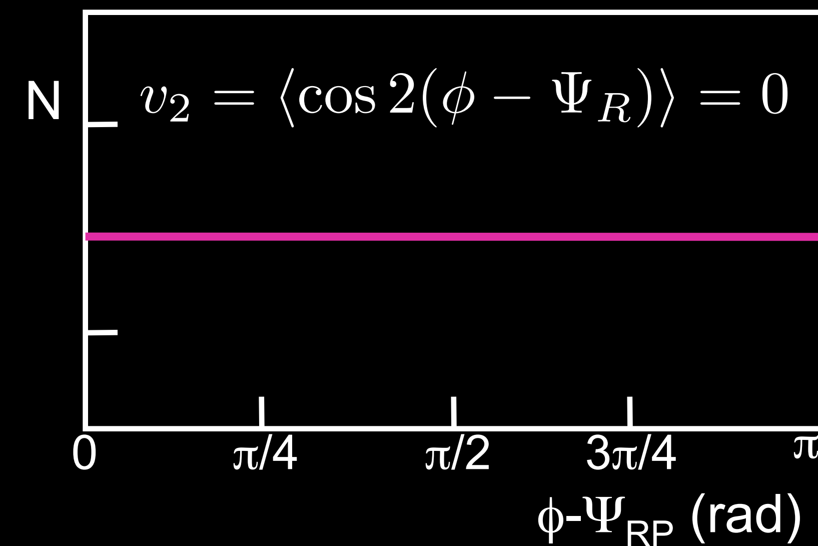
more, faster particles
seen in-plane

Elliptic Flow

1) superposition of independent p+p:

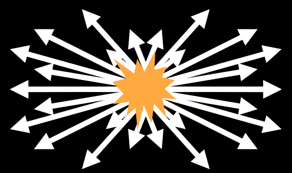
momenta pointed at random
relative to reaction plane



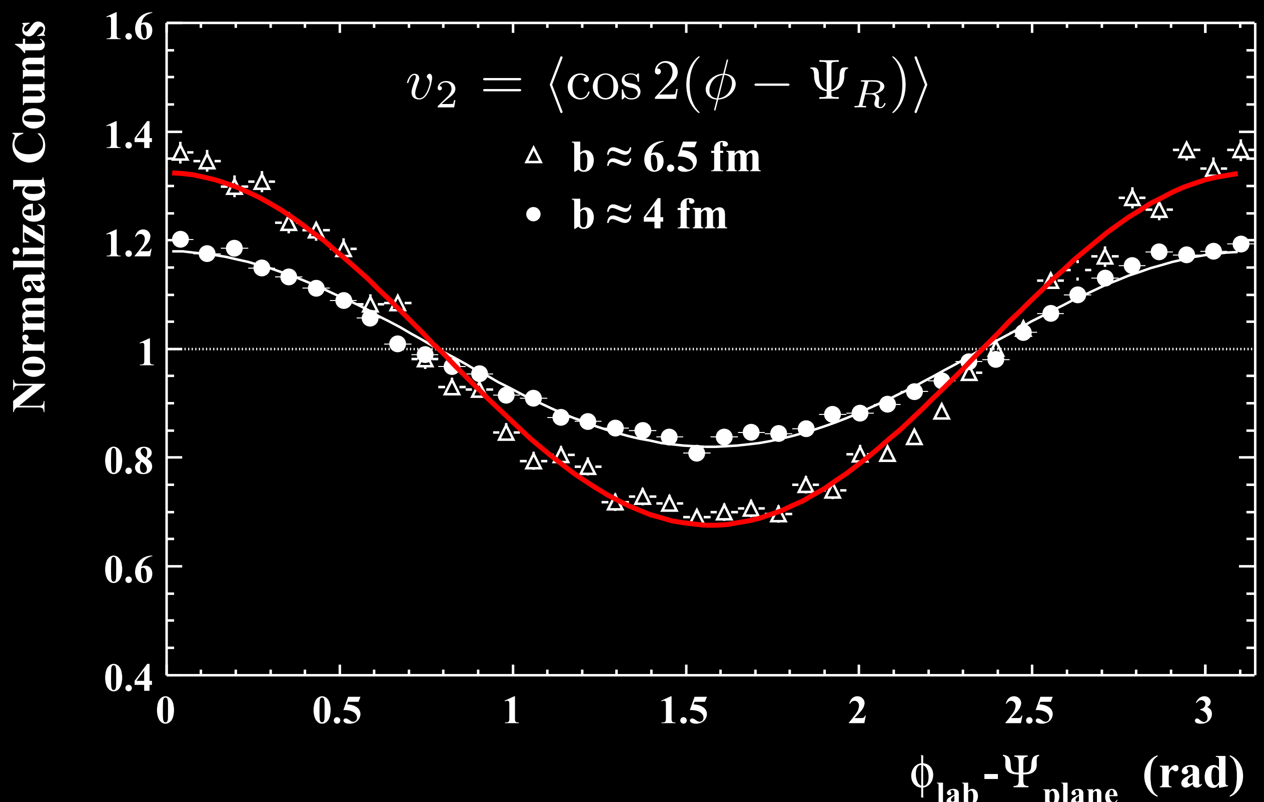


2) evolution as a **bulk <u>system</u>**

pressure gradients (larger in-plane)
push bulk "out" → "flow"



more, faster particles
seen in-plane



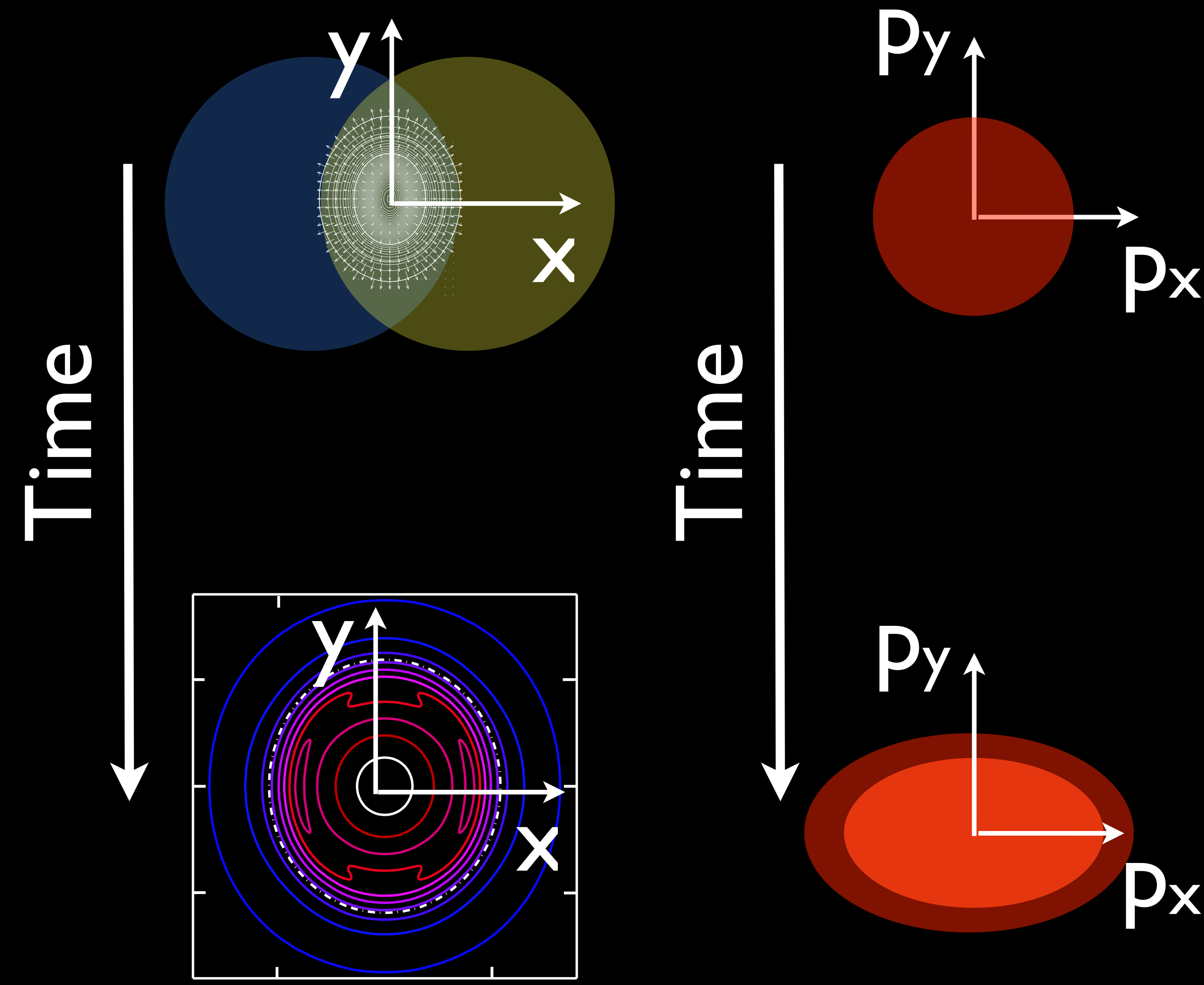
Elliptic Flow

$$\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} \qquad v_2 = \langle \cos 2\phi \rangle$$

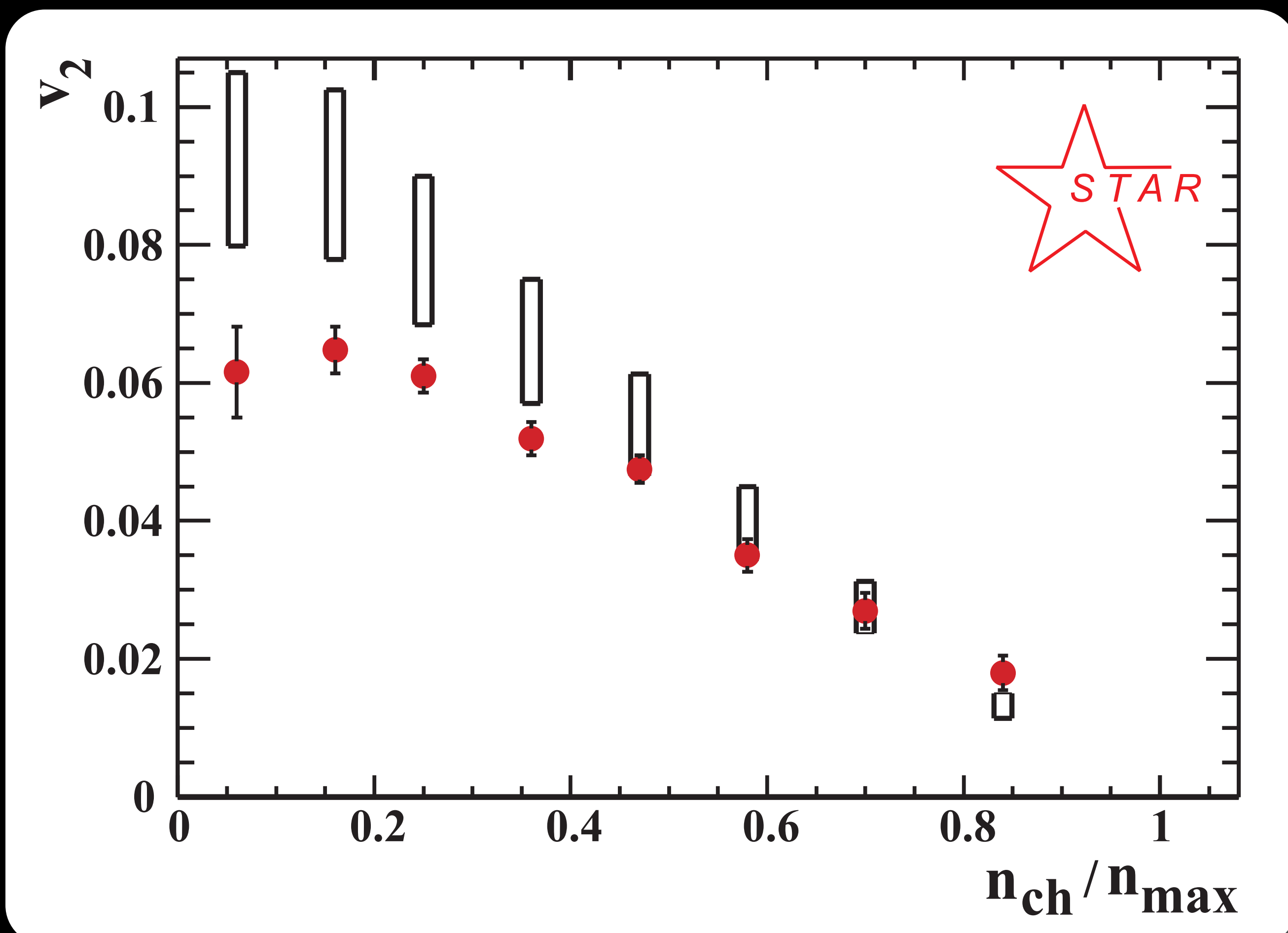
- in non central collisions coordinate space configuration is anisotropic (almond shape). However, initial momentum distribution isotropic (spherically symmetric)

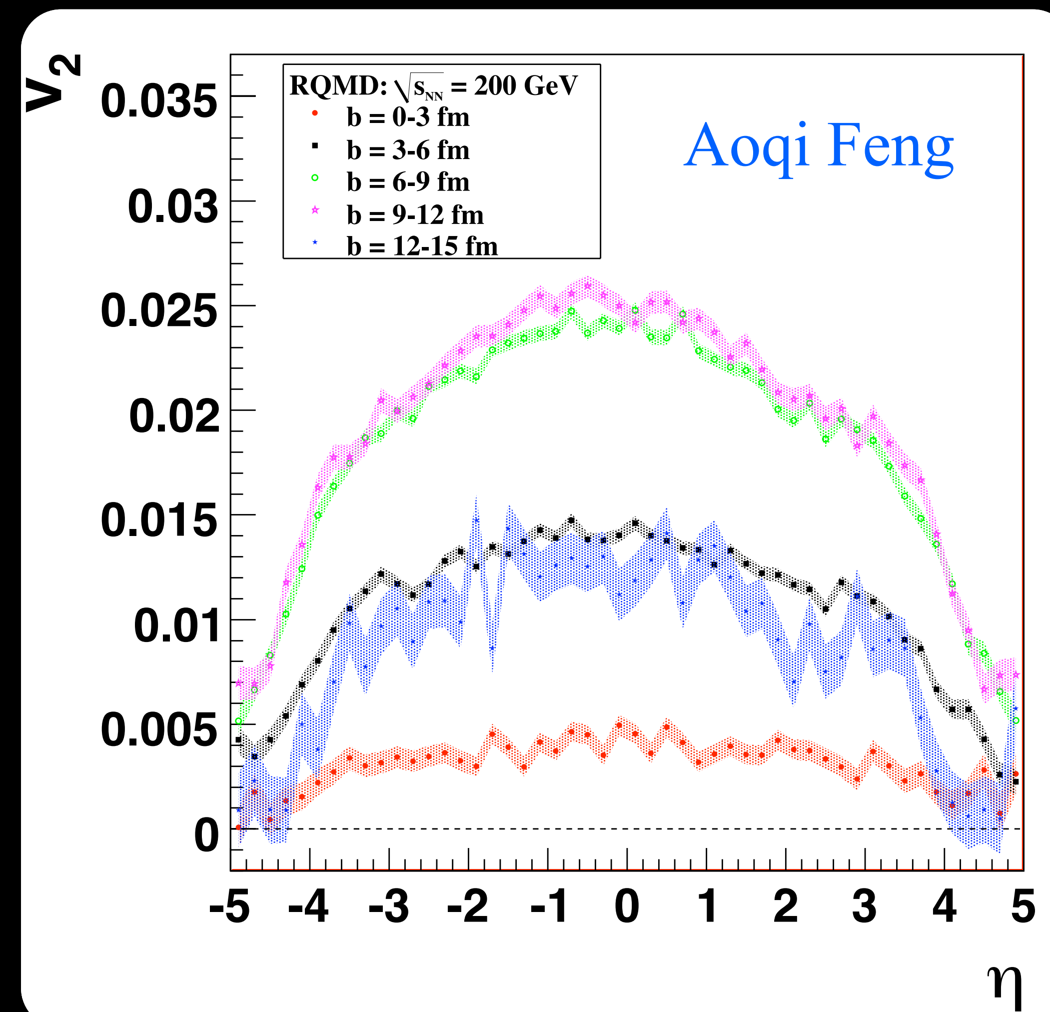
- interactions among constituents generate a pressure gradient which transforms the initial coordinate space anisotropy into the observed momentum space anisotropy → anisotropic flow

- self-quenching → sensitive to early stage



Flow at RHIC



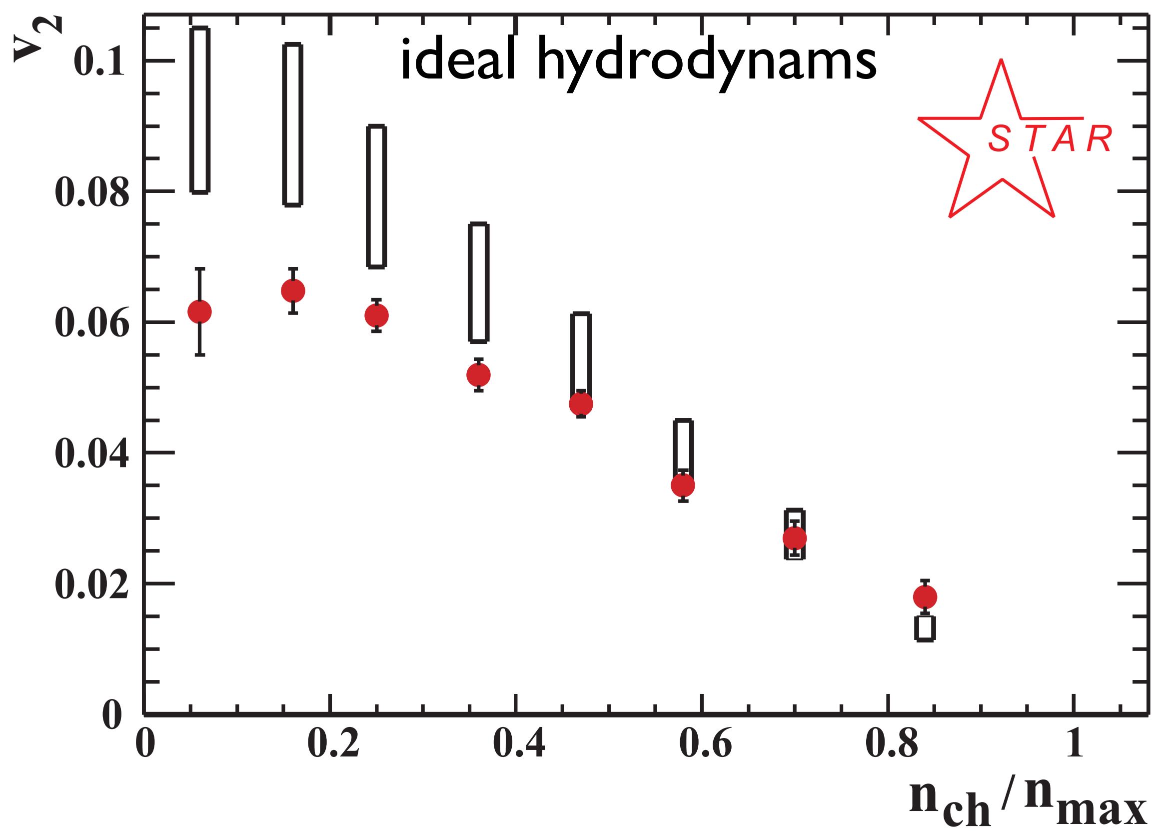


STAR Phys. Rev. Lett. 86, 402–407 (2001)

ideal hydro gets the magnitude for more central collisions

hadron transport calculations are factors 2-3 off

Anisotropic Flow



STAR Phys. Rev. Lett. 86, 402–407 (2001)

R.S., S.Voloshin, A. Poskanzer
(Berkeley 2001)

For an ideal gas (pertubative QGP) the predicted elliptic flow is negligible

Against naive expectations the measured elliptic flow agrees with an ideal liquid (negligible specific shear viscosity η/s~0)

$v_2(p_t)$ and particle mass

- on what freeze-out variables does it depend (simplification)?
  - the average velocity difference in and out of plane (due to Δp)
  - but also
    - the average freeze-out temperature
    - the average transverse flow
    - the average spatial eccentricity

Hydro Motivated Fit

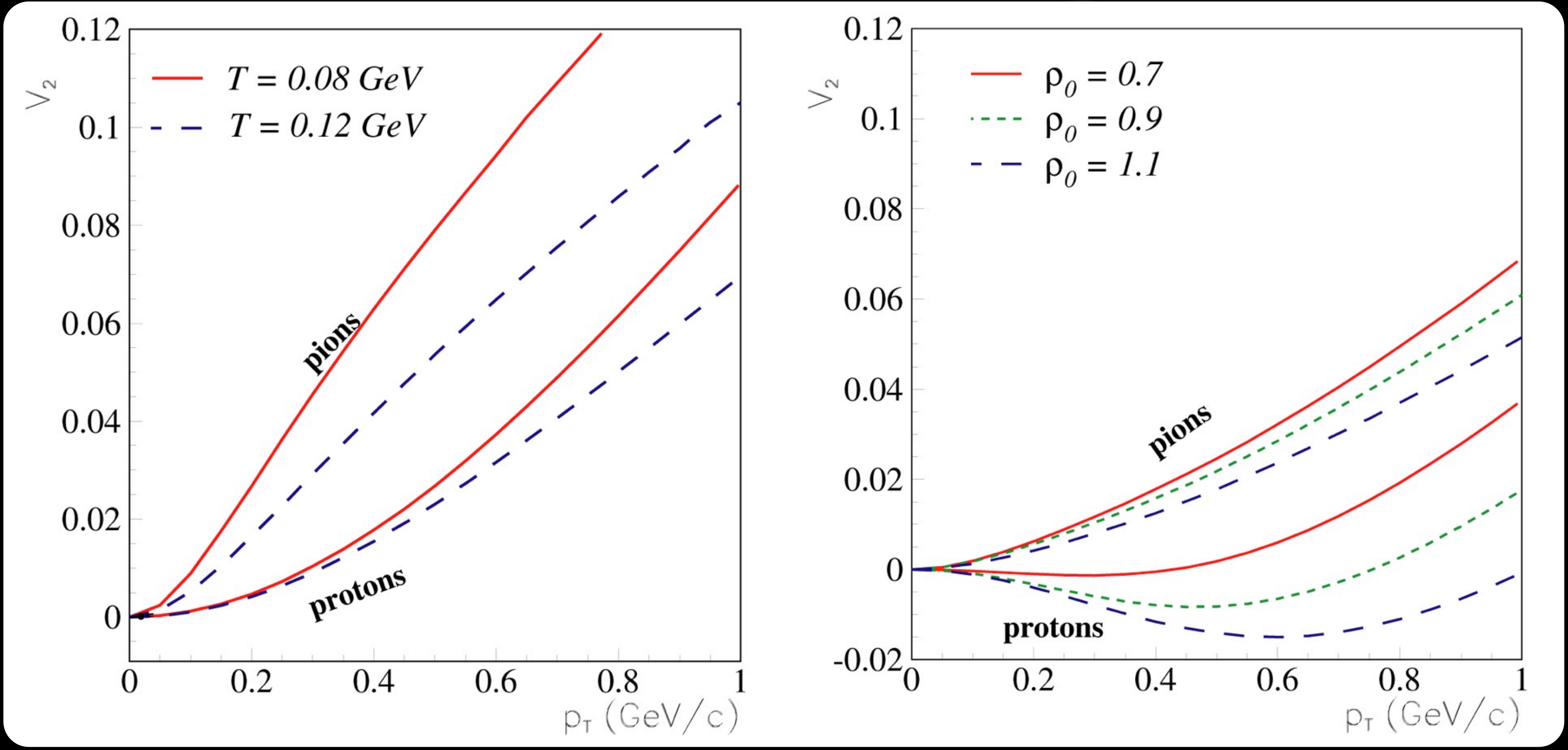
$$v_2(p_t) = \frac{\int_0^{2\pi} d\phi_b \cos(2\phi_b) I_2(\alpha_t) K_1(\beta_t)(1 + 2s_2 \cos(2\phi_b))}{\int_0^{2\pi} d\phi_b I_0(\alpha_t) K_1(\beta_t)(1 + 2s_2 \cos(2\phi_b))}$$

$$\alpha_t(\phi_b) = (\frac{p_t}{T_f}) \sinh(\rho(\phi_b)) \qquad \beta_t(\phi_b) = (\frac{m_t}{T_f}) \cosh(\rho(\phi_b))$$

$$\rho(\phi_b) = \rho_0 + \rho_a \cos(2\phi_b)$$

STAR Phys. Rev. Lett. 87, 182301 (2001)

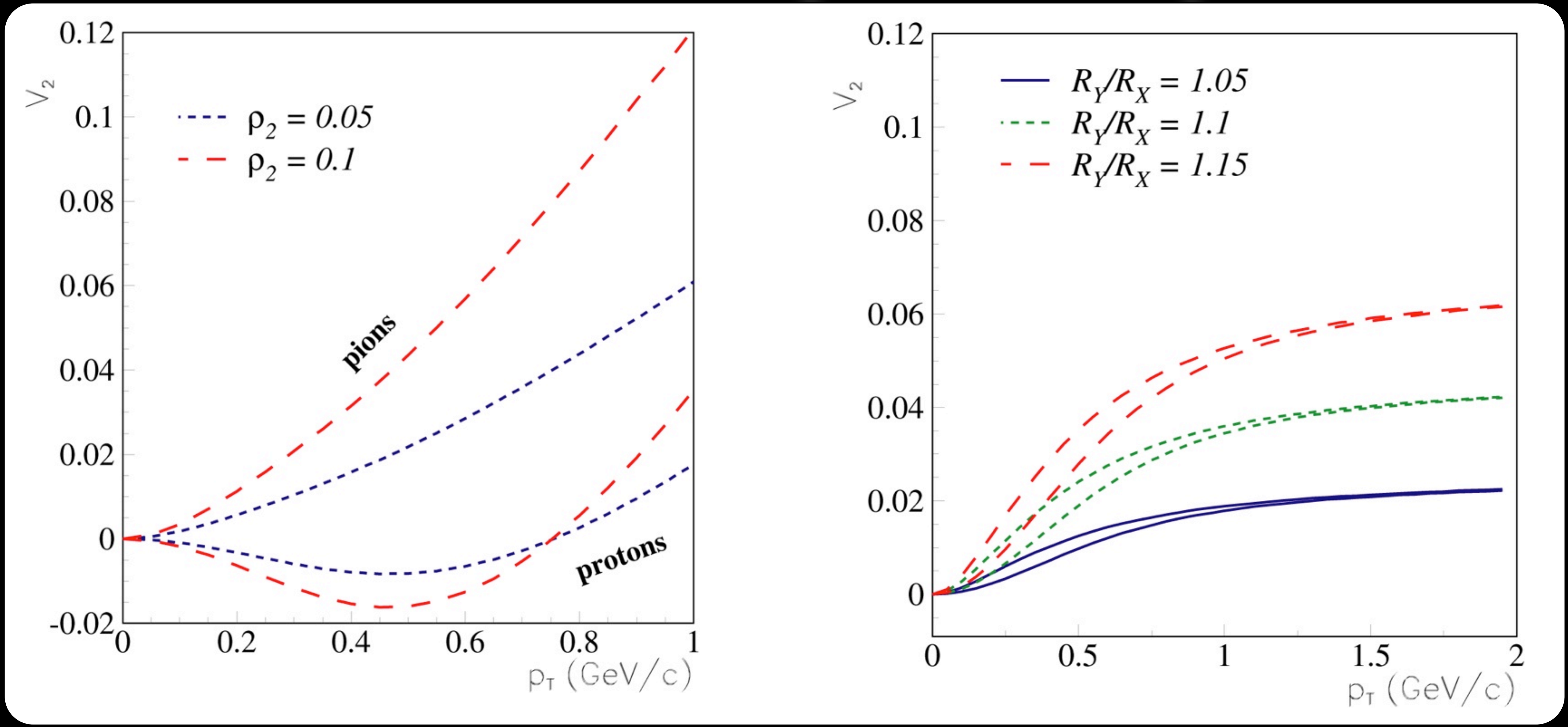
The effect of freeze-out temperature and radial flow on v_2



F. Retiere and M.A. Lisa, Phys.Rev.C70:044907,2004

- **light particle $v_2(p_t)$ very sensitive to temperature**
- **heavier particles $v_2(p_t)$ more sensitive to transverse flow**

The effect of the azimuthal asymmetric flow velocity and shape

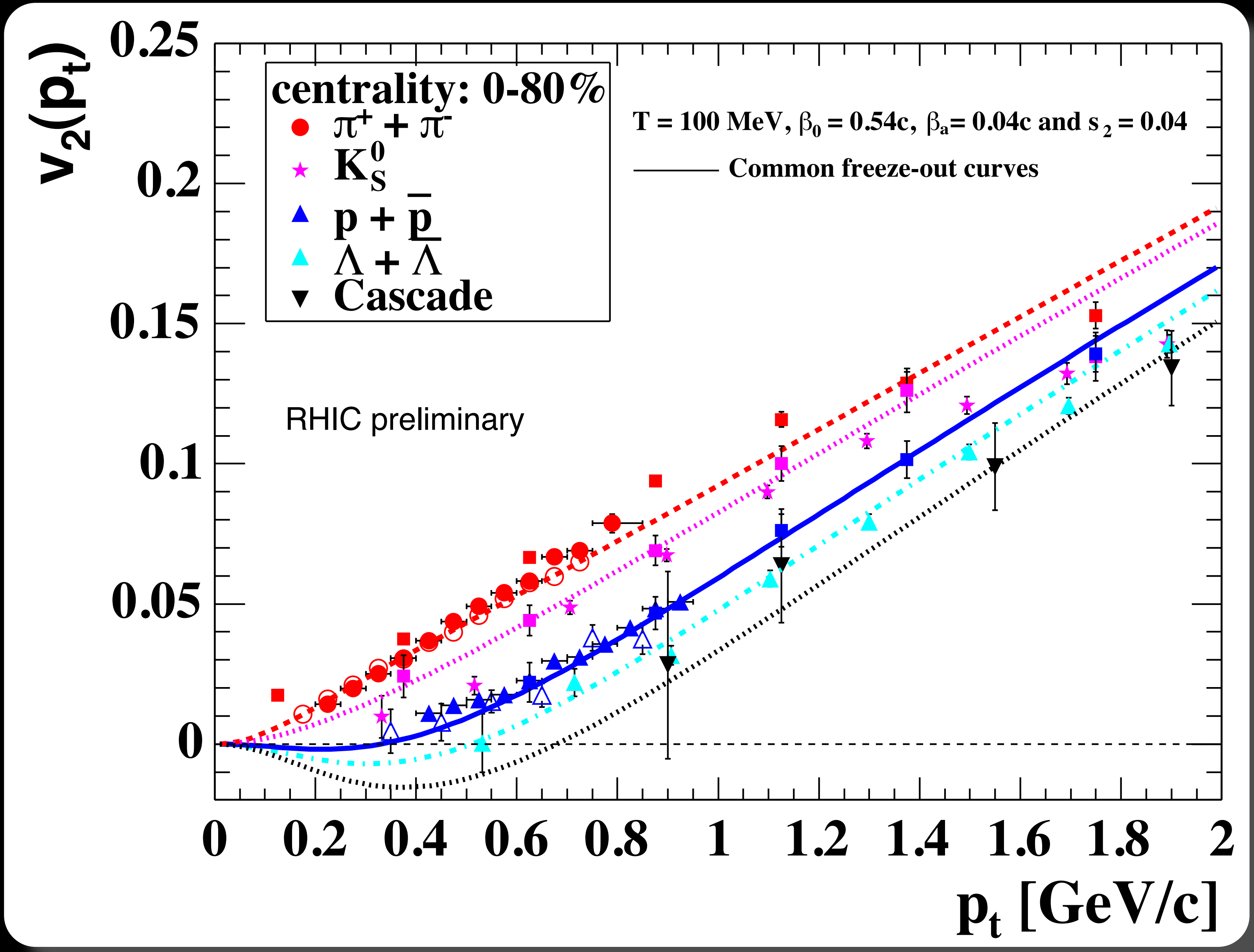


- larger value of the difference in collective velocity in and out of the reaction plane leads to larger slope of $v_2(p_t)$ above ~ <p_t> of the particle

- larger spatial anisotropy leads to larger v_2 with little mass dependence (transverse flow boosts more particles in the reaction plane)

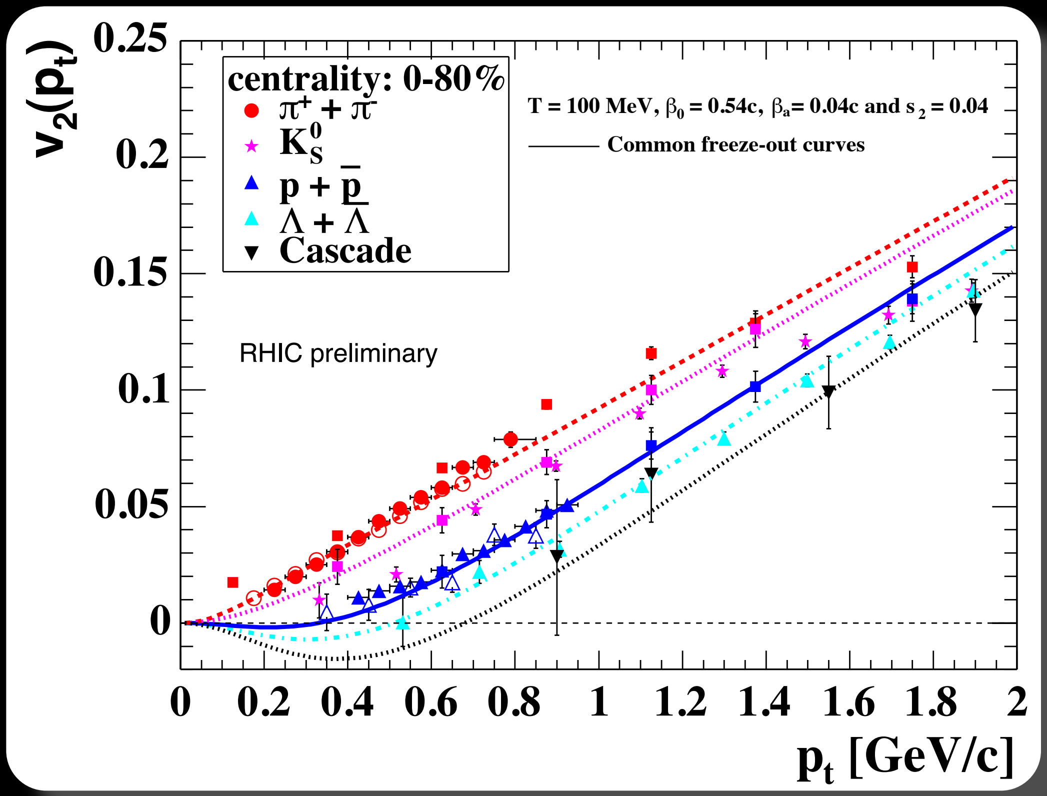
boosted thermal spectra

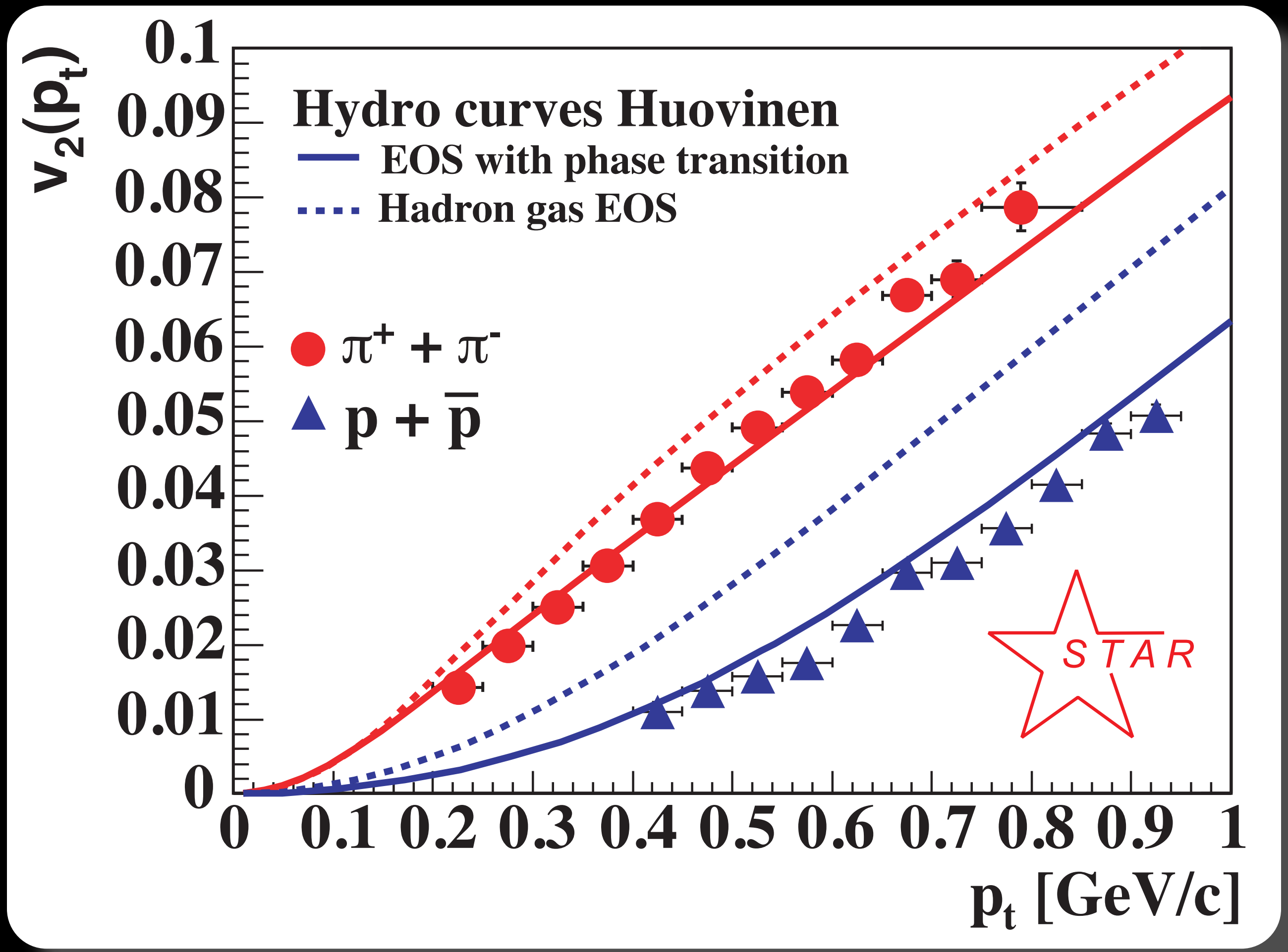
the observed particles are characterized by a single freeze-out temperature and a common azimuthal dependent boost velocity



Fits from STAR Phys. Rev. Lett. 87, 182301 (2001)

The EoS





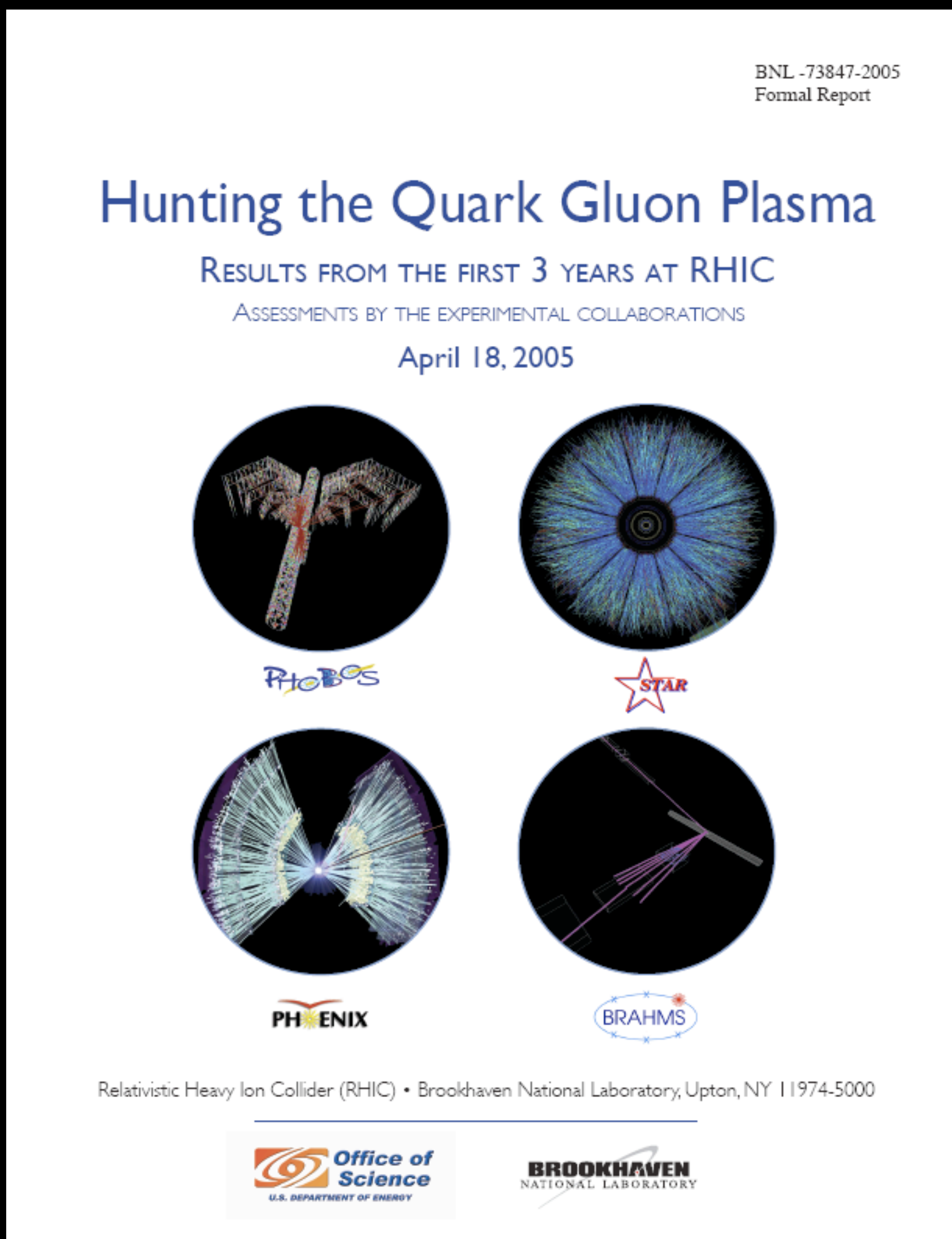
Data: STAR Phys. Rev. Lett. 87, 182301 (2001)

The species dependence is sensitive to the EoS

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005


BNL -73847-2005
Formal Report

Hunting the Quark Gluon Plasma

RESULTS FROM THE FIRST 3 YEARS AT RHIC

ASSESSMENTS BY THE EXPERIMENTAL COLLABORATIONS

April 18, 2005

PHOBOS

STAR

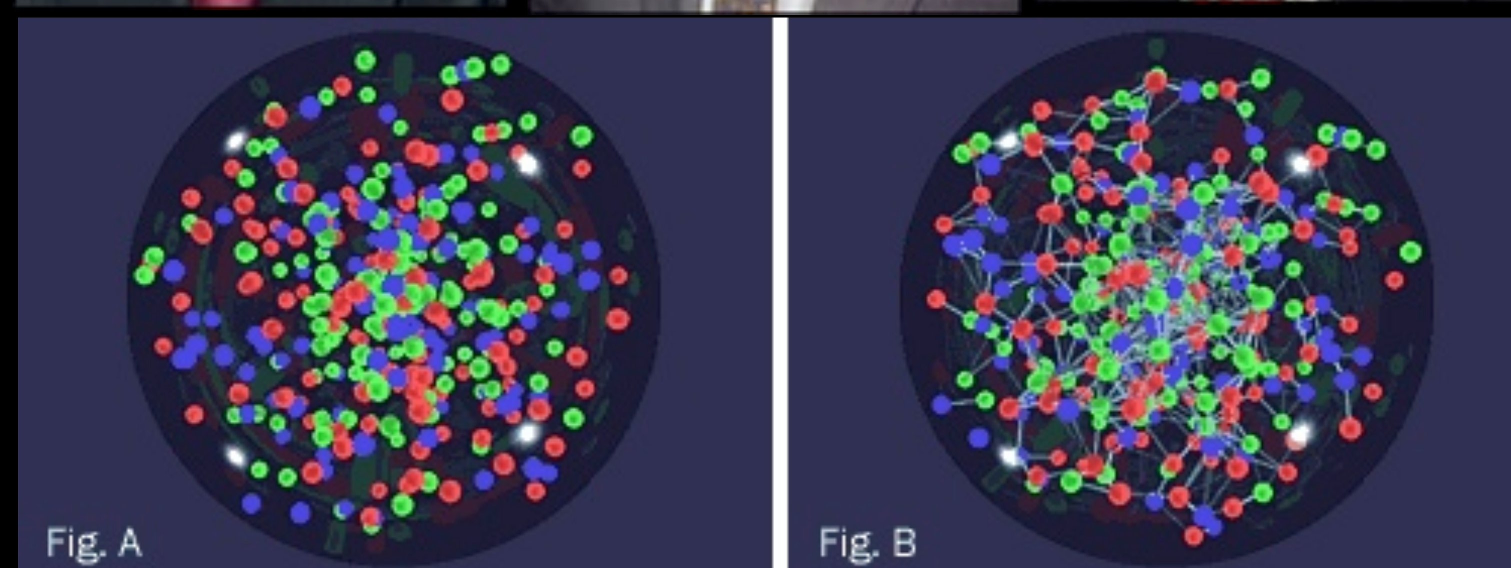
PHENIX

BRAHMS

Relativistic Heavy Ion Collider (RHIC) • Brookhaven National Laboratory, Upton, NY 11974-5000

Office of Science
U.S. DEPARTMENT OF ENERGY

BROOKHAVEN
NATIONAL LABORATORY



change description from a weakly coupled to strongly coupled system

Early Universe Went With the Flow

Posted April 18, 2005 5:57PM

Between 2000 and 2003 the lab's Relativistic Heavy Ion Collider repeatedly smashed the nuclei of gold atoms together with such force that their energy briefly generated trillion-degree temperatures. Physicists think of the collider as a time machine, because those extreme temperature conditions last prevailed in the universe less than 100 millionths of a second after the big bang.

Early Universe was a liquid

Quark-gluon blob surprises particle physicists.

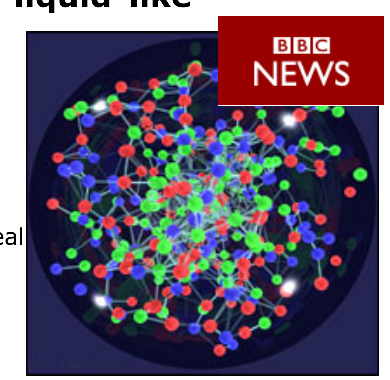
Mark Peplow

nature

The Universe consisted of a perfect liquid in its first moments, according to results from an atom-smashing experiment.

Early Universe was 'liquid-like'

Physicists say they have created a new state of hot, dense matter by crashing together the nuclei of gold atoms.



The impression is of matter that is more strongly interacting than predicted

The high-energy collisions prised open the nuclei to reveal their most basic particles, known as quarks and gluons.

The researchers, at the US Brookhaven National Laboratory, say these particles were seen to behave as an almost perfect "liquid".

Universe May Have Begun as Liquid, Not Gas

Associated Press
Tuesday, April 19, 2005; Page A05

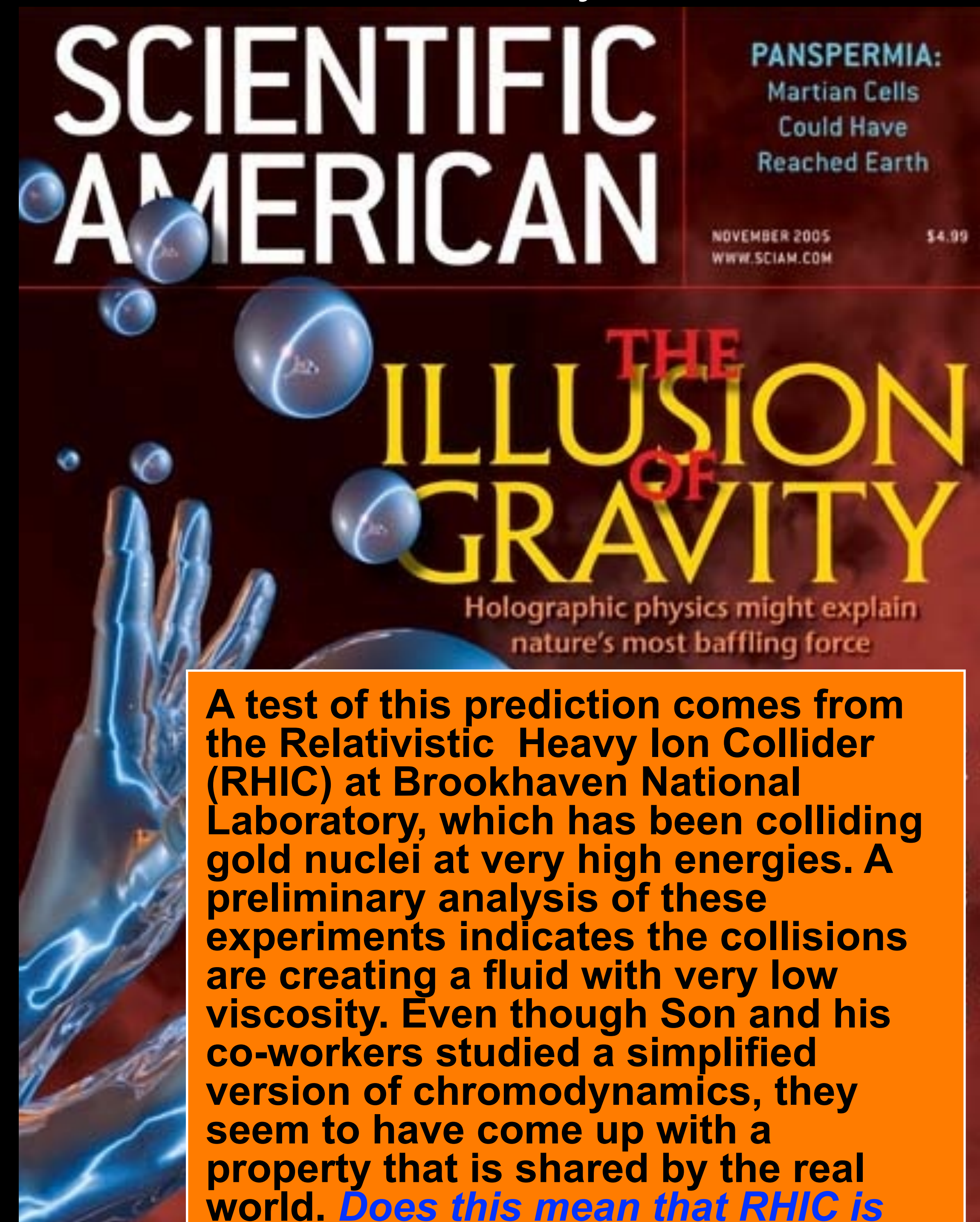
The Washington Post

New results from a particle collider suggest that the universe behaved like a liquid in its earliest moments, not the fiery gas that was thought to have pervaded the first microseconds of existence.

Allows for using 'AdS/CFT'-correspondence to calculate transport properties like the specific shear viscosity

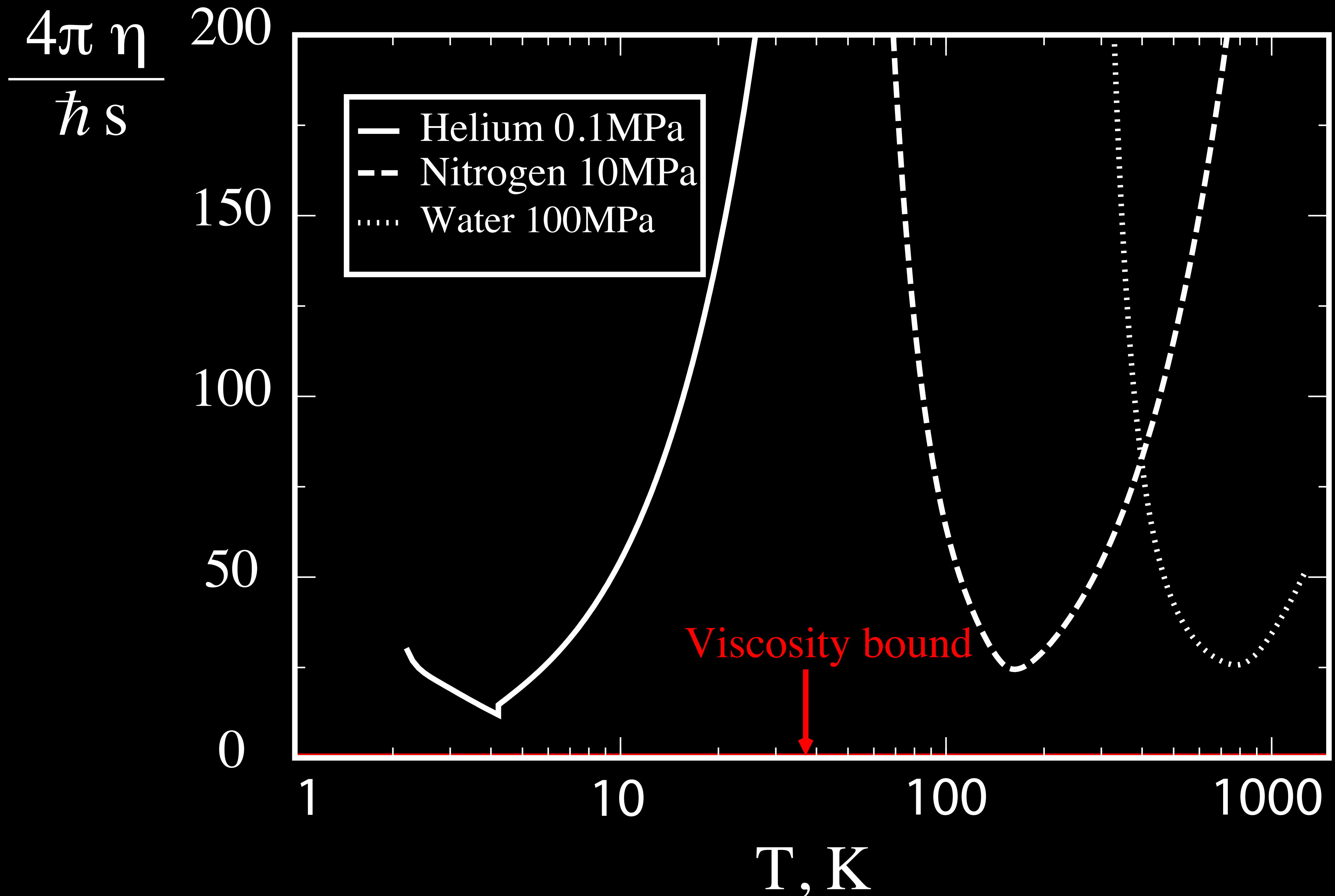
AdS/CFT calculations established a strong coupling lower limits to the specific shear viscosity which seem to be very close to the maximum allowed by the elliptic flow data

"The Illusion of Gravity" J. Maldacena



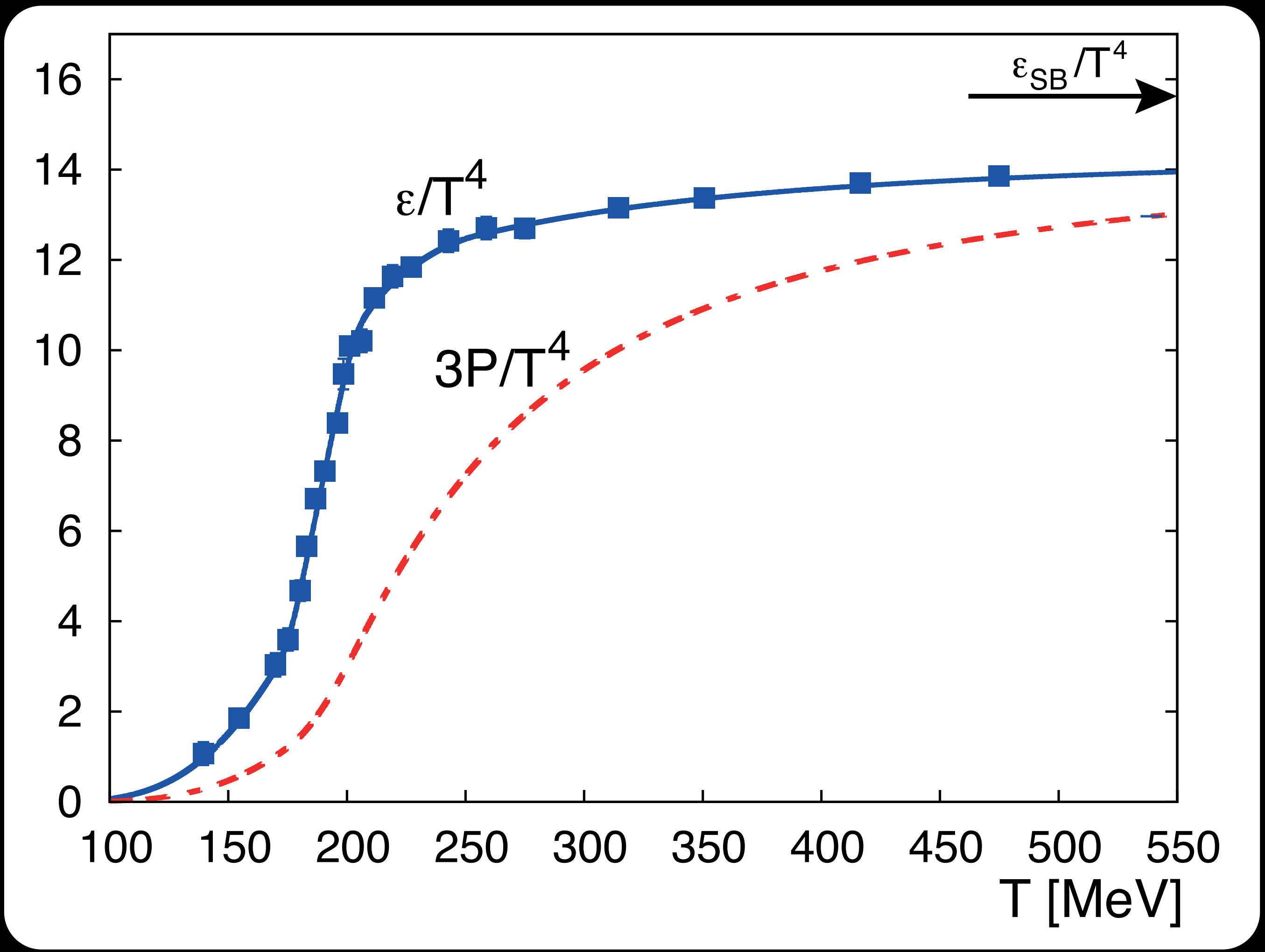
A test of this prediction comes from the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory, which has been colliding gold nuclei at very high energies. A preliminary analysis of these experiments indicates the collisions are creating a fluid with very low viscosity. Even though Son and his co-workers studied a simplified version of chromodynamics, they seem to have come up with a property that is shared by the real world. *Does this mean that RHIC is creating small five-dimensional black holes? It is really too early to tell,* both experimentally and theoretically.

AdS/CFT



Kovtun, Son, Starinets, PRL 94 (2005) 111601

QCD properties are in principle calculable from the QCD Lagrangian using Lattice QCD



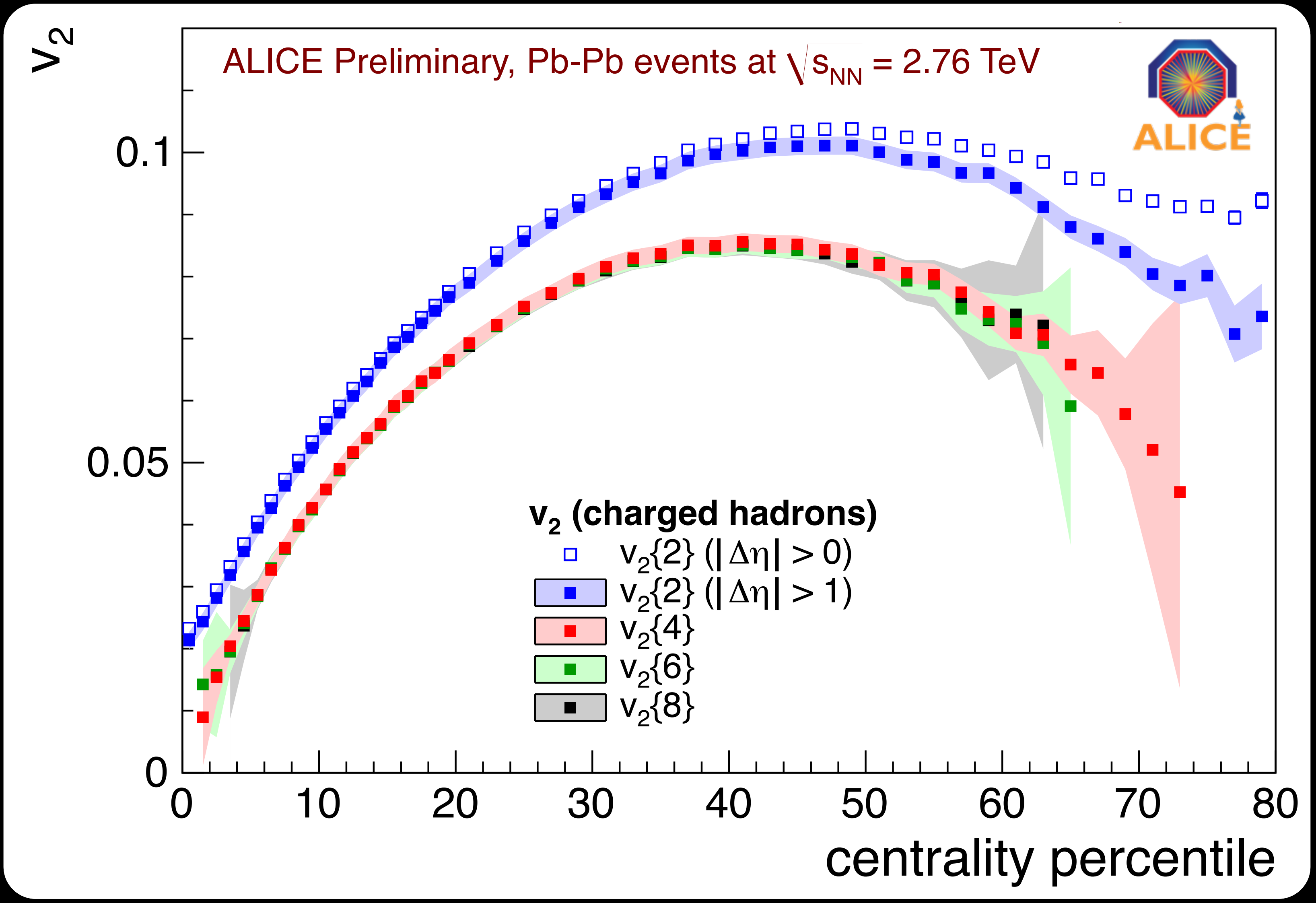
- ✓ not an ideal gas!!!!
- ✓ small difference in ϵ/T^4, still a very different system
  - ✓ ideal gas → strongly coupled liquid
- ✓ AdS/CFT reaches indeed also 0.75 of the SB limit

$$
\begin{aligned}
p &= \frac{1}{3}\epsilon = g\frac{\pi^2}{90}T^4 \\
g_{\mathrm{H}} &\approx 3 \quad g_{\mathrm{QGP}} \approx 37 \\
g &= 2_{\mathrm{spin}} \times 8_{\mathrm{gluons}} + \frac{7}{8} \times 2_{\mathrm{flavors}} \times 2_{q\bar{q}} \times 2_{\mathrm{spin}} \times 3_{\mathrm{color}}
\end{aligned}
$$

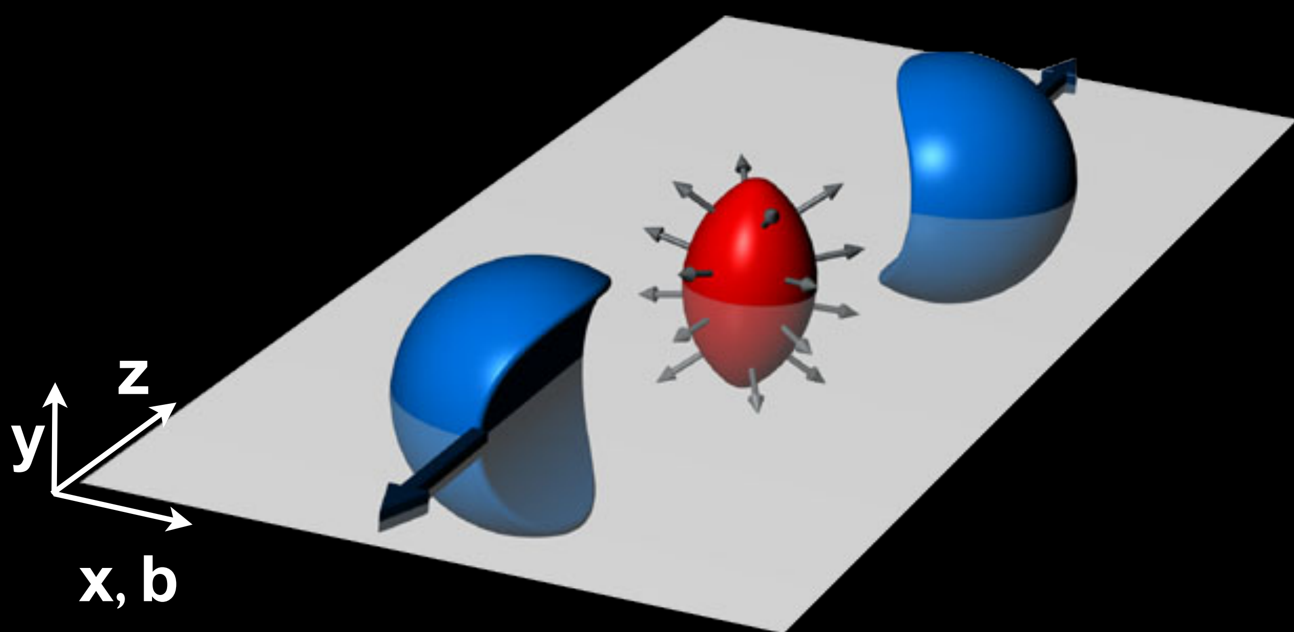
How to measure flow?

the event plane method

(multi)-particle correlations and cumulants



Anisotropic Flow



Azimuthal distributions of particles measured with respect to the reaction plane (spanned by impact parameter vector and beam axis) are not isotropic.

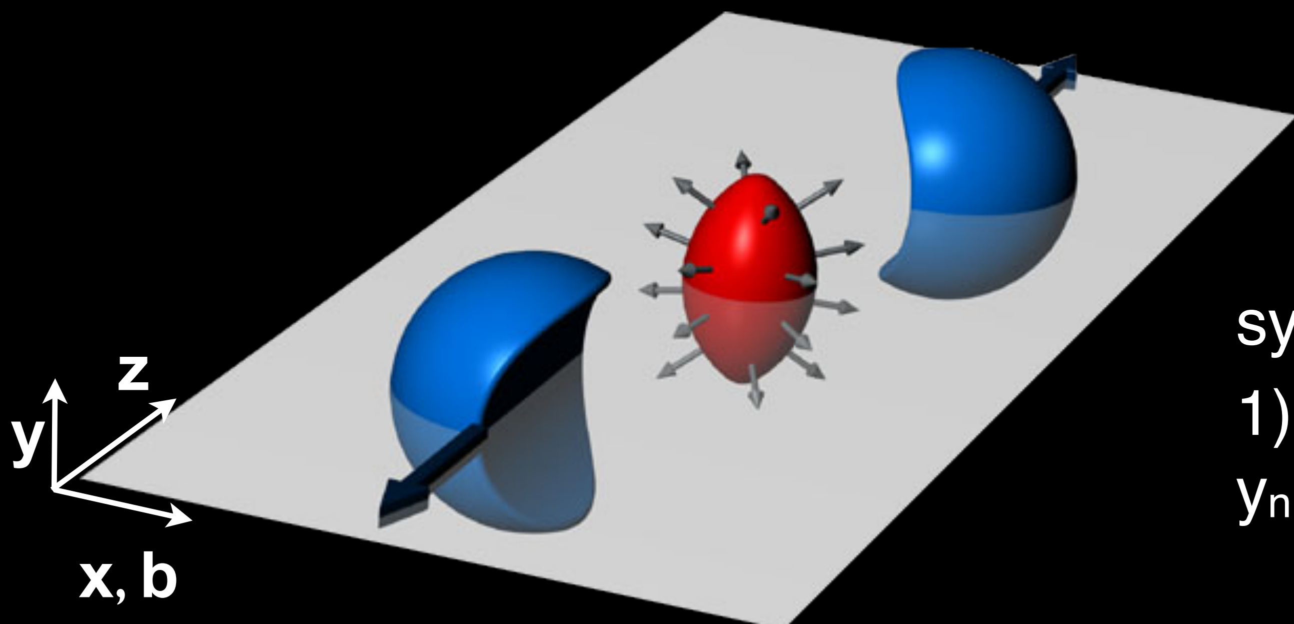
$$E\frac{d^3N}{d^3\vec{p}} = \frac{1}{2\pi}\frac{d^2N}{p_T\,dp_T\,dy}\left(1+\sum_{n=1}^{\infty}2v_n\cos\left(n\left(\phi-\Psi_{\mathrm{RP}}\right)\right)\right)$$

$$v_n = \langle \cos n(\phi - \Psi_{\mathrm{RP}})\rangle$$

harmonics v_n quantify anisotropic flow

S. Voloshin and Y. Zhang (1996)

Azimuthal distributions



$$r(\varphi) = \frac{x_0}{2\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty}[x_n \cos(n\varphi) + y_n \sin(n\varphi)]$$

symmetries reduce the number of parameters

1) particle yield at ϕ and -ϕ should be equal ->
y_n = 0 (no sin terms)

$$\frac{dN}{d\varphi} = \frac{1}{2\pi}\left[1 + \sum_{n=1}^{\infty} 2v_n \cos[n(\varphi - \Psi_R)]\right]$$

2) particle yield at ϕ and ϕ+π should be equal ->
cos(nϕ) = 0 for odd n

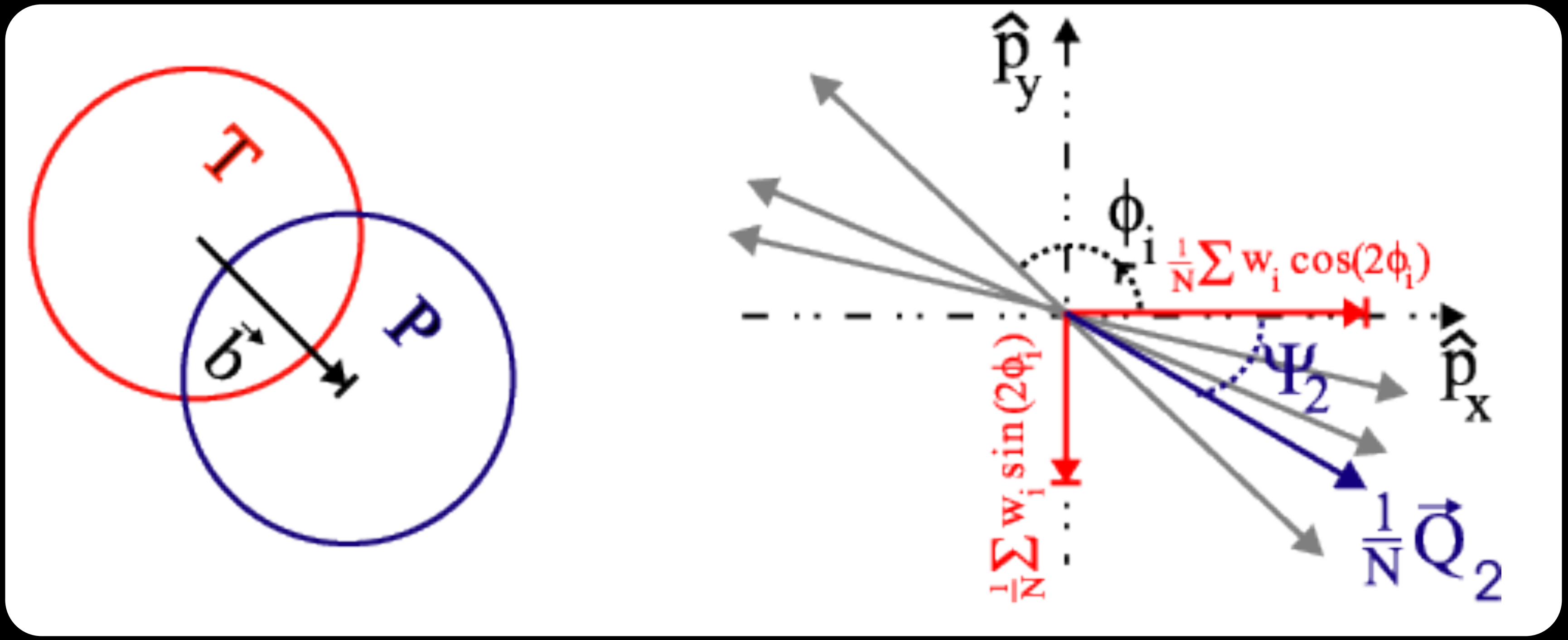
only even harmonics at mid-rapidity, v_2, v_4, v_6, etc

$$v_n = \langle\cos[n(\varphi - \Psi_R)]\rangle$$

The Event Plane Method

Event Plane Method

the event plane is an experimental estimate of the reaction plane



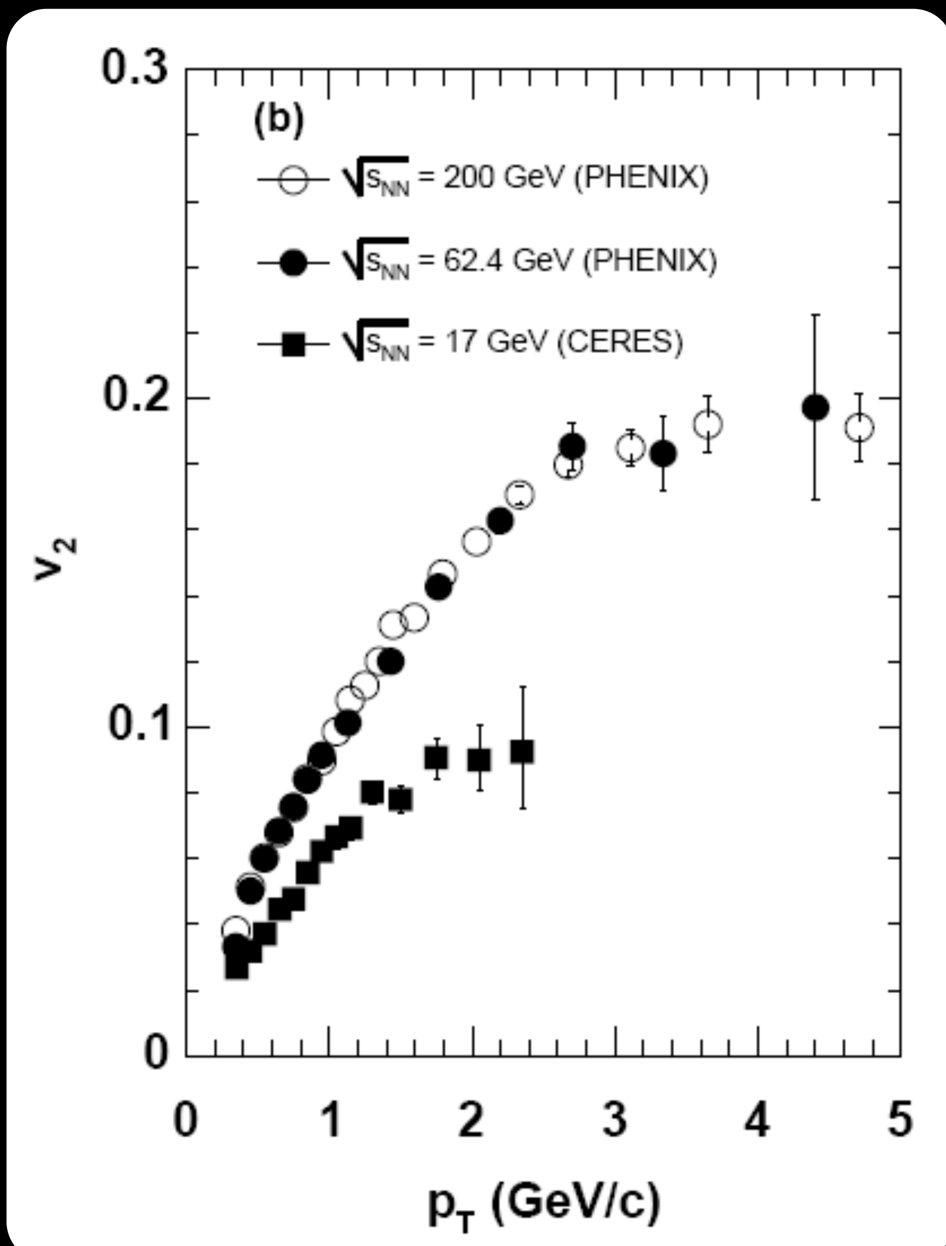
$$Q_{nx} = \sum_i w_i \cos\left(n\phi_i\right)$$

$$Q_{ny} = \sum_i w_i \sin\left(n\phi_i\right)$$

$$\Psi_n^{EP} = \frac{1}{n}\tan^{-1}\left(\frac{Q_{ny}}{Q_{nx}}\right)$$

Event Plane Method

weights to optimize the Q-vector



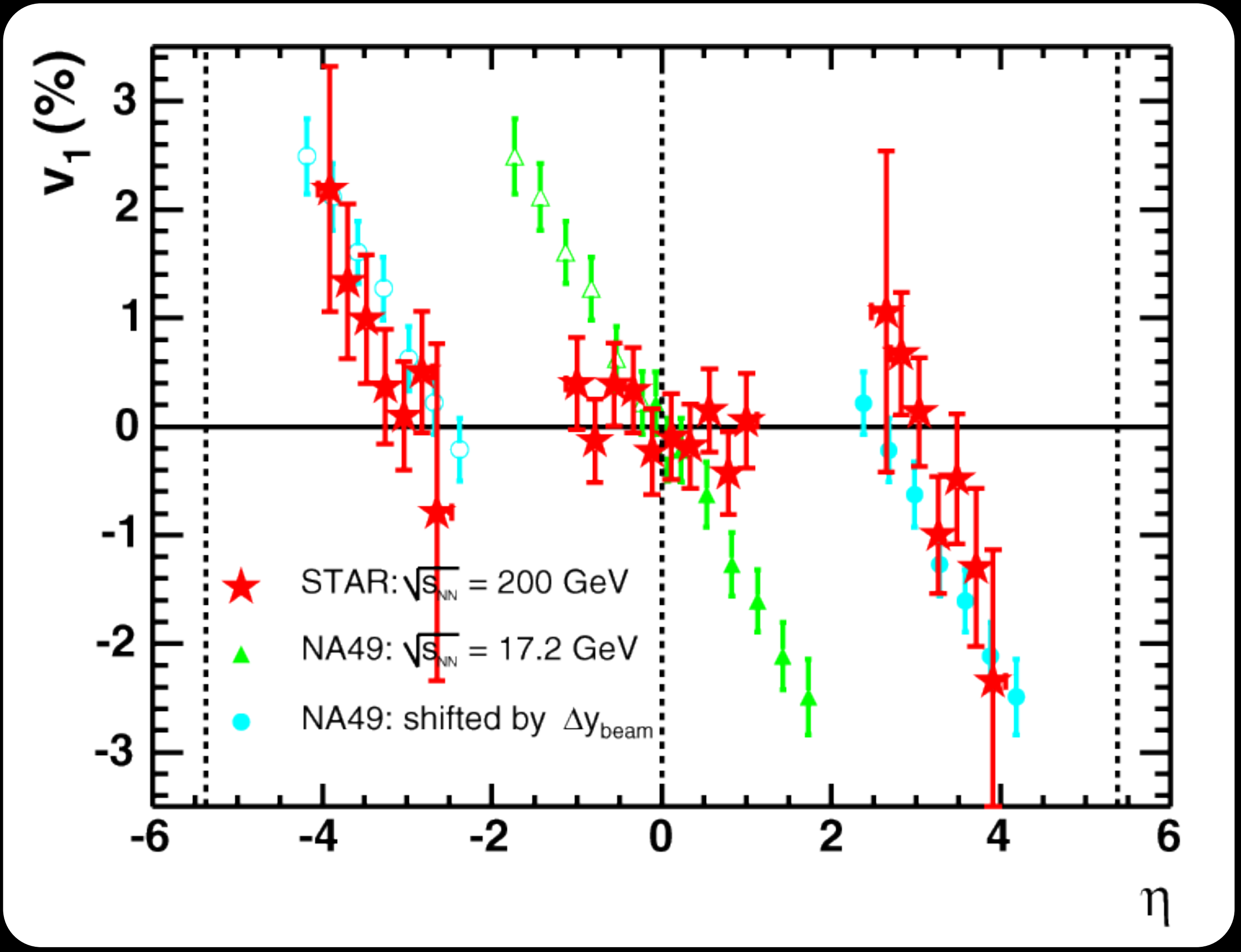
$$Q_{nx} = \sum_i w_i \cos(n\phi_i)$$

$$Q_{ny} = \sum_i w_i \sin(n\phi_i)$$

weights are in general p_t dependent

Event Plane Method

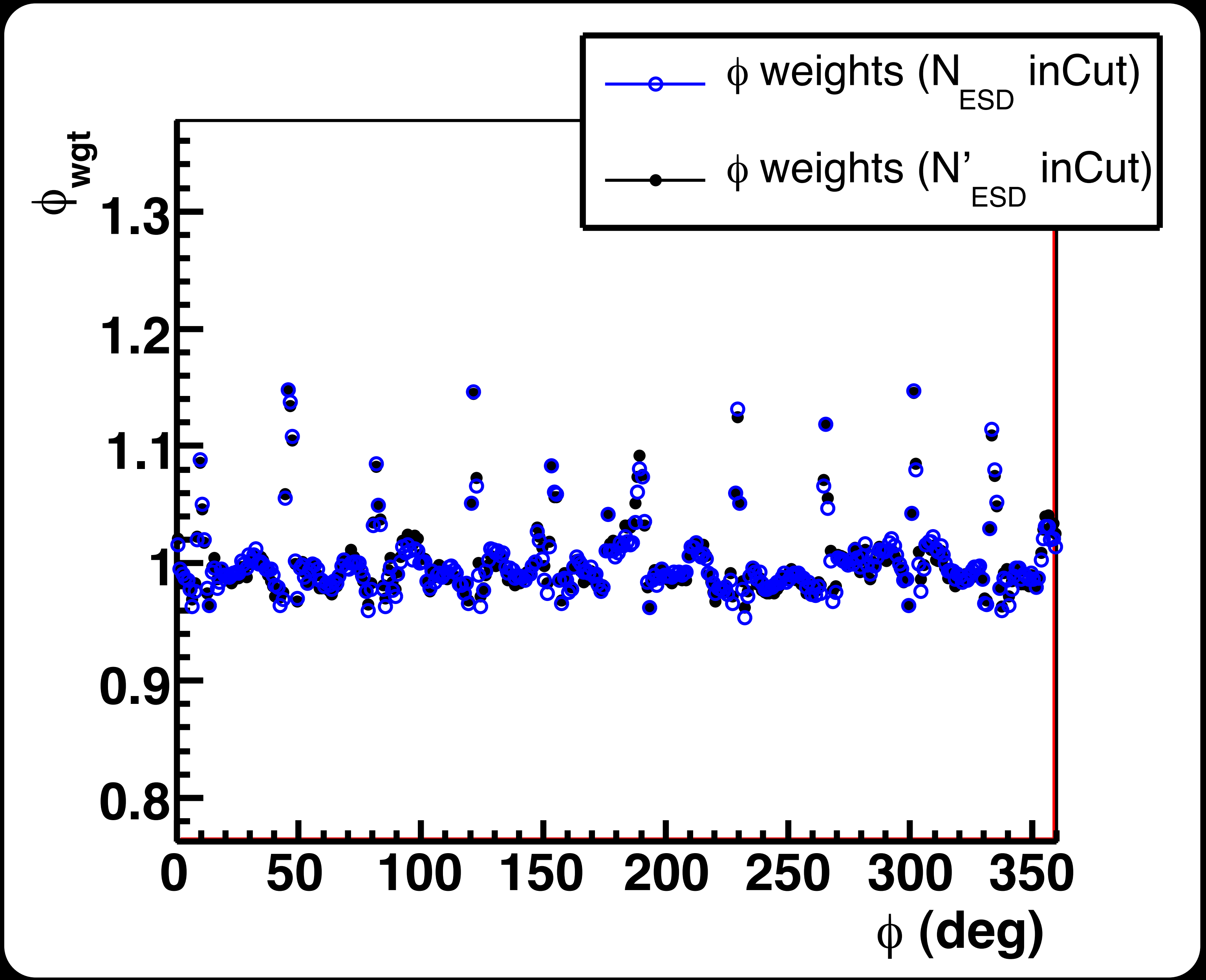
weights to optimize the Q-vector



weights are in general η dependent (in magnitude and for odd harmonics in sign)

Event Plane Method

weights to correct for detector asymmetries



requires two passes over the data

resolution and subevents

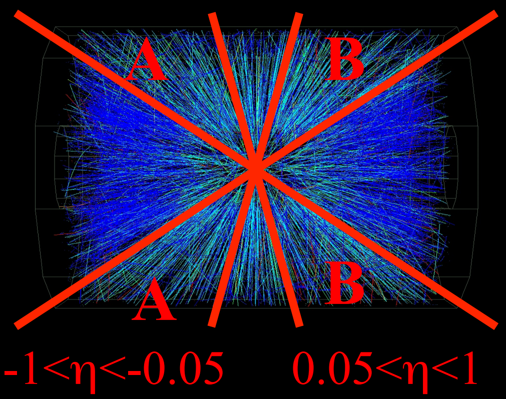
- due to the finite number of detected particles there is a limited resolution in the event plane angle

$$v_n^{\mathrm{obs}} = \left\langle \cos n \left(\phi_i - \Psi_n^{EP} \right) \right\rangle$$

$$v_n = \frac{v_n^{\mathrm{obs}}}{\langle \cos n \left(\Psi_n^{EP} - \Psi_R \right) \rangle}$$

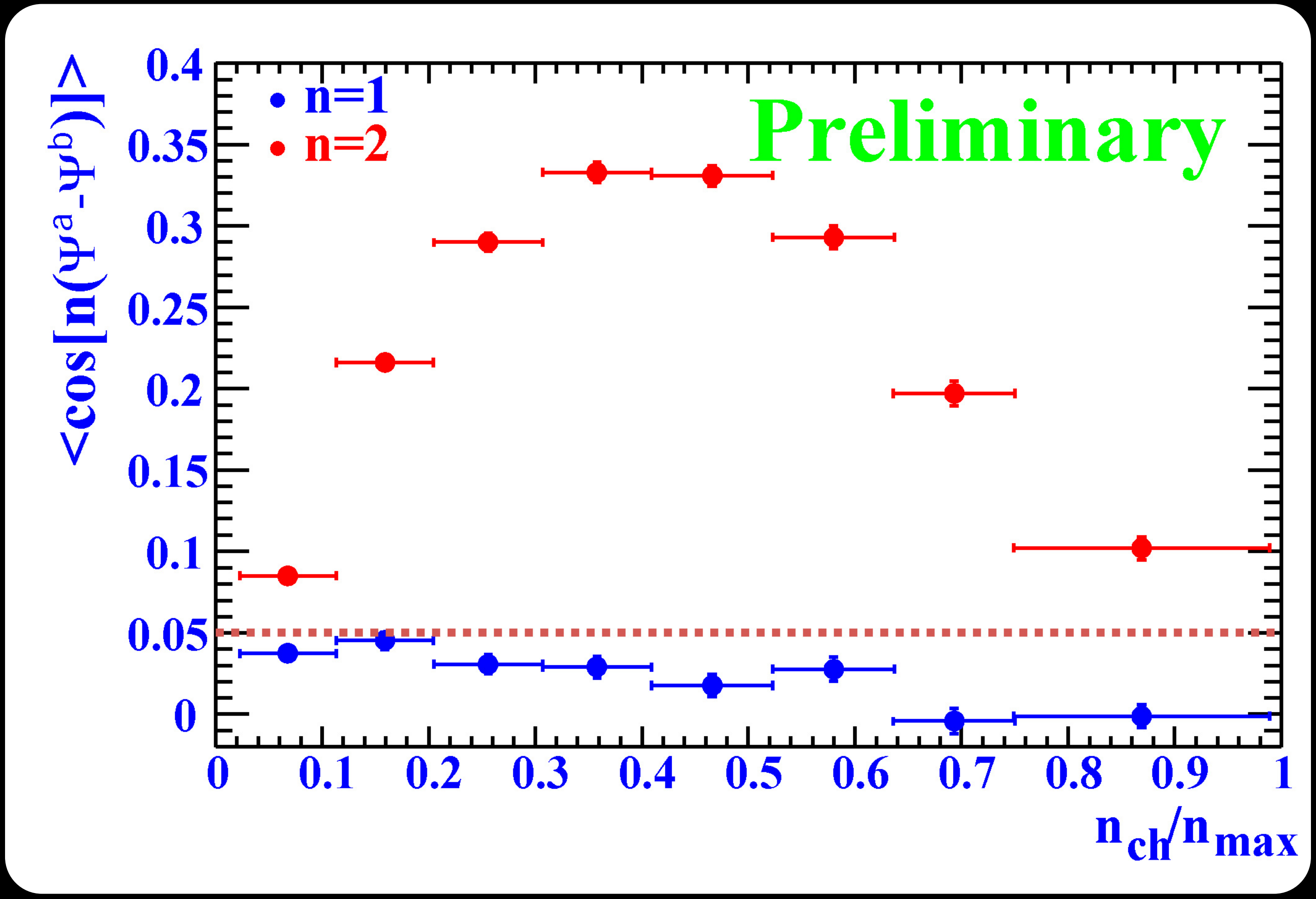
- one can correct for that with subevents

$$\left\langle \cos n \left(\Psi_n^{EP} - \Psi_R \right) \right\rangle = \mathrm{C} \times \sqrt{\langle \cos n \left(\Psi_n^a - \Psi_n^b \right) \rangle}$$



resolution and subevents

- when dominated by flow the event plane resolution scales with $M^{1/2} \times v_2$ (when not too close to 1)
- gives very characteristic dependence on centrality
- nonflow will scale very different: the red line was first STAR estimate of nonflow



STAR, PRL 86, (2001) 402, Nucl. Phys. A698 (2002) 193

Methods using directly (multi)-particle correlations

measure anisotropic flow

$$v_n \equiv \langle e^{in(\varphi - \Psi_R)} \rangle$$

- since reaction plane cannot be measured event-by-event, consider quantities which do not depend on it's orientation: multi-particle azimuthal correlations

$$\left\langle e^{in(\phi_1-\phi_2)} \right\rangle = \left\langle e^{in\phi_1} \right\rangle \left\langle e^{-in\phi_2} \right\rangle + \left\langle e^{in(\phi_1-\phi_2)} \right\rangle_{\text{corr}}$$

zero for symmetric detector when averaged over many events

$$\begin{aligned}
\langle\langle 2 \rangle\rangle \equiv \left\langle\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\right\rangle &= \left\langle\!\left\langle e^{in(\phi_1-\Psi_{\mathrm{RP}}-(\phi_2-\Psi_{\mathrm{RP}}))} \right\rangle\!\right\rangle \\
&= \left\langle\!\left\langle e^{in(\phi_1-\Psi_{\mathrm{RP}})} \right\rangle \left\langle e^{-in(\phi_2-\Psi_{\mathrm{RP}})} \right\rangle\!\right\rangle \\
&= \left\langle v_n^2 \right\rangle
\end{aligned}$$

- assuming that only correlations with the reaction plane are present

intermezzo

- why do we define the correlations like this:

$$\Big\langle \langle x \rangle_{\text{particles in single event}} \Big\rangle_{\text{over events}}$$

  - easy to relate to v_n

$$\left\langle\left\langle e^{in(\phi_1 - \phi_2)} \right\rangle\right\rangle$$

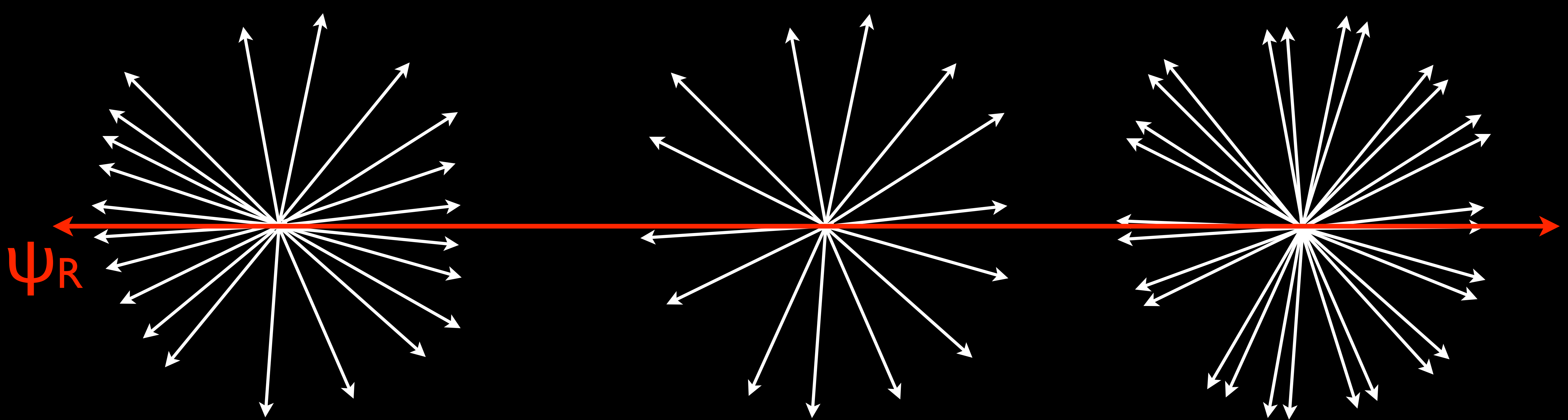
$$\left\langle\left\langle e^{in(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \right\rangle\right\rangle$$

  - vanishes for independent particles
  - do not depend on frame $\Phi + \alpha$ (shifting all particles by fixed angle) gives same answer for the correlation

nonflow

- however, there are other sources of correlations between the particles which are not related to the reaction plane which break the factorization, lets call those δ_2 for two particle correlations

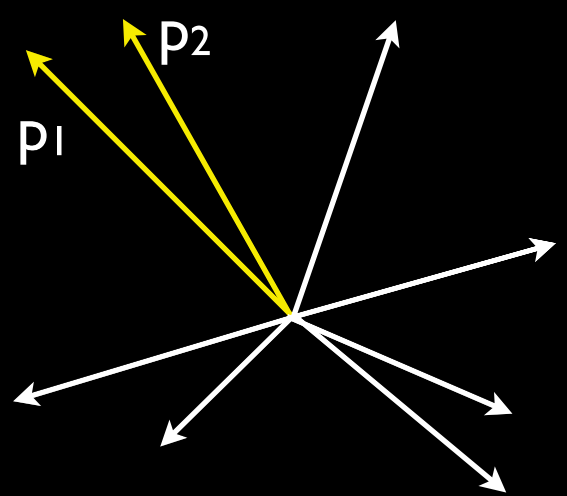
$$\left\langle\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\right\rangle = \langle v_n^2 \rangle + \delta_2$$



$v_2 > 0$, $v_2\{2\} > 0$ $v_2 = 0$, $v_2\{2\} = 0$ $v_2 = 0$, $v_2\{2\} > 0$

nonflow

$$\left\langle\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\right\rangle = \langle v_n^2 \rangle + \delta_2$$



particle 1 coming from the resonance. Out of remaining M-1 particles there is only one which is coming from the same resonance, particle 2. Hence a probability that out of M particles we will select two coming from the same resonance is ~ 1/(M-1). From this we can draw a conclusion that for large multiplicity: $\delta_2 \sim 1/M$

- therefore to reliably measure flow:

$$v_n^2 \gg 1/M \quad \Rightarrow \quad v_n \gg 1/M^{1/2}$$

- not easily satisfied: M=200 v_n >> 0.07

can we do better?

- use the fact that flow is a correlation between all particles: use multi-particle correlations

$$\left\langle\!\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\!\right\rangle = v_n^2 + \delta_2$$

$$\left\langle\!\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle\!\right\rangle = v_n^4 + 4v_n^2\delta_2 + 2\delta_2^2 + \delta_4$$

- not so clear if we gained something

Cumulants

JOURNAL OF THE PHYSICAL SOCIETY OF JAPAN, Vol. 17, No. 7, JULY 1962

Generalized Cumulant Expansion Method*

Ryogo KUBO
Department of Physics, University of Tokyo

(Received April 11, 1962)

The moment generating function of a set of stochastic variables defines the cumulants or the semi-invariants and the cumulant function. It is possible, simply by formal properties of exponential functions, to generalize to a great extent the concepts of cumulants and cumulant function. The stochastic variables to be considered need not be ordinary c-numbers but they may be q-numbers such as used in quantum mechanics. The exponential function which defines a moment generating function may be any kind of generalized exponential, for example an ordered exponential with a certain prescription for ordering q-number variables. The definition of average may be greatly generalized as far as the condition is fulfilled that the average of unity is unity. After statements of a few basic theorems these generalizations are discussed here with certain examples of application. This generalized cumulant expansion provides us with a point of view from which many existent methods in quantum mechanics and statistical mechanics can be unified.

$$
\begin{aligned}
&\langle X_j\rangle_c=\langle X_j\rangle\\
&\langle X_j^2\rangle_c=\langle X_j^2\rangle-\langle X_j\rangle^2\\
&\langle X_jX_l\rangle_c=\langle X_jX_l\rangle-\langle X_j\rangle\langle X_l\rangle\\
&\langle X_jX_kX_l\rangle_c=\langle X_jX_kX_l\rangle\\
&\qquad-\{\langle X_j\rangle\langle X_kX_l\rangle+\langle X_k\rangle\langle X_lX_j\rangle+\langle X_l\rangle\langle X_jX_k\rangle\}\\
&\qquad+2\langle X_j\rangle\langle X_k\rangle\langle X_l\rangle\\
&\langle X_jX_kX_lX_m\rangle_c=\langle X_jX_kX_lX_m\rangle\\
&\quad-\{\langle X_j\rangle\langle X_kX_lX_m\rangle+\langle X_k\rangle\langle X_jX_lX_m\rangle+\langle X_l\rangle\langle X_jX_kX_m\rangle+\langle X_m\rangle\langle X_jX_kX_l\rangle\}\\
&\quad-\{\langle X_jX_k\rangle\langle X_lX_m\rangle+\langle X_jX_l\rangle\langle X_kX_m\rangle+\langle X_jX_m\rangle\langle X_kX_l\rangle\}\\
&\quad+2\{\langle X_j\rangle\langle X_k\rangle\langle X_lX_m\rangle+\langle X_j\rangle\langle X_l\rangle\langle X_kX_m\rangle+\langle X_j\rangle\langle X_m\rangle\langle X_kX_l\rangle\\
&\qquad+\langle X_jX_k\rangle\langle X_l\rangle\langle X_m\rangle+\langle X_jX_l\rangle\langle X_k\rangle\langle X_m\rangle+\langle X_jX_m\rangle\langle X_k\rangle\langle X_l\rangle\}\\
&\quad-6\langle X_j\rangle\langle X_k\rangle\langle X_l\rangle\langle X_m\rangle
\end{aligned}
\tag{2.8}
$$

cumulants allow us to see if there are correlations in the system between particles (cumulants nonzero only mathematical proof)

Cumulants

The most general decomposition of the 2-particle correlation is given by:

$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c$$

The 2-particle correlation is the product of the single particle distribution
+ a genuine 2-particle correlation

There is no way to measure the genuine two particle correlation directly
but by rewriting the equation we can obtain them
$<..>_c$ are the cumulants

The second order cumulants is rather trivial:

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

Cumulants

The most general decomposition of the 3-particle correlation is given by:

$$\langle X_1X_2X_3\rangle = \langle X_1\rangle\langle X_2\rangle\langle X_3\rangle+\langle X_1X_2\rangle_c\langle X_3\rangle+\langle X_1X_3\rangle_c\langle X_2\rangle+\langle X_2X_3\rangle_c\langle X_1\rangle+ +\langle X_1X_2X_3\rangle_c$$

observables are the three particle correlation, the single particle distributions and the second order cumulant we already extracted before from the two-particle correlation and single particle distributions

With these measurements we extract the 3rd order cumulant $<X_1X_2X_3>c$:

$$\langle X_1X_2X_3\rangle_c = \langle X_1X_2X_3\rangle - (\langle X_1\rangle\langle X_2X_3\rangle + \langle X_2\rangle\langle X_3X_1\rangle + \langle X_3\rangle\langle X_1X_2\rangle) + 2\langle X_1\rangle\langle X_2\rangle\langle X_3\rangle$$

Following this strategy we can extract also all higher order cumulants

If higher order cumulants are zero there is no way to prove that there are genuine multi-particle correlation, no matter what two or multiparticles show (only way to prove this mathematically)

Can we do better?

- build cumulants with the multi-particle correlations

Ollitrault and Borghini

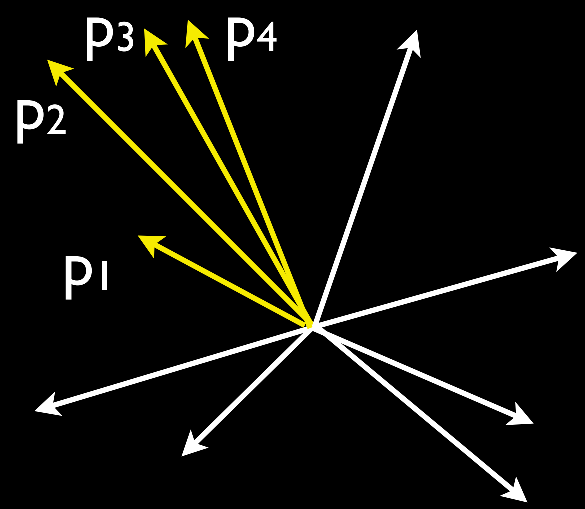
- for detectors with uniform acceptance 2nd and 4th cumulant are given by:

$$c_n\{2\} \equiv \left\langle\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\right\rangle = v_n^2 + \delta_2$$

$$\begin{aligned} c_n\{4\} &\equiv \left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \right\rangle\right\rangle - 2\left\langle\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\right\rangle^2 \\ &= v_n^4 + 4v_n^2\delta_2 + 2\delta_2^2 - 2(v_n^2+\delta_2)^2 + \delta_4 \\ &= -v_n^4 + \delta_4 \end{aligned}$$

- got rid of two particle non-flow correlations!

Can we do better?



Particle 1 coming from the mini-jet. To select particle 2 we can make a choice out of remaining M-1 particles; once particle 2 is selected we can select particle 3 out of remaining M-2 particles and finally we can select particle 4 out of remaining M-3 particles. Hence the probability that we will select randomly four particles coming from the same resonance is 1/(M-1)(M-2)(M-3). From this we can draw a conclusion that for large multiplicity: $\delta_2 \sim 1/M, \qquad \delta_4 \sim 1/M^3$

- therefore to reliably measure flow:

$$v_n^2 \gg 1/M \quad \Rightarrow \quad v_n \gg 1/M^{1/2}$$
$$v_n^4 \gg 1/M^3 \quad \Rightarrow \quad v_n \gg 1/M^{3/4}$$

Can we do better?

- it is possible to extend this:

$$v_n^{2k} \gg 1/M^{2k-1} \Rightarrow v_n \gg 1/M^{\frac{2k-1}{2k}}$$

- for large k

$$v_n \gg 1/M$$

- as an example: M=200 v_n >> 0.005 (more than order of magnitude better than two particle correlations)

- to reliably measure small flow in presence of other correlations one needs to use multi-particle correlations!

Calculate Correlations

(using nested loops)

To evaluate average 2-particle correlation

$$\langle 2 \rangle \equiv \left\langle e^{in(\phi_1 - \phi_2)} \right\rangle = \frac{1}{\binom{M}{2} 2!} \sum_{\substack{i,j=1 \\ (i \neq j)}}^{M} e^{in(\phi_i - \phi_j)}$$

in a nested loop # operations $\quad \dfrac{1}{2!} \dfrac{M!}{(M-2)!}$

- With M=1000, this approach already for 4-particle correlations gives 1.2×10^{12} operations per event!
- calculation of average 6-particle correlation requires roughly 1.4×10^{17} operations, and of average 8-particle correlation roughly 8.4×10^{21} operations per event
- clearly not the way to go

Calculate Correlations

(using Q-cumulants)

A. Bilandzic, RS, S. Voloshin (2011)

azimuthal two particle correlations:

$$\langle 2 \rangle \equiv \left\langle e^{in(\phi_1 - \phi_2)} \right\rangle = \frac{1}{\binom{M}{2} 2!} \sum_{\substack{i,j=1 \\ (i \neq j)}}^{M} e^{in(\phi_i - \phi_j)}$$

definition of Q vector of harmonic n

$$Q_n \equiv \sum_{i=1}^{M} e^{in\phi_i}$$

can write two particle correlation in terms of Q vector of harmonic n

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

Calculate Correlations

(using Q-cumulants)

as we saw before in case of only
flow correlations

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)} = v_n^2$$

for zero flow, we have a random walk

$$|Q_n| \sim \sqrt{M} \qquad \langle 2 \rangle = \frac{M - M}{M(M-1)} = 0$$

and as we saw if there is no flow and
only nonflow we get

$$\langle 2 \rangle = \delta_2$$

Calculate Correlations

(using Q-cumulants)

two particle correlations can be expressed in Q vectors

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

but also four particle correlations (and more)

note the mixed harmonics

$$\langle 4 \rangle = \frac{|Q_n|^4 + |Q_{2n}|^2 - 2 \cdot \mathbf{Re}\left[Q_{2n} Q_n^* Q_n^*\right] - 4(M-2) \cdot |Q_n|^2}{M(M-1)(M-2)(M-3)} + \frac{2}{(M-1)(M-2)}$$

with this it becomes trivial to make cumulants again

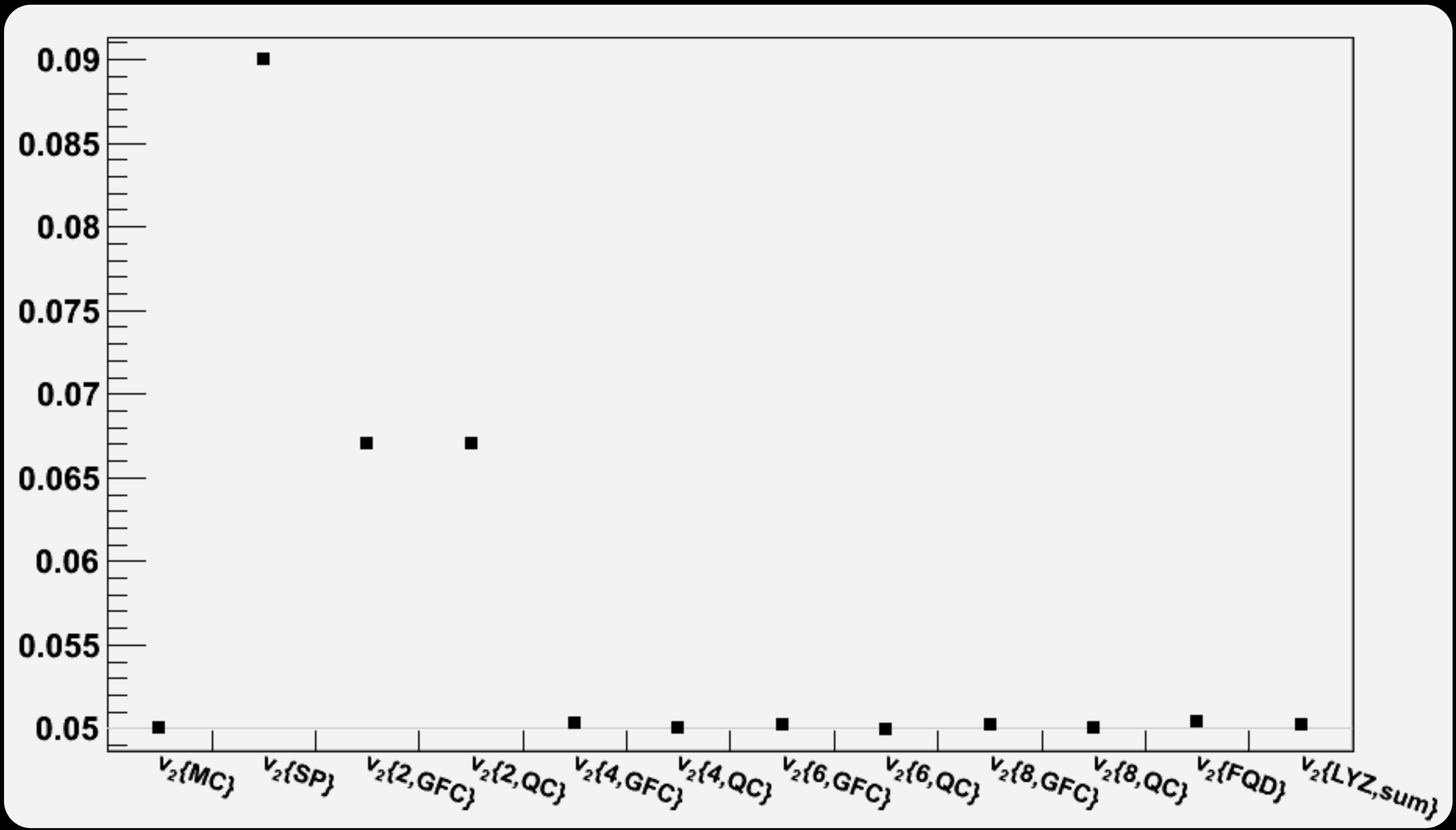
Calculate Correlations

(using Q-cumulants)

- pros Q-cumulants
  - exact solutions, give same answer as nested loops
  - one loop over data enough to calculate all multi-particle correlations
  - number of operations to get all multi-particle correlations up to 8^{th} order is 4 x 2 x Multiplicity
    - for multiplicities of ~ 1000 the number of operations is reduced by a factor 10^{18} !!

nonflow example

Example: input $v_2 = 0.05$, $M = 500$, $N = 5 \times 10^6$ and simulate nonflow by taking each particle twice



as expected only two particle methods are biased

Flow Fluctuations

Both two and multi-particle correlations have an extra feature one has to keep in mind!

- By using multi-particle correlations to estimate flow we are actually estimating the averages of various powers of flow

$$\langle\langle 2\rangle\rangle = \left\langle v^2 \right\rangle, \quad \langle\langle 6\rangle\rangle = \left\langle v^6 \right\rangle$$
$$\langle\langle 4\rangle\rangle = \left\langle v^4 \right\rangle, \quad \langle\langle 8\rangle\rangle = \left\langle v^8 \right\rangle$$

- But what we are after is: $\langle v \rangle$

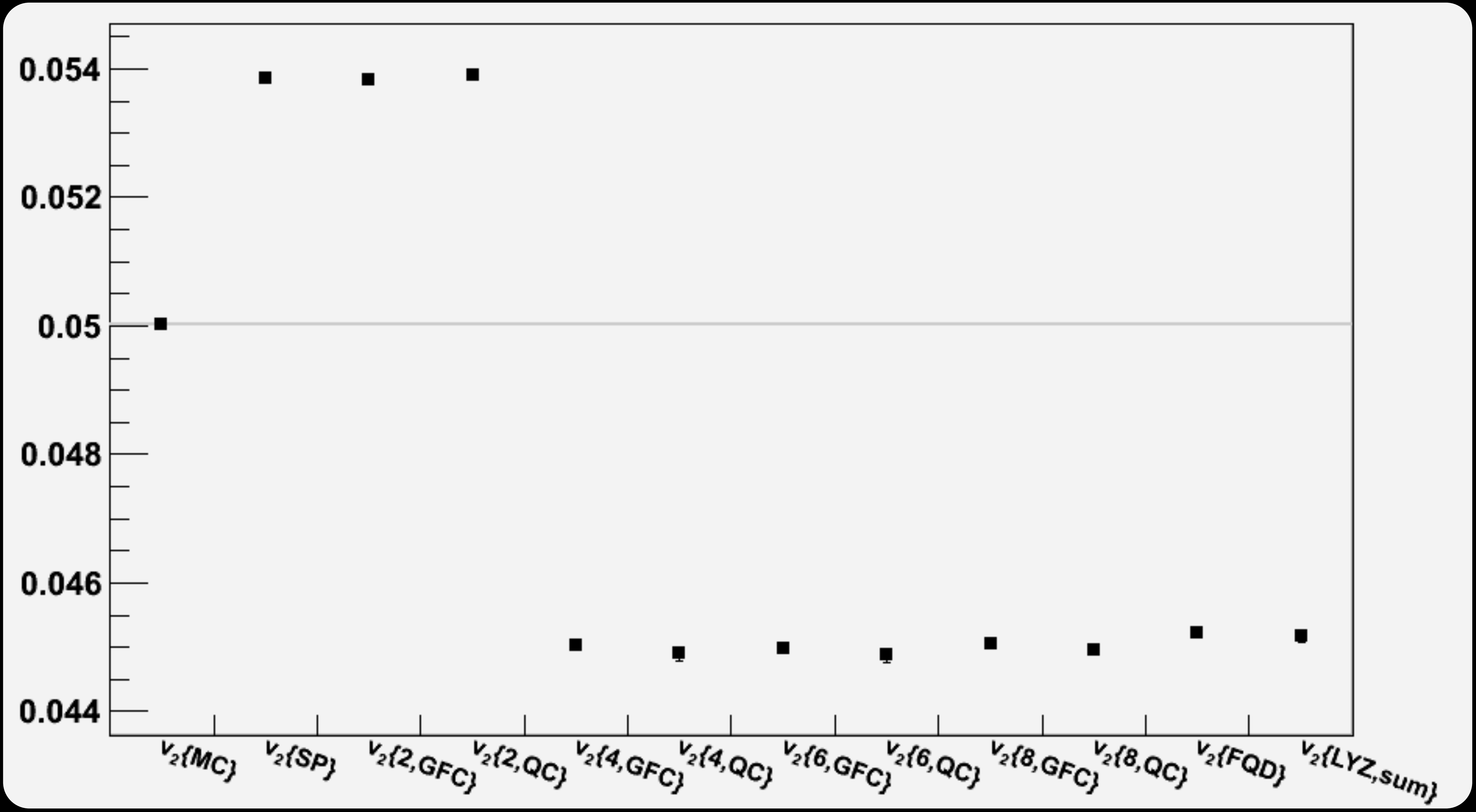
Flow Fluctuations

$$\begin{aligned} v\{2\} &= \langle v\rangle + \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle} \\ v\{4\} &= \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle} \\ v\{6\} &= \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle} \\ v\{8\} &= \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle} \end{aligned}$$

- for $\sigma_v << \langle v \rangle$ this is a general result to order σ^2

Flow Fluctuations

Example: input v_2 = 0.05 +/- 0.02 (Gausian), M = 500, $N = 1 \times 10^6$



Gaussian fluctuation behave as predicted

non-uniform acceptance

- To correct for the bias on the correlations from a non uniform detector various techniques have been developed over the years
- Initially for analysis (e.g. using the reaction plane method) approaches such as flattening, re-centering, etc were developed
  - they required a second run over the data (bad idea nowadays)
  - some have problems with gaps in the acceptance
- For Q-cumulants we calculate the bias explicitly
  - works to high precision and can be done without extra run over data

recap ϕ-weights

- first perform a run over the data to get the azimuthal distribution in the laboratory (with the cuts on the tracks which one uses in the analysis!)
  - this should be flat for a good detector
- use in the second run the inverse of the distribution as a weight for constructing the weights

$$Q_n = \sum_{i=1}^{M} w_{\phi_i} e^{in\phi_i}$$

limitations: needs second run and cannot handle big gaps in the detector!

Non-uniform Acceptance

- remember when we started the derivation for the multi-particle correlations we removed the terms which are zero for a perfect detector

$$\left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle = \langle\langle e^{in\phi_1}\rangle\rangle\,\langle\langle e^{-in\phi_2}\rangle\rangle + \left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle_{\mathrm{corr}}$$

zero for symmetric detector

- now we gonna keep track of them

$$\left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle_{\mathrm{corr}} = \left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle - \langle\langle e^{in\phi_1}\rangle\rangle\,\langle\langle e^{-in\phi_2}\rangle\rangle$$

Non-uniform Acceptance

for the real part

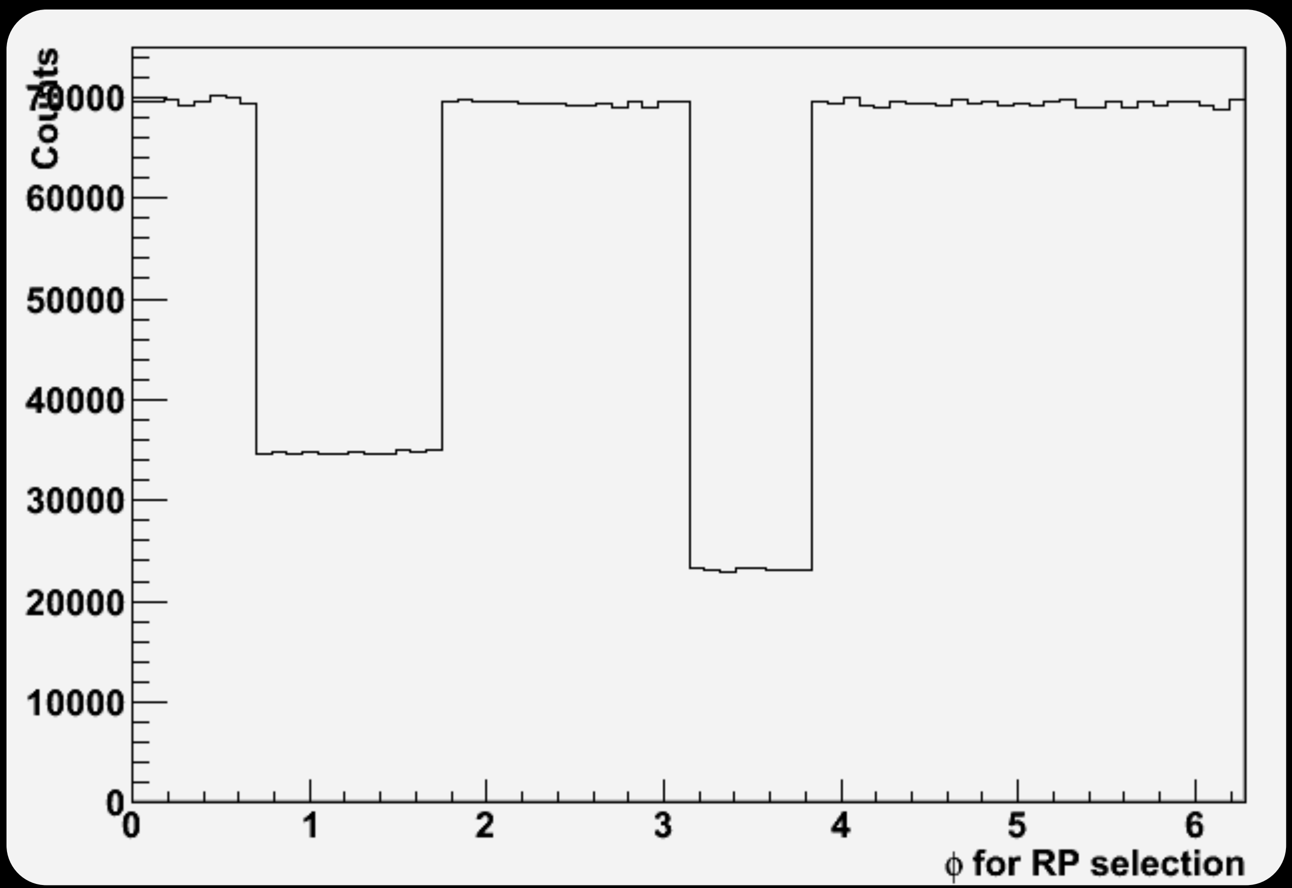
$$QC\{2\} = \langle\langle \cos n(\phi_1 - \phi_2)\rangle\rangle - \langle\langle \cos n\phi_1 \rangle\rangle^2 - \langle\langle \sin n\phi_1 \rangle\rangle^2$$

$$\begin{aligned}
QC\{4\} \quad = \quad & \langle\langle \cos n(\phi_1+\phi_2-\phi_3-\phi_4)\rangle\rangle - 2\cdot\langle\langle \cos n(\phi_1-\phi_2)\rangle\rangle^2 \\
- \quad & 4\cdot\langle\langle \cos n\phi_1\rangle\rangle\,\langle\langle \cos n(\phi_1-\phi_2-\phi_3)\rangle\rangle \\
+ \quad & 4\cdot\langle\langle \sin n\phi_1\rangle\rangle\,\langle\langle \sin n(\phi_1-\phi_2-\phi_3)\rangle\rangle \\
- \quad & \langle\langle \cos n(\phi_1+\phi_2)\rangle\rangle^2 - \langle\langle \sin n(\phi_1+\phi_2)\rangle\rangle^2 \\
+ \quad & 4\cdot\langle\langle \cos n(\phi_1+\phi_2)\rangle\rangle\left[\langle\langle \cos n\phi_1\rangle\rangle^2 - \langle\langle \sin n\phi_1\rangle\rangle^2\right] \\
+ \quad & 8\cdot\langle\langle \sin n(\phi_1+\phi_2)\rangle\rangle\,\langle\langle \sin n\phi_1\rangle\rangle\,\langle\langle \cos n\phi_1\rangle\rangle \\
+ \quad & 8\cdot\langle\langle \cos n(\phi_1-\phi_2)\rangle\rangle\left[\langle\langle \cos n\phi_1\rangle\rangle^2 + \langle\langle \sin n\phi_1\rangle\rangle^2\right] \\
- \quad & 6\cdot\left[\langle\langle \cos n\phi_1\rangle\rangle^2 + \langle\langle \sin n\phi_1\rangle\rangle^2\right]^2
\end{aligned}$$

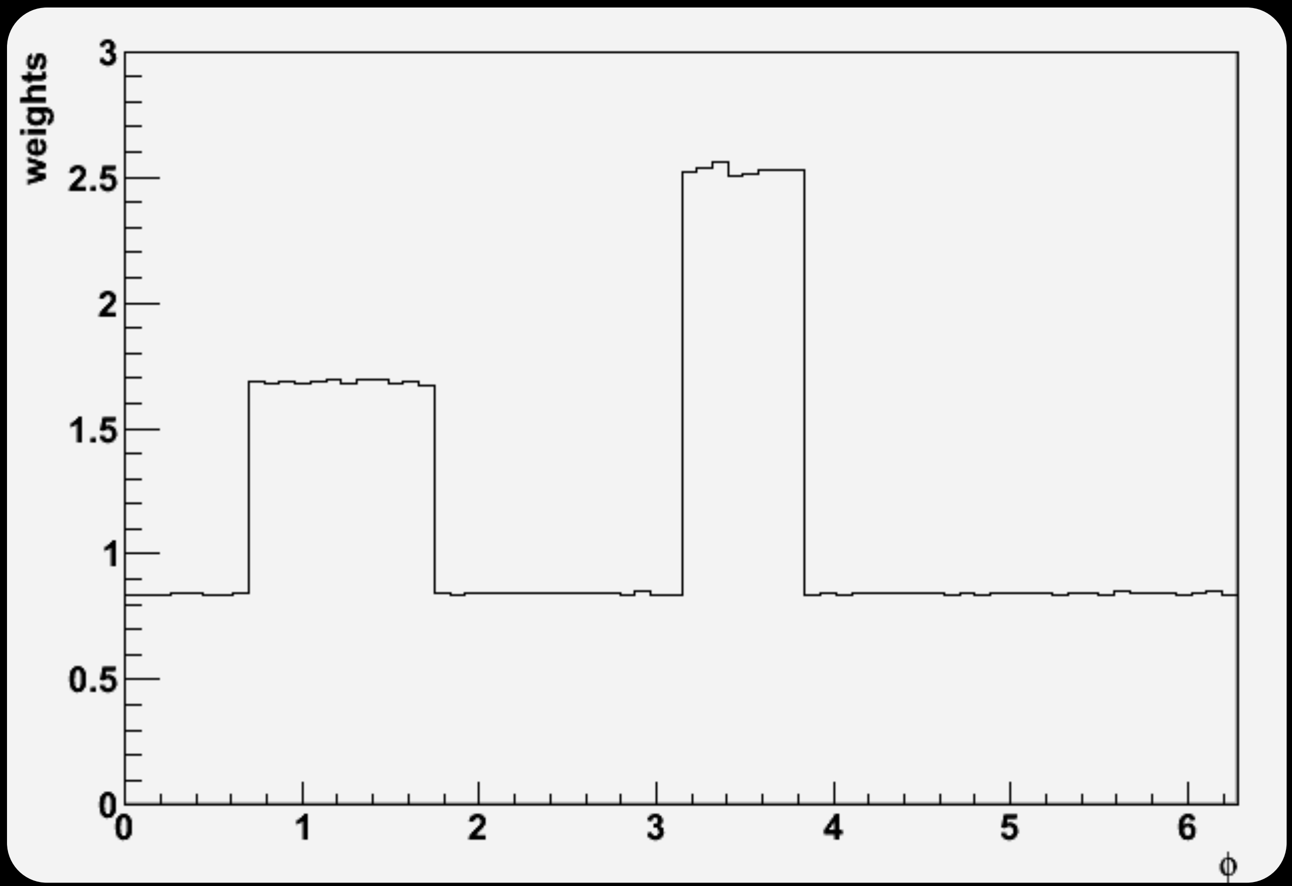
The yellow terms correct for non-uniform acceptance

Non-uniform Acceptance

when using ϕ-weight is possible



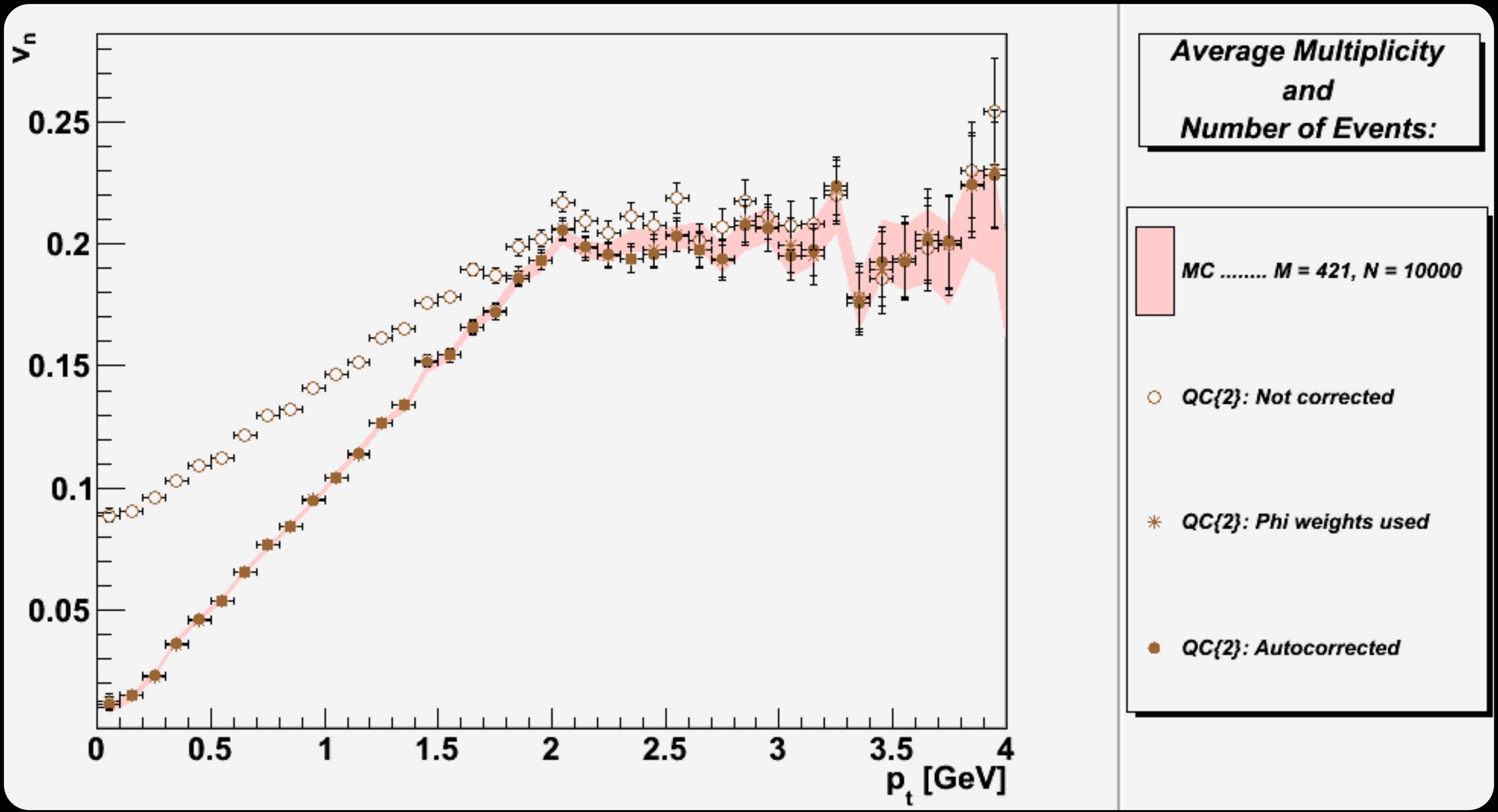
azimuthal distribution in the lab frame



ϕ-weights

Non-uniform Acceptance

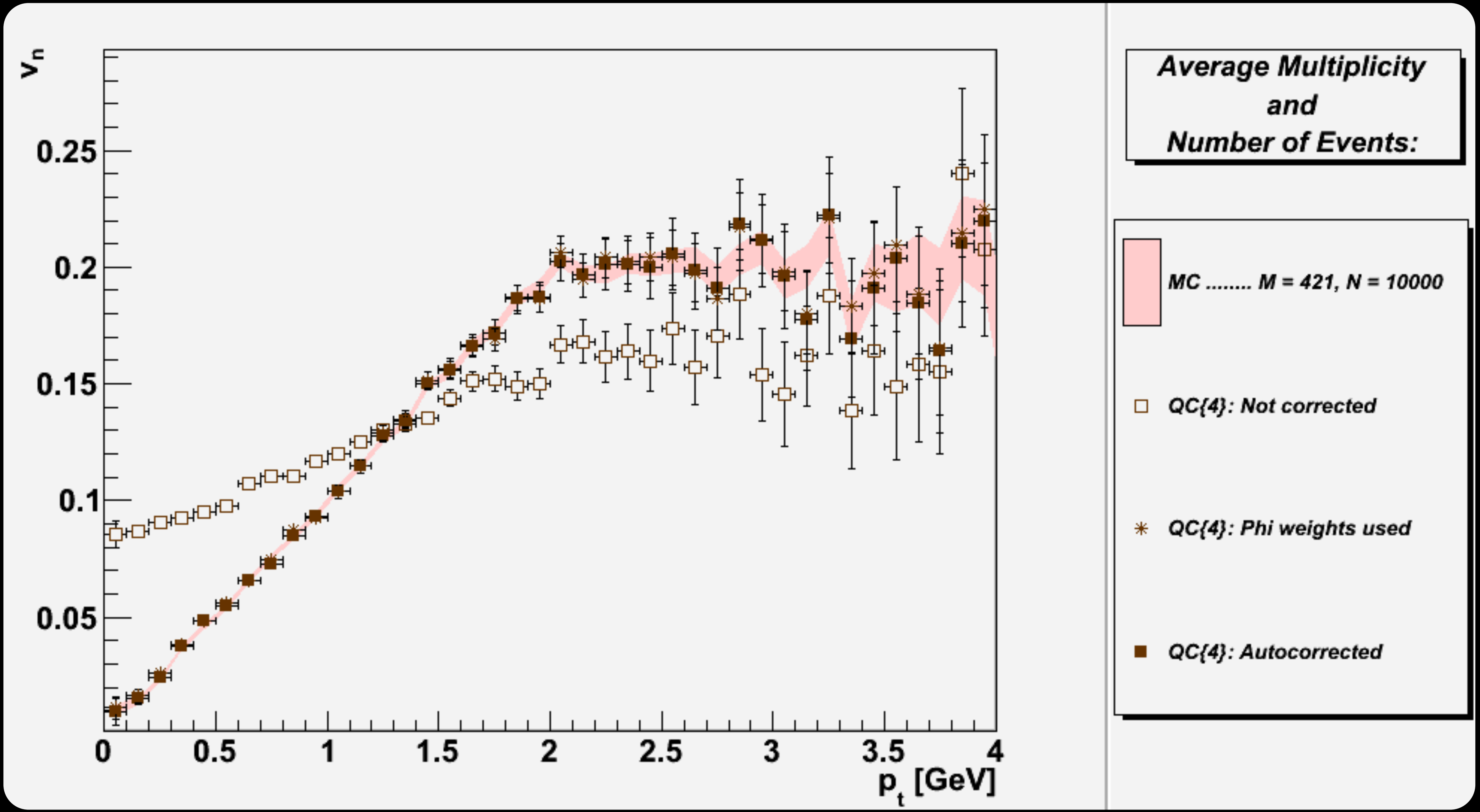
when using ϕ-weight is possible



For 2nd order both corrections work (and agree)!

Non-uniform Acceptance

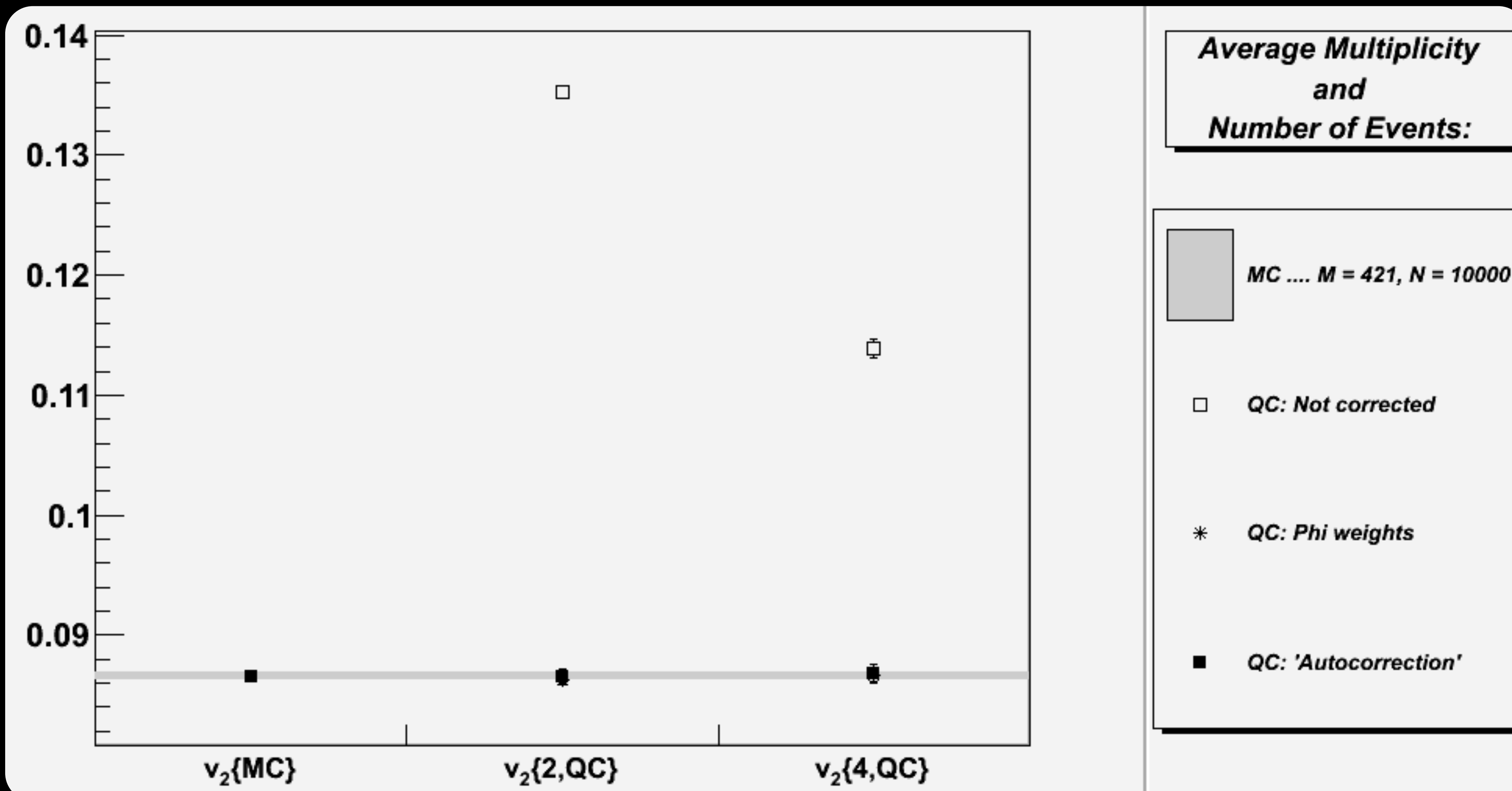
when using ϕ-weight is possible



For 4^{th} order both corrections work (and agree)!

Non-uniform Acceptance

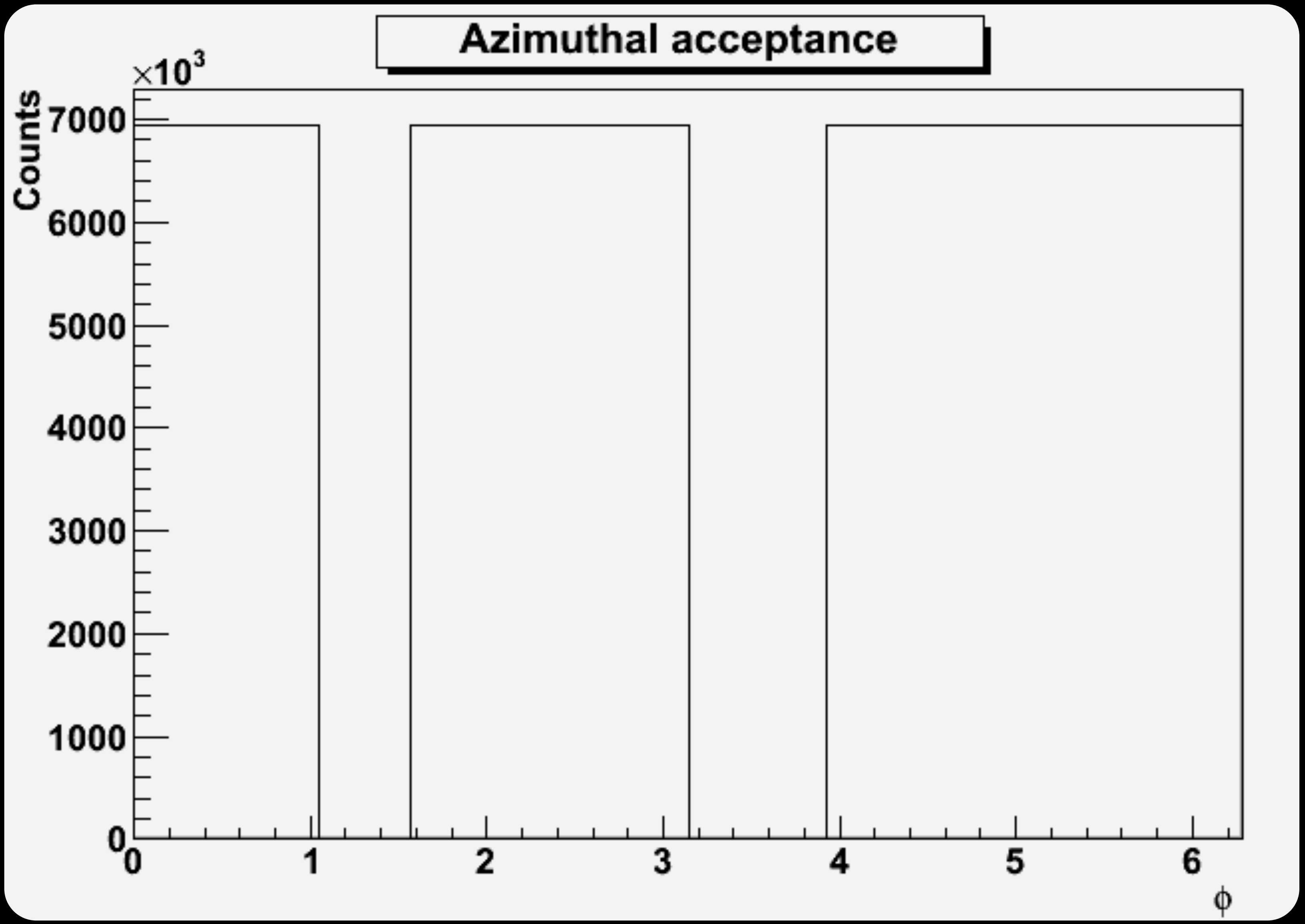
when using ϕ-weight is possible



corrections are sizable, but work!

Non-uniform Acceptance

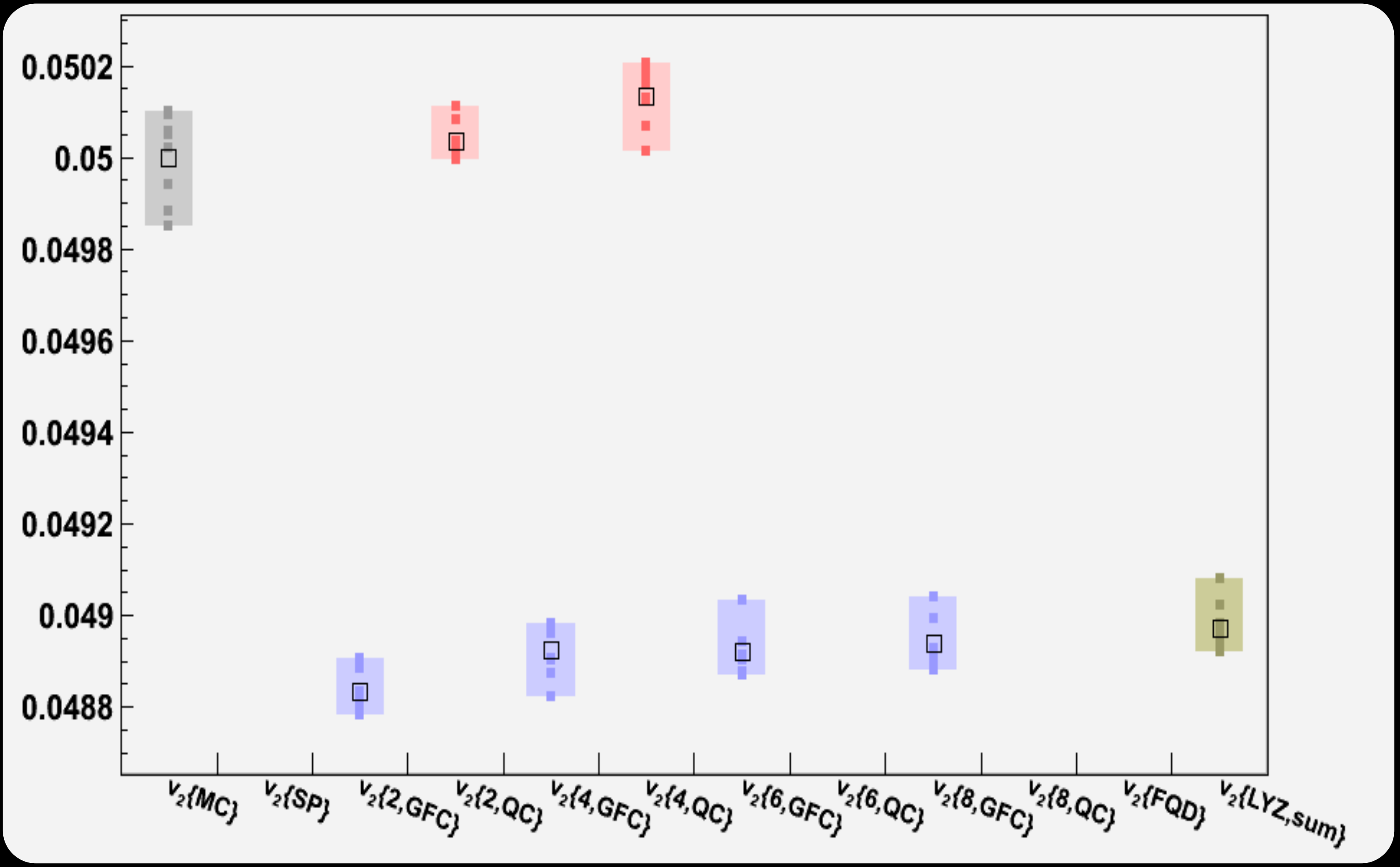
when using ϕ-weight is not possible



sizable gaps, can be closed by rebinning but then you have really large bins

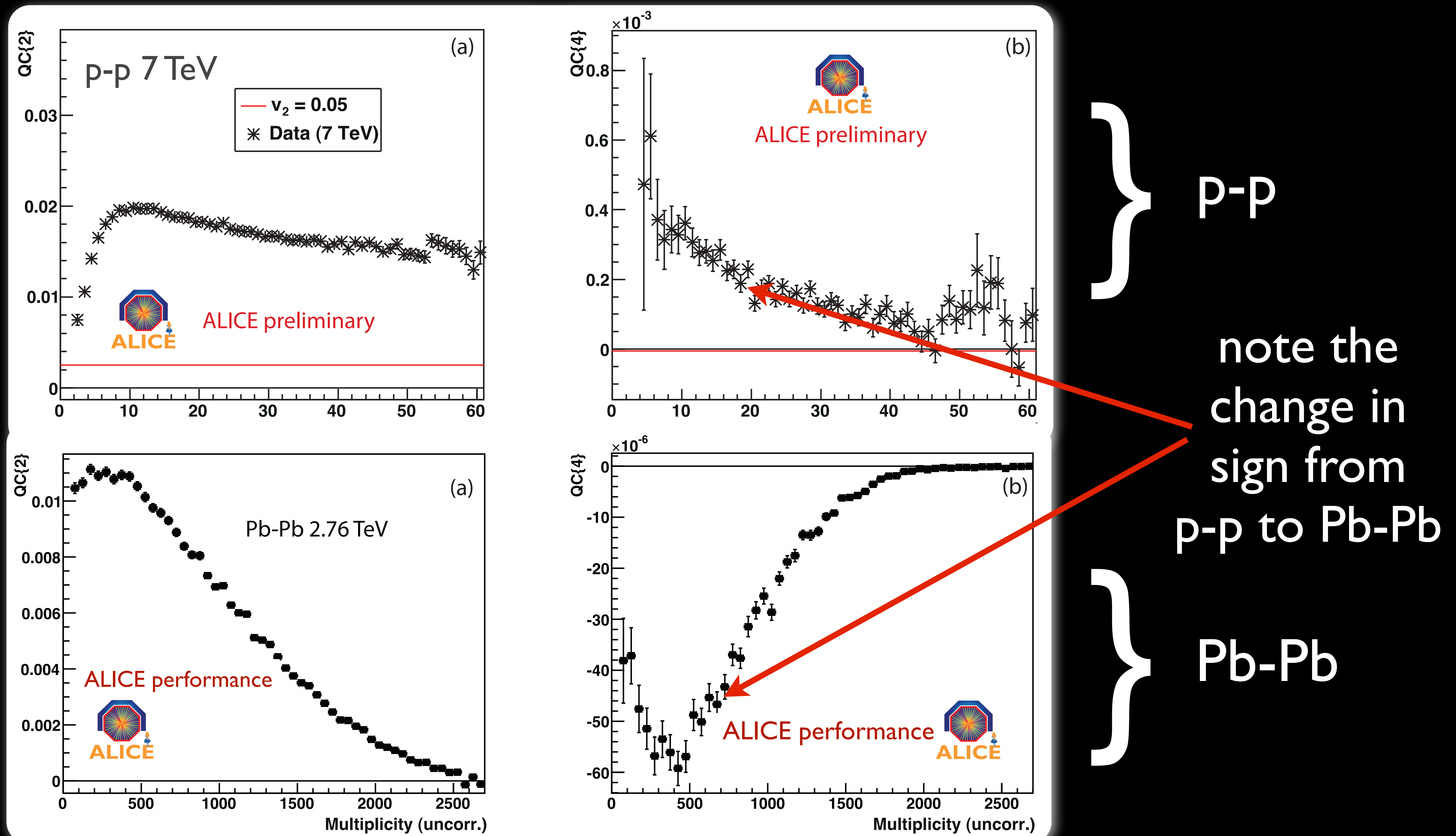
Non-uniform Acceptance

when using ϕ-weight is not possible



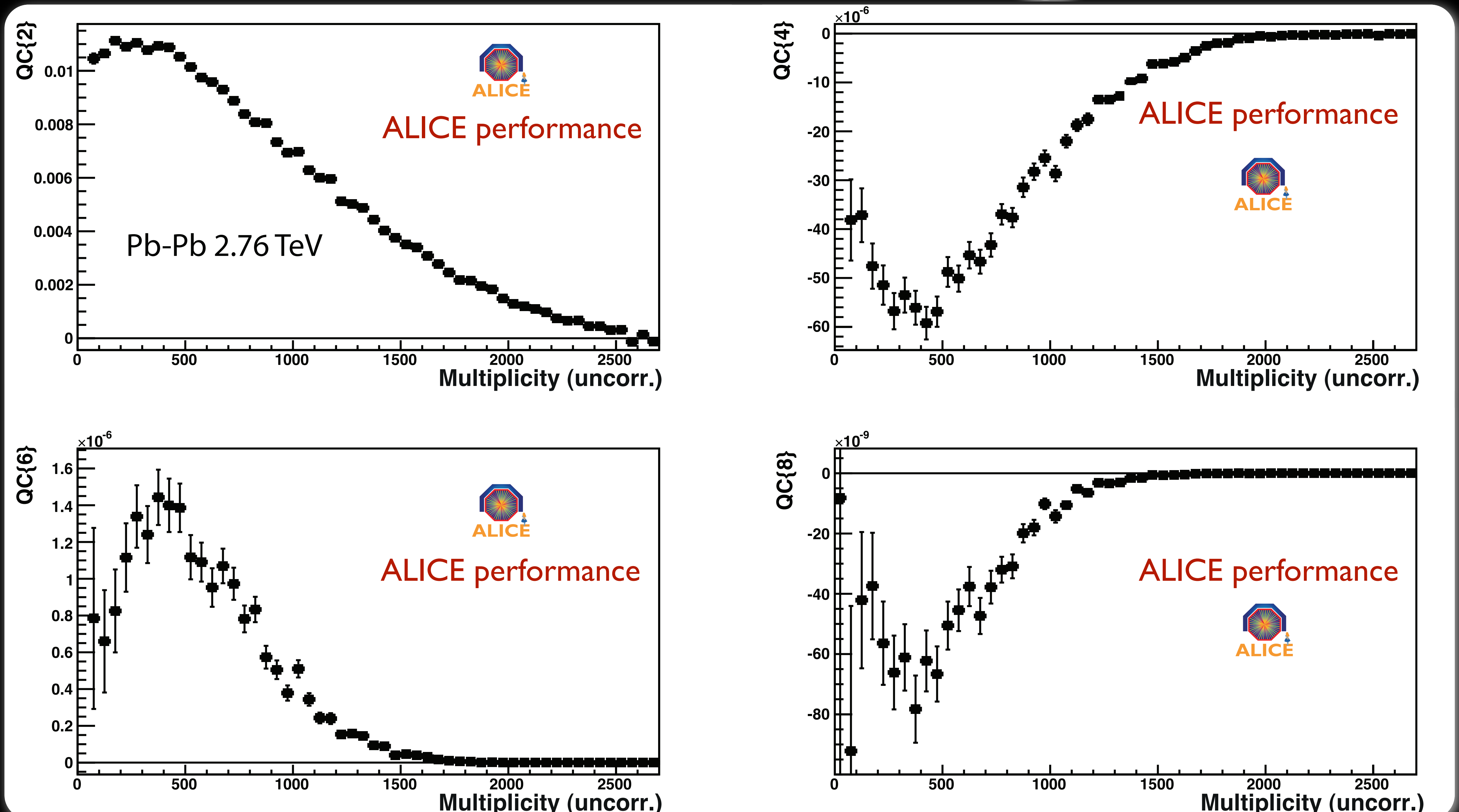
SP and FQD are off the plot. {QC,2} and {QC,4} are in agreement with the Monte Carlo. The projection on fixed angles (GFC and LYZ) do a good job (but not as good as the full correction)

Cumulants in Pb-Pb



remember QC{2} = v^2, QC{4} = $-v^4$

Flow at first sight!

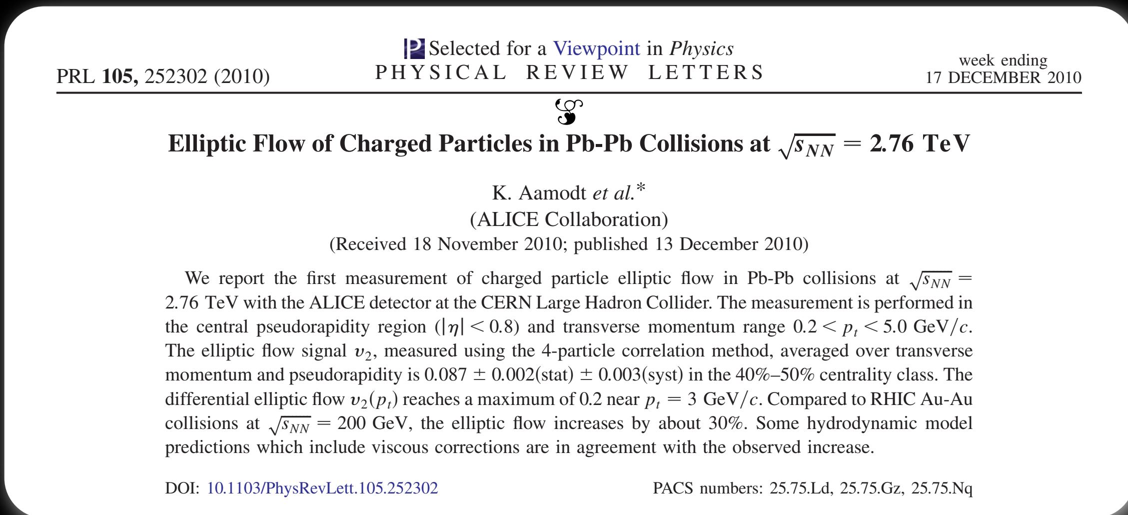


remember QC{2} = v^2, QC{4} = $-v^4$, QC{6} = $4v^6$. and QC{8} = $-33v^8$

cumulants show strong collective flow!

perfect liquid

K. Aamodt et al. (ALICE Collaboration) PRL 105, 252302 (2010)


PRL 105, 252302 (2010) Selected for a Viewpoint in *Physics* PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2010

Elliptic Flow of Charged Particles in Pb-Pb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV

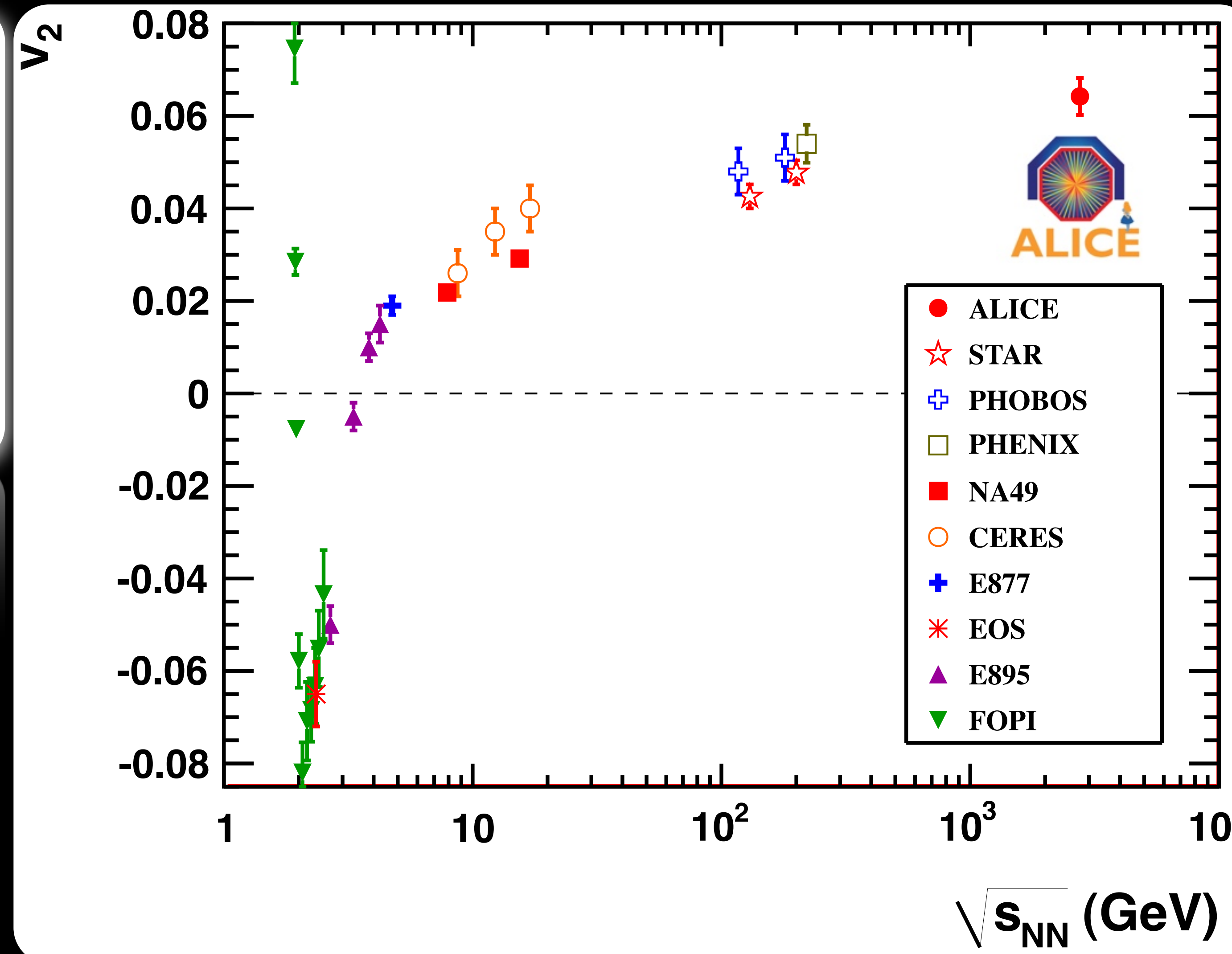
K. Aamodt *et al.*[*]
(ALICE Collaboration)
(Received 18 November 2010; published 13 December 2010)

We report the first measurement of charged particle elliptic flow in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV with the ALICE detector at the CERN Large Hadron Collider. The measurement is performed in the central pseudorapidity region ($|\eta| < 0.8$) and transverse momentum range $0.2 < p_t < 5.0$ GeV/c. The elliptic flow signal v_2, measured using the 4-particle correlation method, averaged over transverse momentum and pseudorapidity is $0.087 \pm 0.002(\text{stat}) \pm 0.003(\text{syst})$ in the 40%–50% centrality class. The differential elliptic flow $v_2(p_t)$ reaches a maximum of 0.2 near $p_t = 3$ GeV/c. Compared to RHIC Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV, the elliptic flow increases by about 30%. Some hydrodynamic model predictions which include viscous corrections are in agreement with the observed increase.

DOI: 10.1103/PhysRevLett.105.252302 PACS numbers: 25.75.Ld, 25.75.Gz, 25.75.Nq


A. Bilandzic

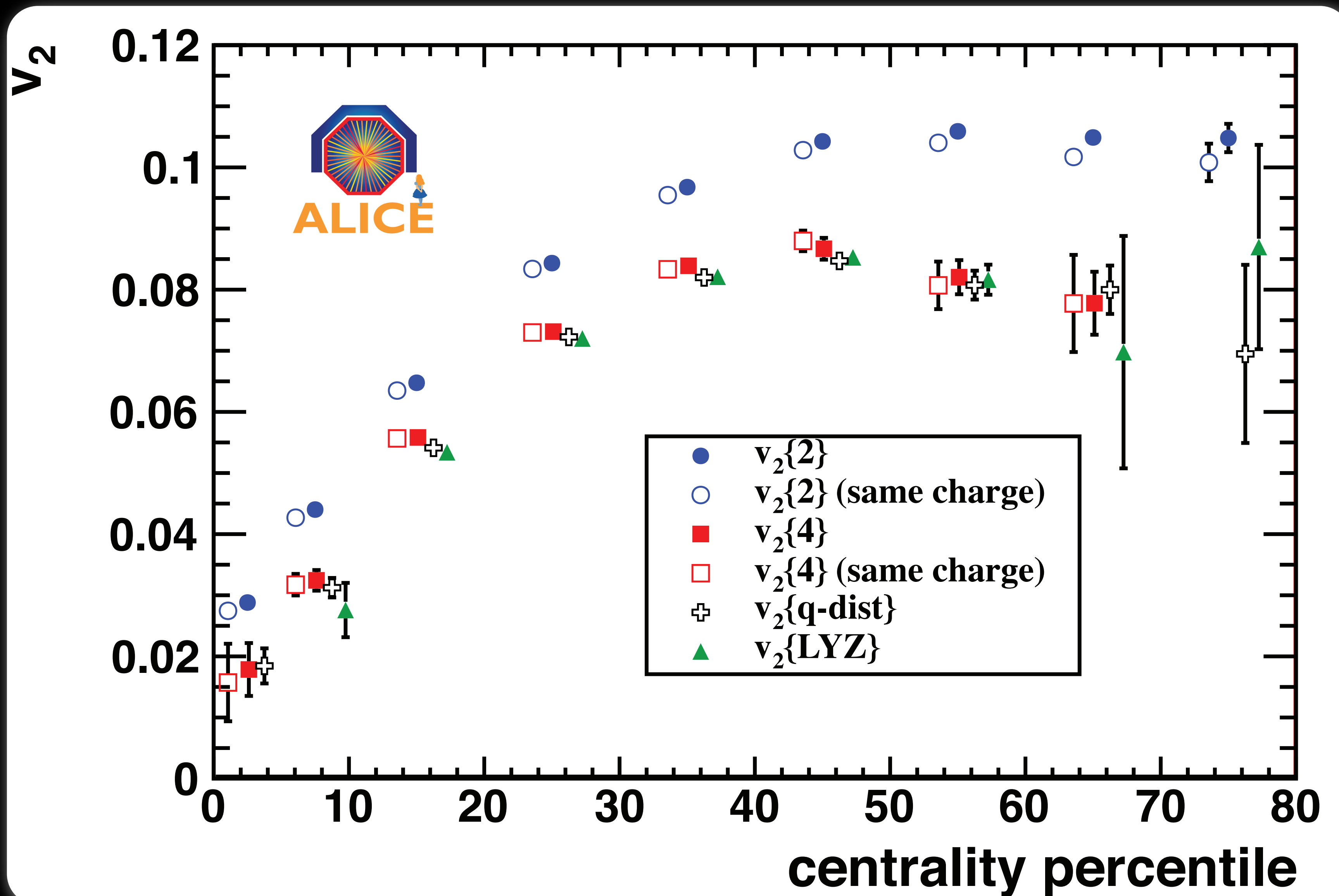

M. Krzewicky



First LHC heavy-ion physics paper <10 days after first collisions

CERN, November 26, 2010:
'the much hotter plasma produced at the LHC behaves as a very low viscosity liquid..'

v_2 in ALICE



K. Aamodt et al. (ALICE Collaboration)
PRL 105, 252302 (2010)

expected difference between two and multi-particle estimates

multi-particle estimates agree within uncertainties as is expected for collective flow!

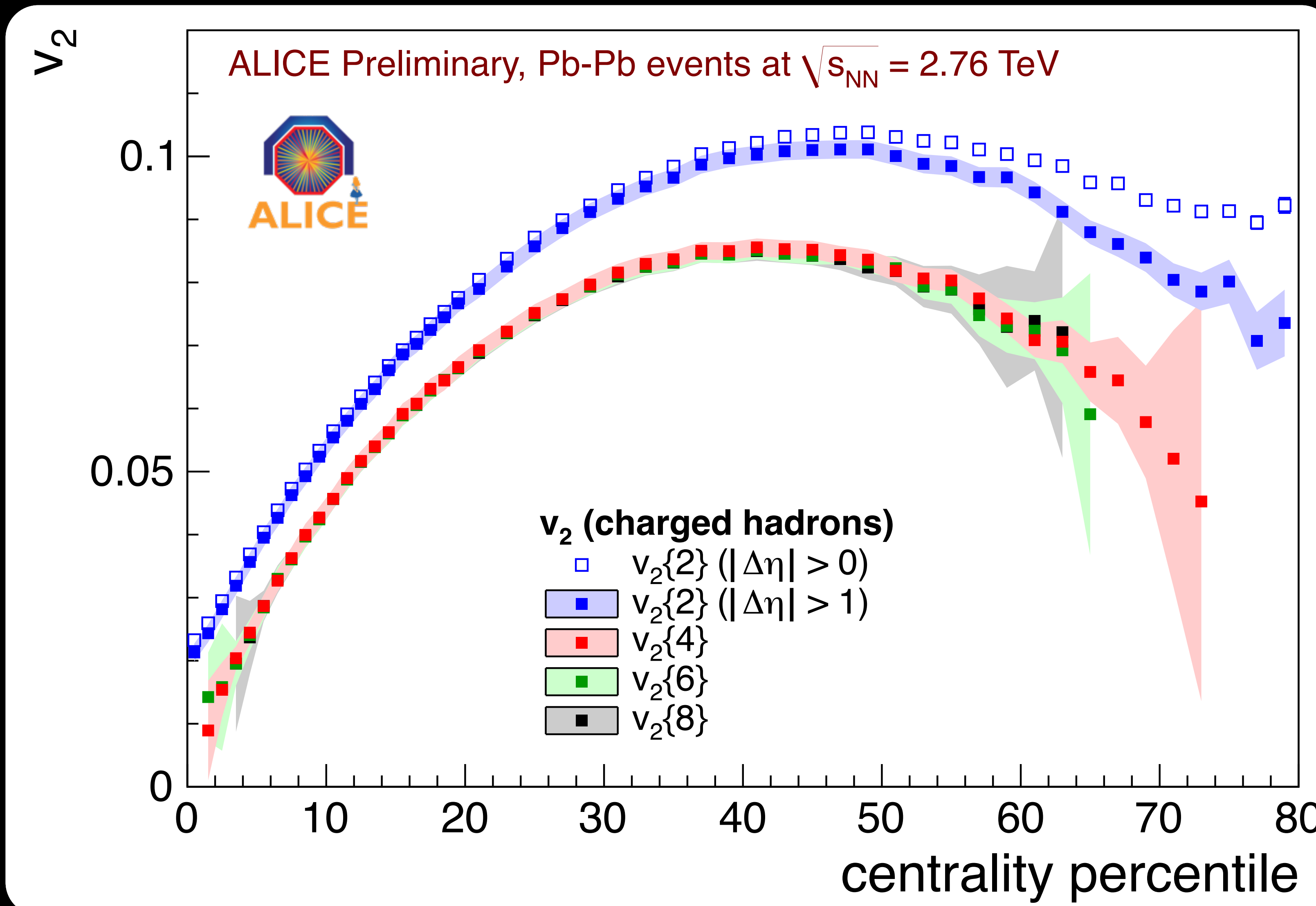
v_2 versus centrality in ALICE

$$v\{2\} = \langle v \rangle + \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle}$$

$$v\{4\} = \langle v \rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle}$$

$$v\{6\} = \langle v \rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle}$$

$$v\{8\} = \langle v \rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v \rangle}$$



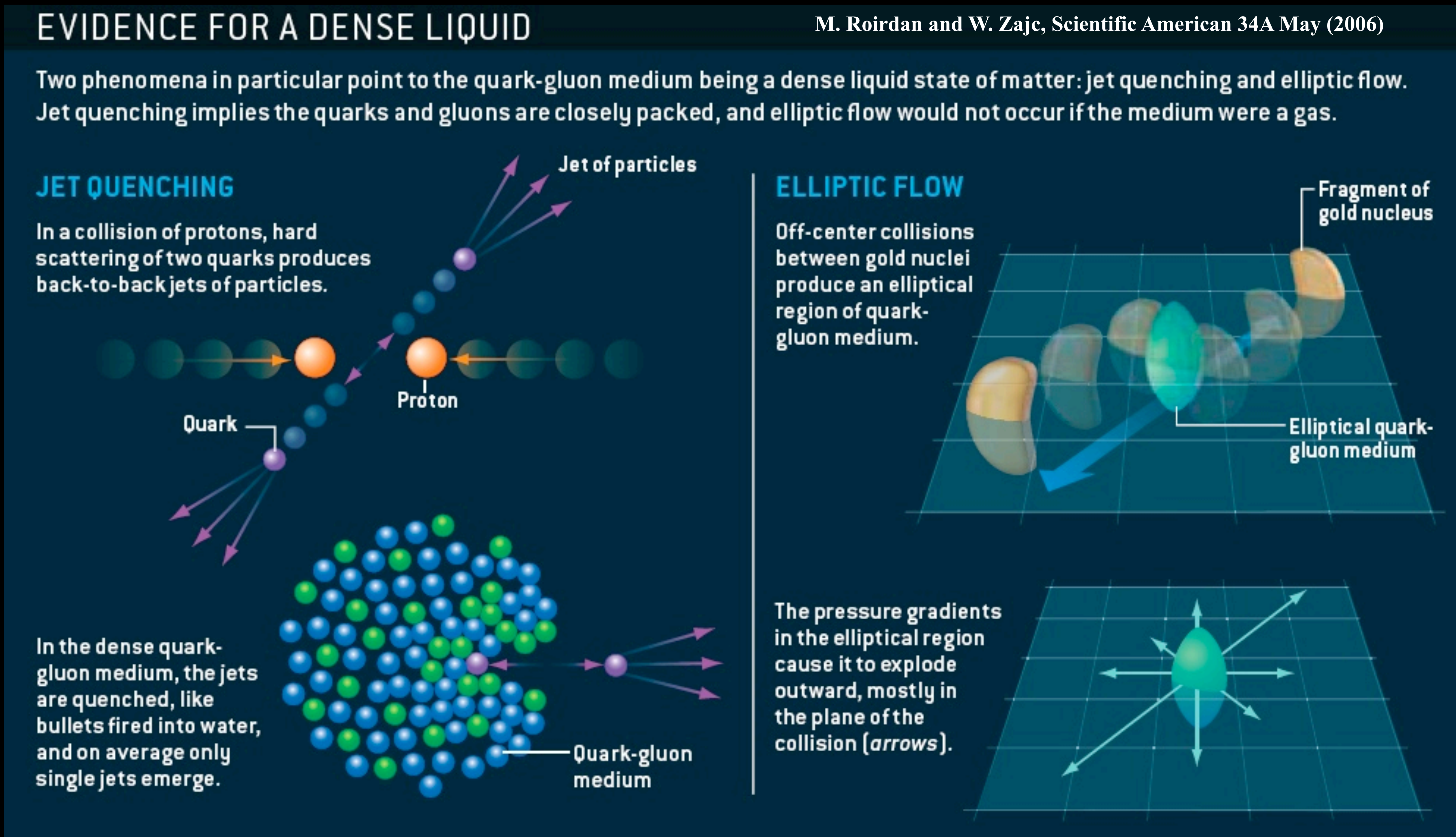
Clear separation between $v_2\{2\}$ and higher order cumulants
Higher order cumulant v_2 estimates are consistent within uncertainties

End Lecture 11

recap lecture 1 and 2

- At high temperatures, ~150 MeV, we expect a phase transition from normal nuclear matter to a QGP
- Properties of this matter teaches us about QCD in the nonperturbative domain and give some insights in the properties of the early universe
- We saw how we characterize the various collisions
- Looked at proposed smoking guns (SPS)
- Characterized the properties of the created system in heavy-ions; energy density, chemical temperature, kinetic temperature, collective expansion, ...
- anisotropy in collective expansion and how we measure this via correlations

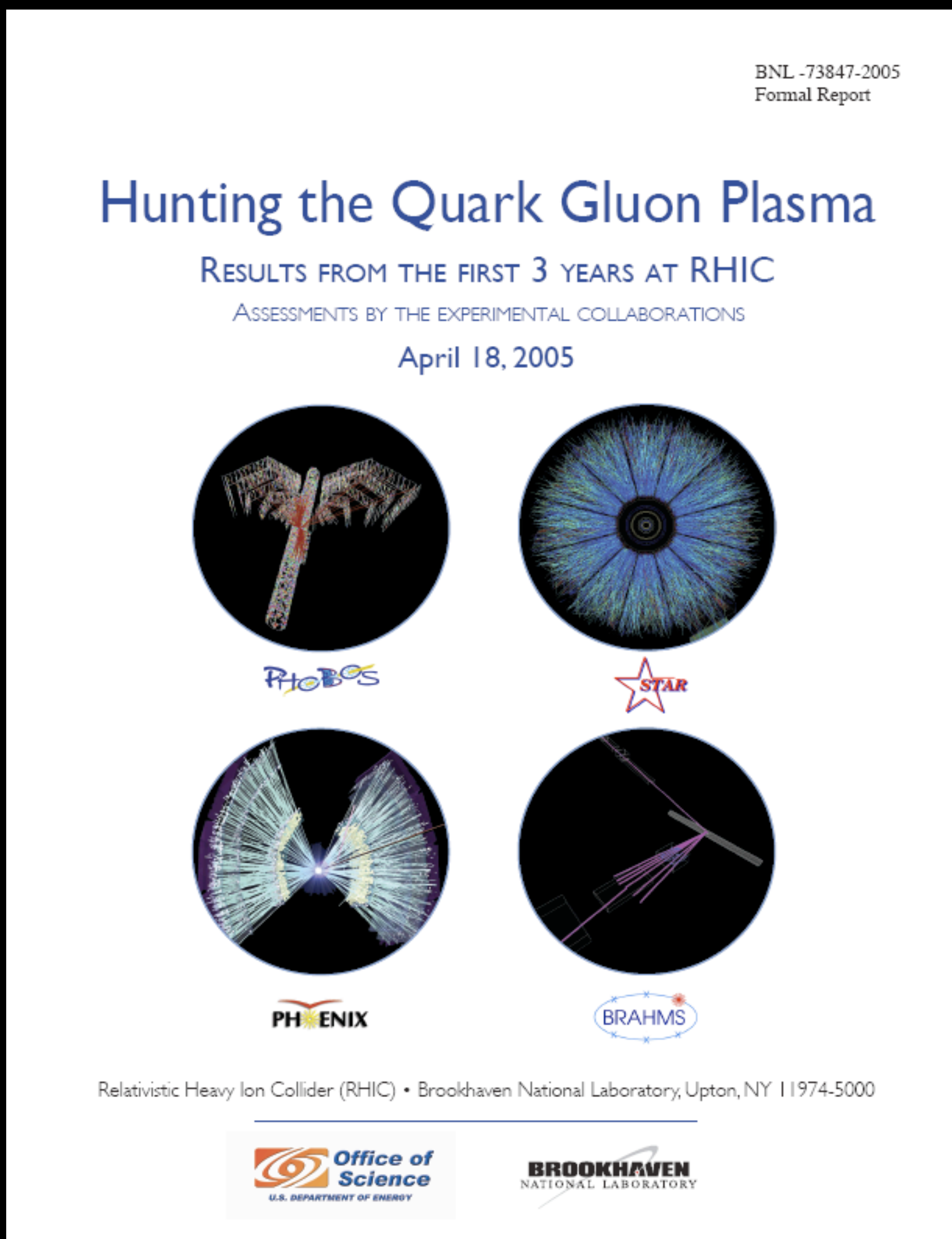
highlights at RHIC



RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005


BNL -73847-2005
Formal Report

Hunting the Quark Gluon Plasma

Results from the first 3 years at RHIC

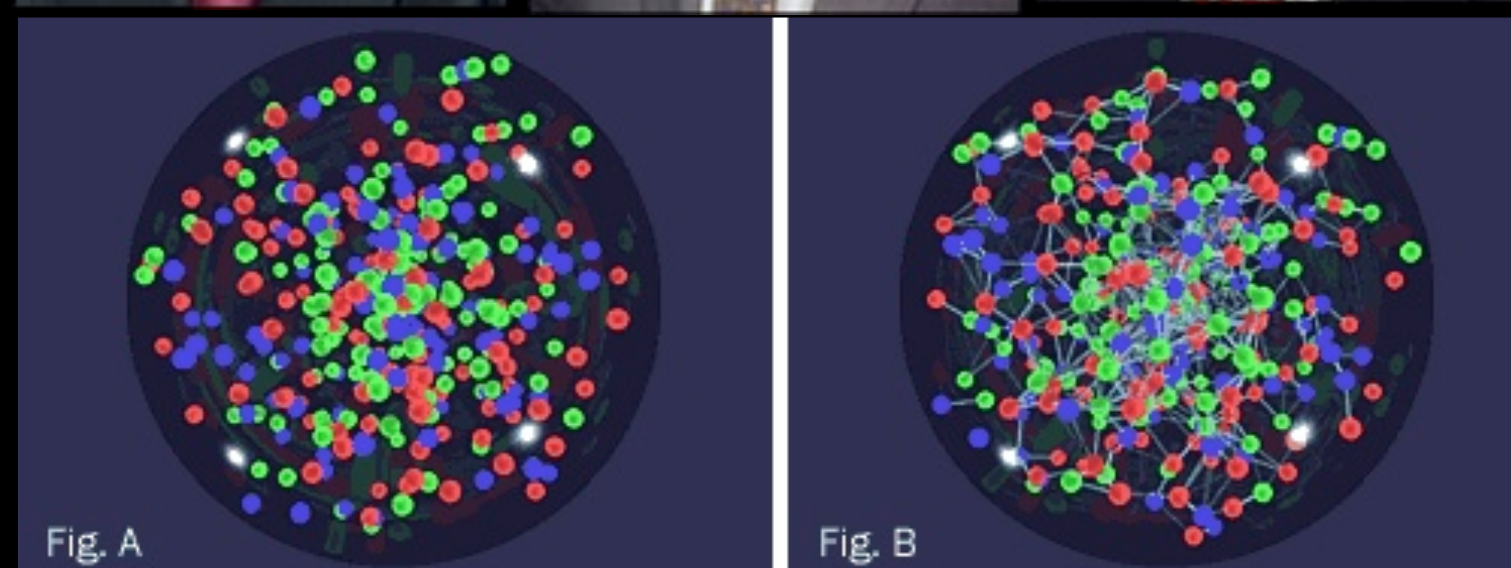
Assessments by the experimental collaborations

April 18, 2005

Relativistic Heavy Ion Collider (RHIC) • Brookhaven National Laboratory, Upton, NY 11974-5000

Office of Science U.S. Department of Energy

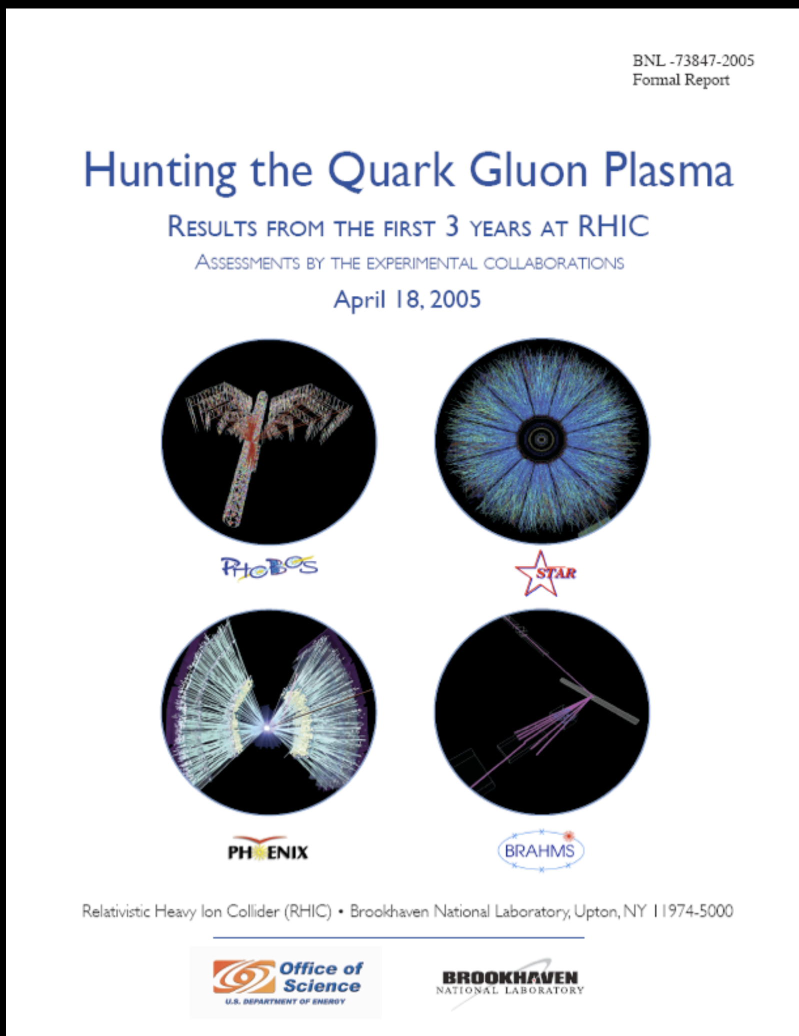
Brookhaven National Laboratory



RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted - raising many new questions - April 18, 2005

ALICE


BNL -73847-2005
Formal Report

Hunting the Quark Gluon Plasma

Results from the first 3 years at RHIC

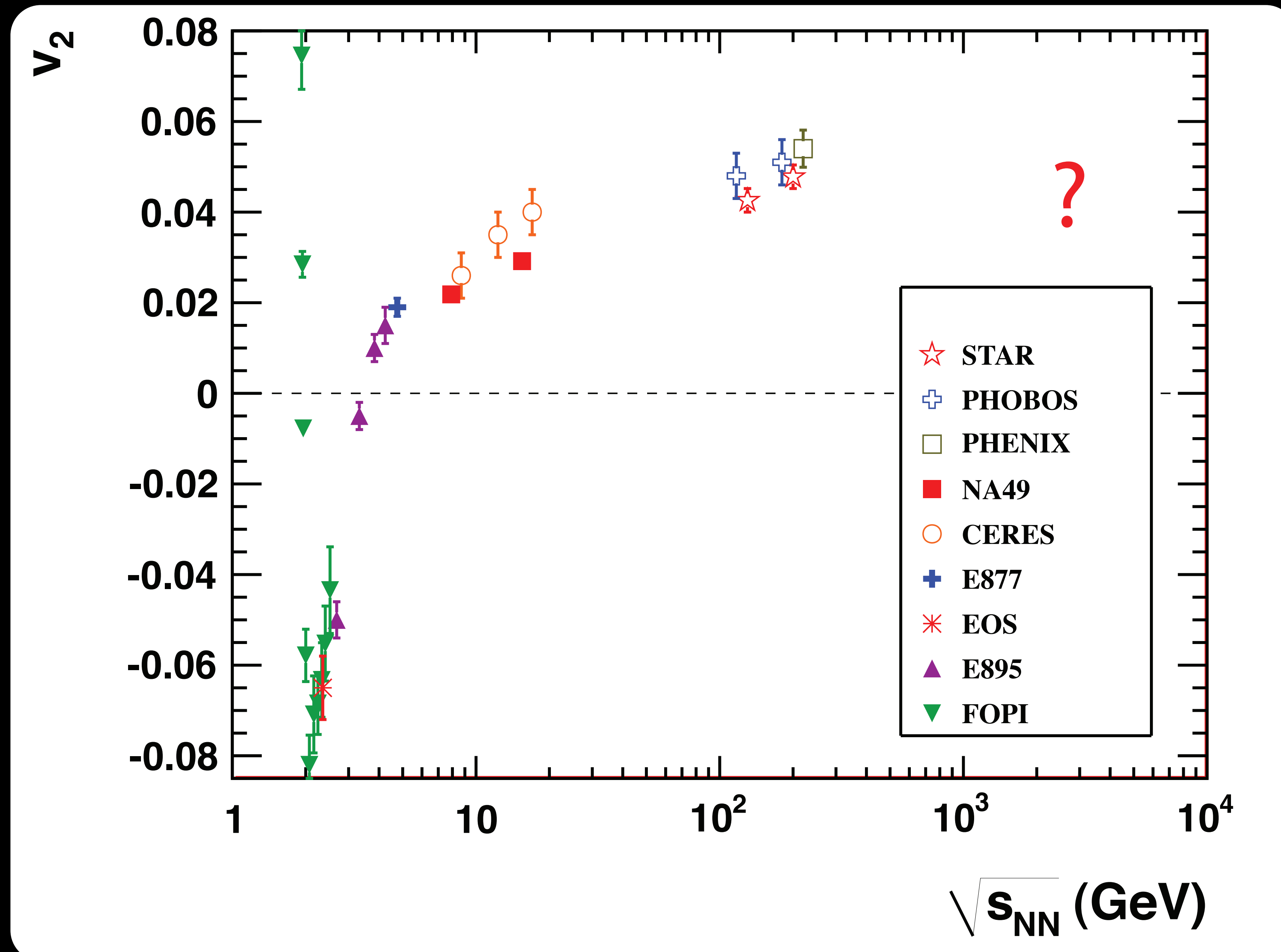
Assessments by the experimental collaborations

April 18, 2005

PHOBOS STAR PHENIX BRAHMS

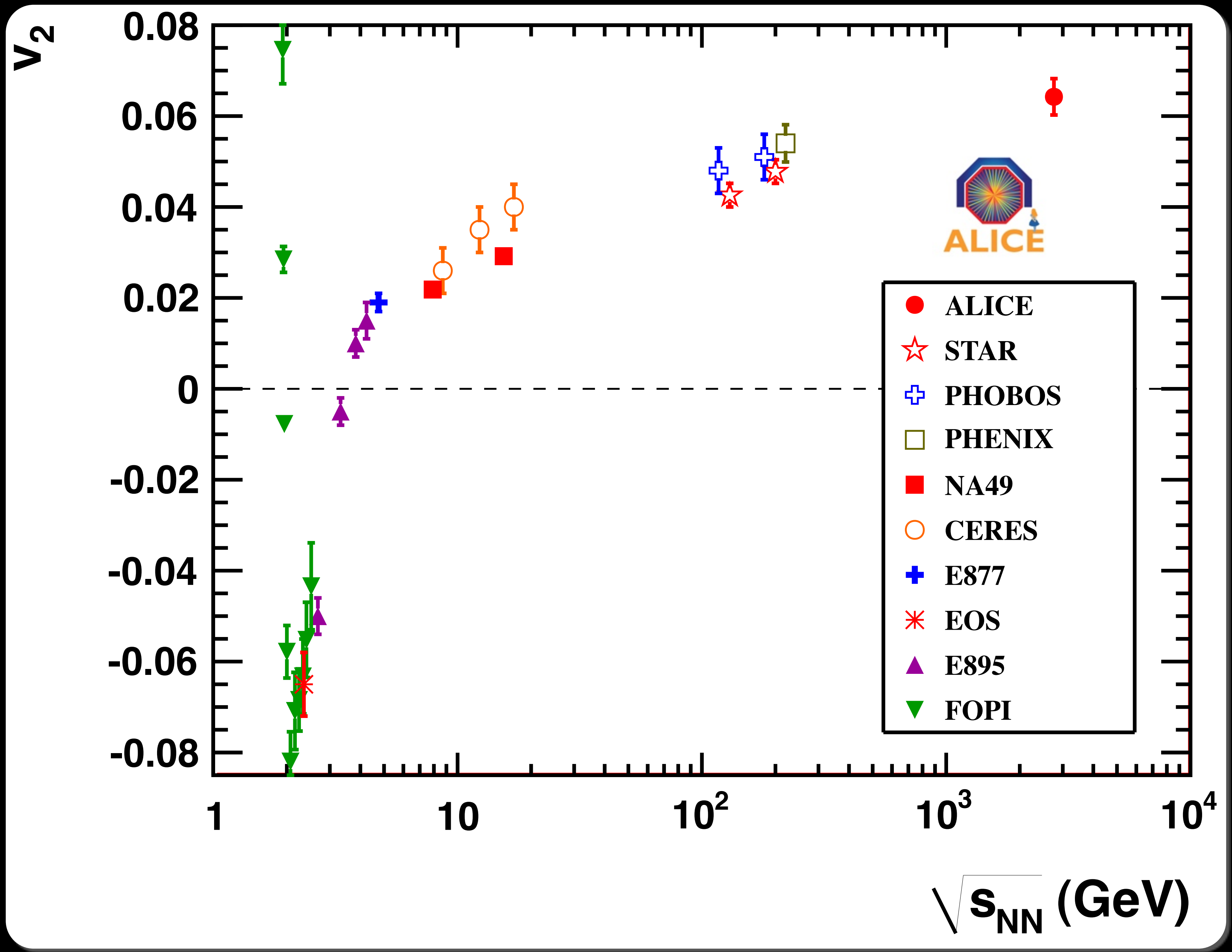
Relativistic Heavy Ion Collider (RHIC) • Brookhaven National Laboratory, Upton, NY 11974-5000

Office of Science U.S. Department of Energy BROOKHAVEN NATIONAL LABORATORY



What to expect at the LHC: still the perfect liquid or are we approaching the viscous ideal gas?

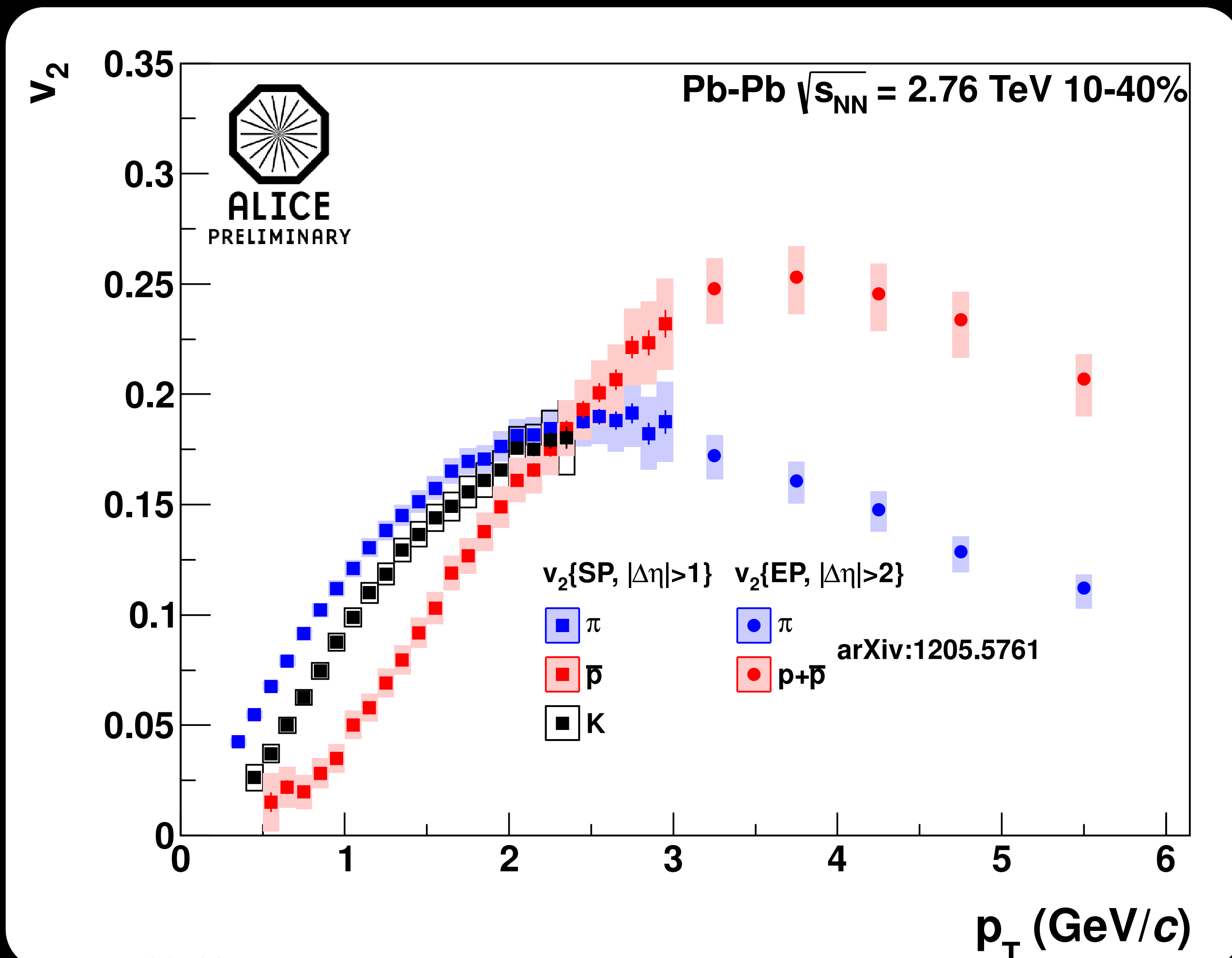
The Perfect Liquid

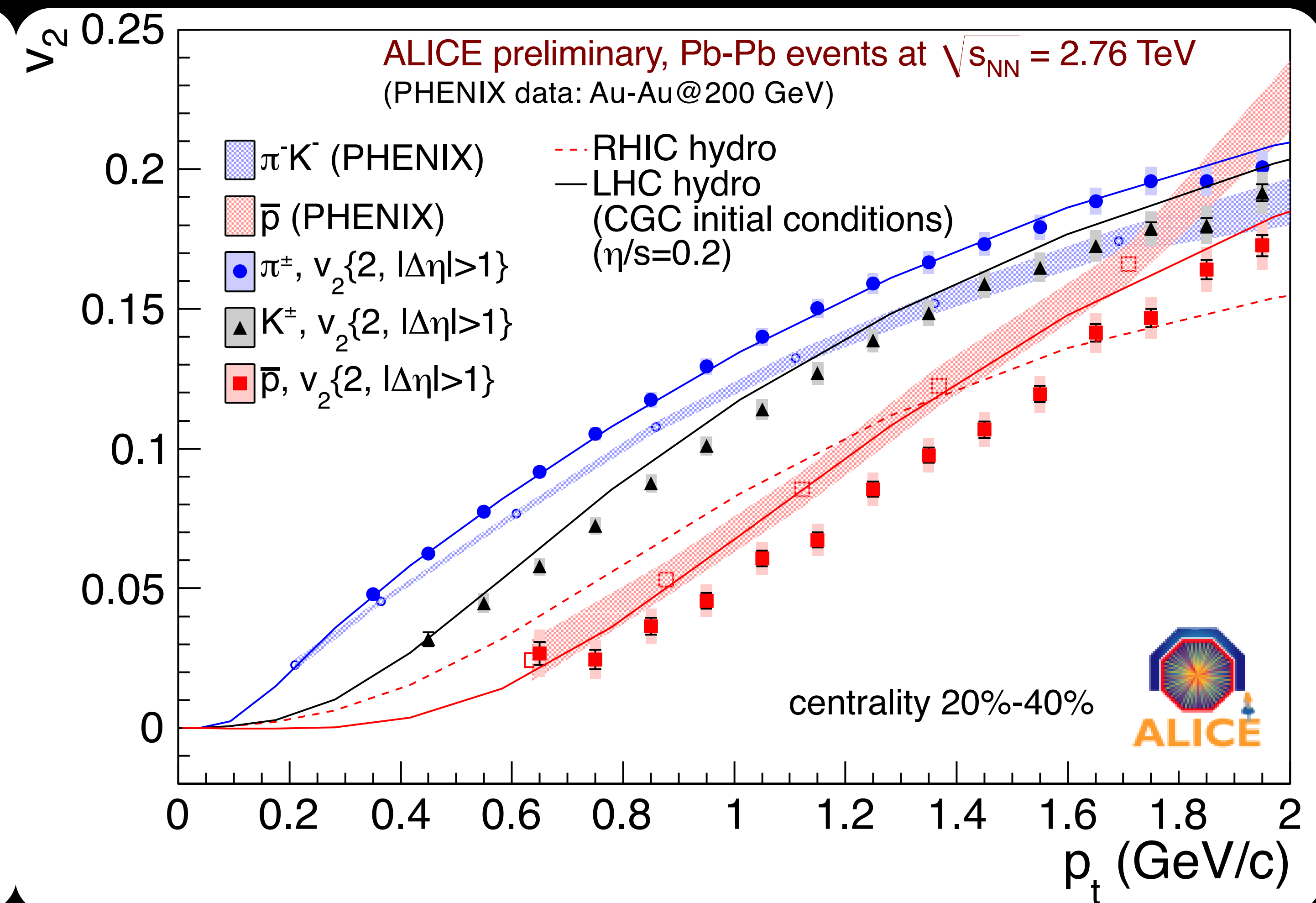


K. Aamodt et al. (ALICE Collaboration)
PRL 105, 252302 (2010)

The system produced at the LHC behaves as a very low viscosity fluid (a perfect fluid)

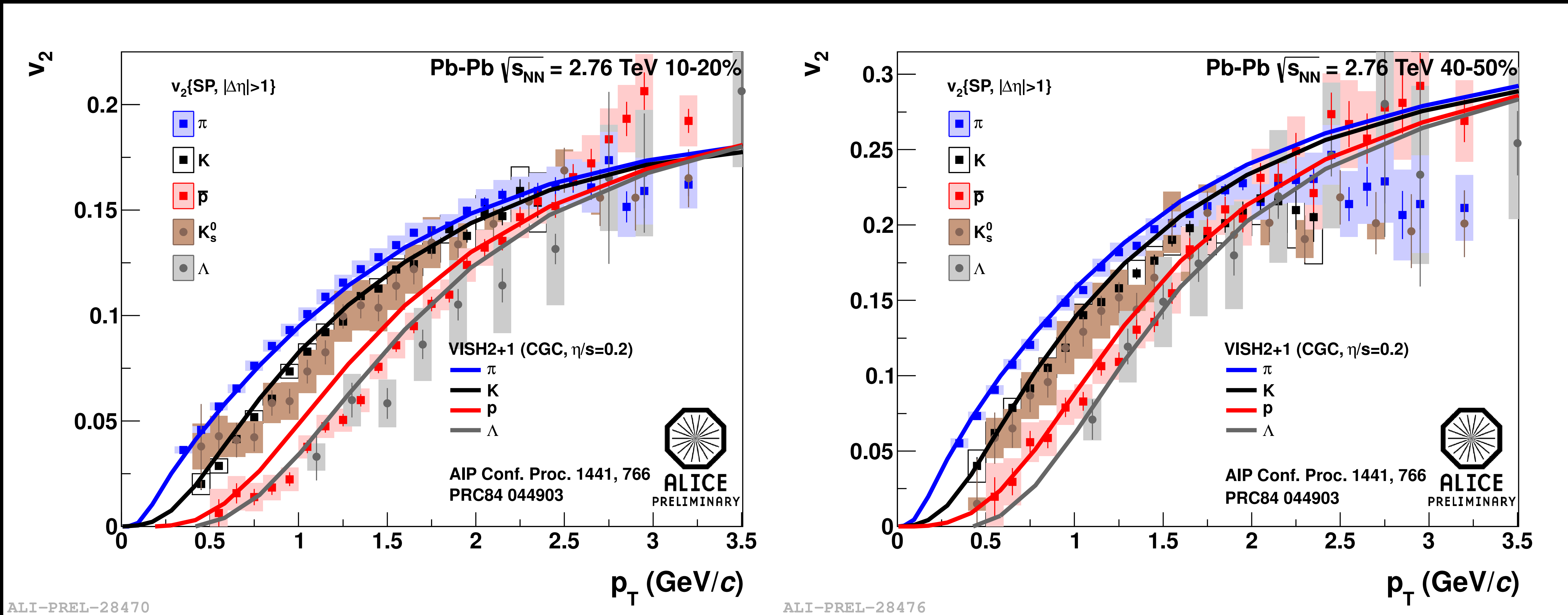
v_2 in ALICE





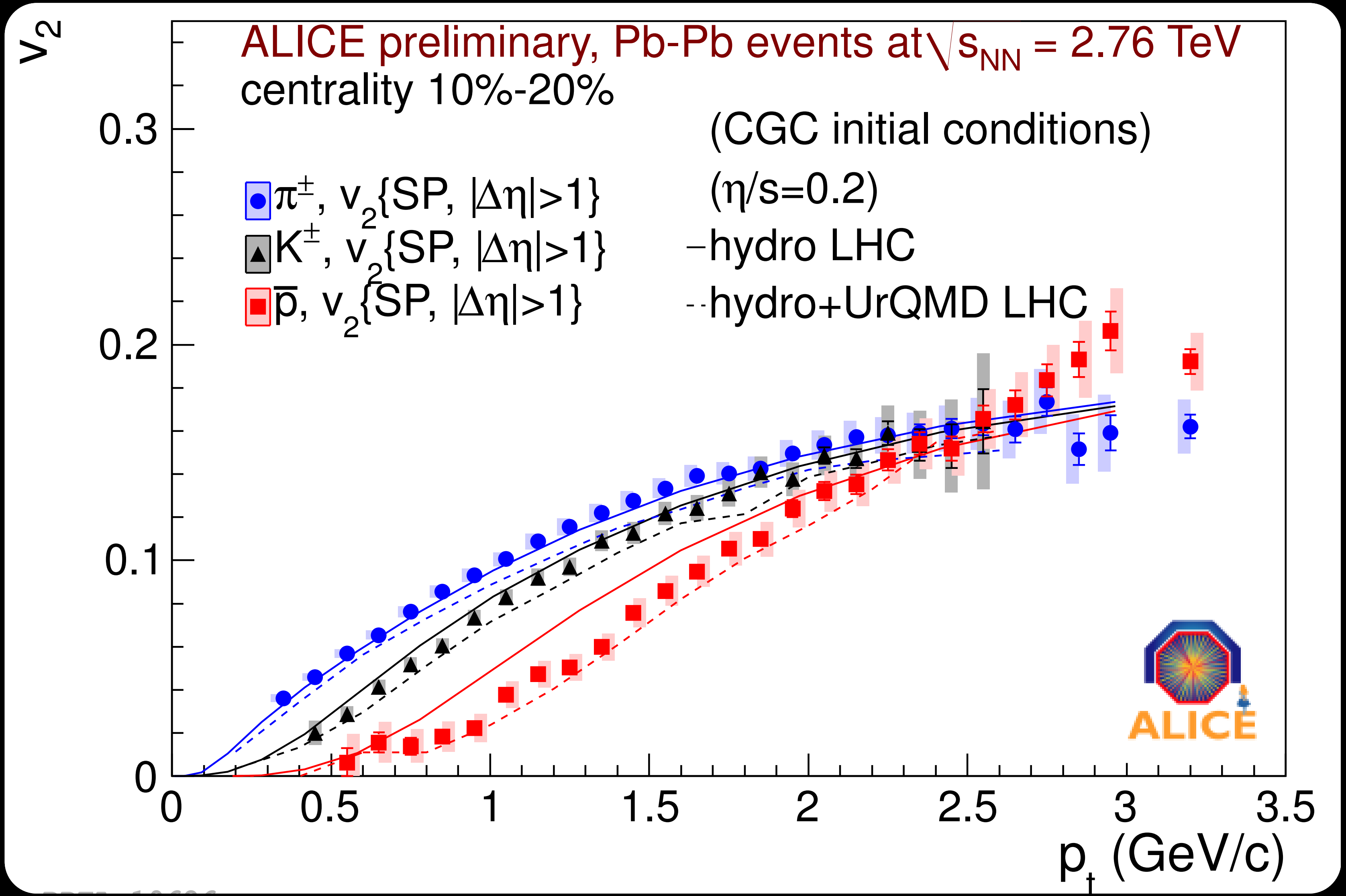
Stronger radial flow at the LHC

v_2 in ALICE



Stronger radial flow but pure hydro calculations do not describe well the most central collisions

v_2 in ALICE



Radial flow build up in the hadronic phase has to be taken into account, models have to be more sophisticated

initial conditions

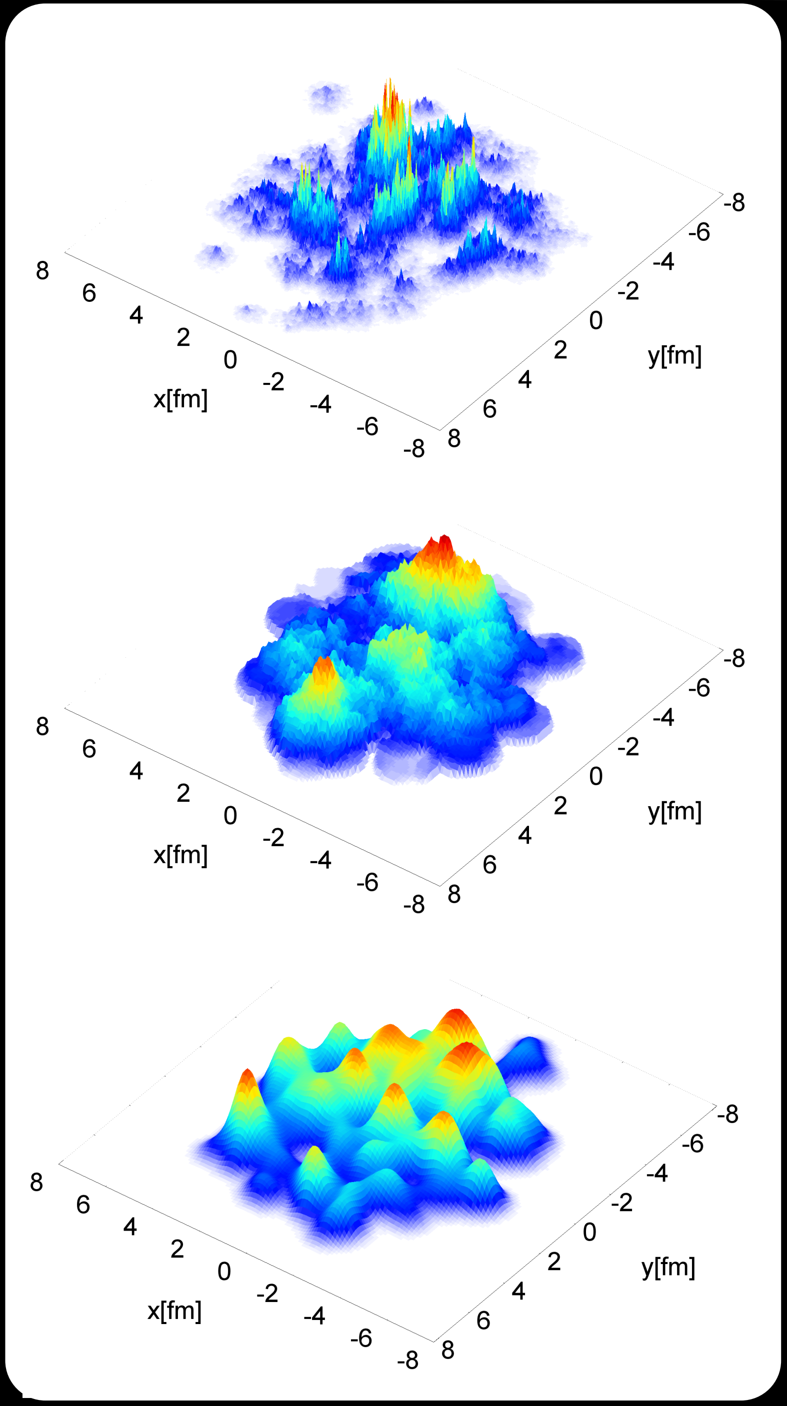
IP-Glasma

gluons + fluctuations

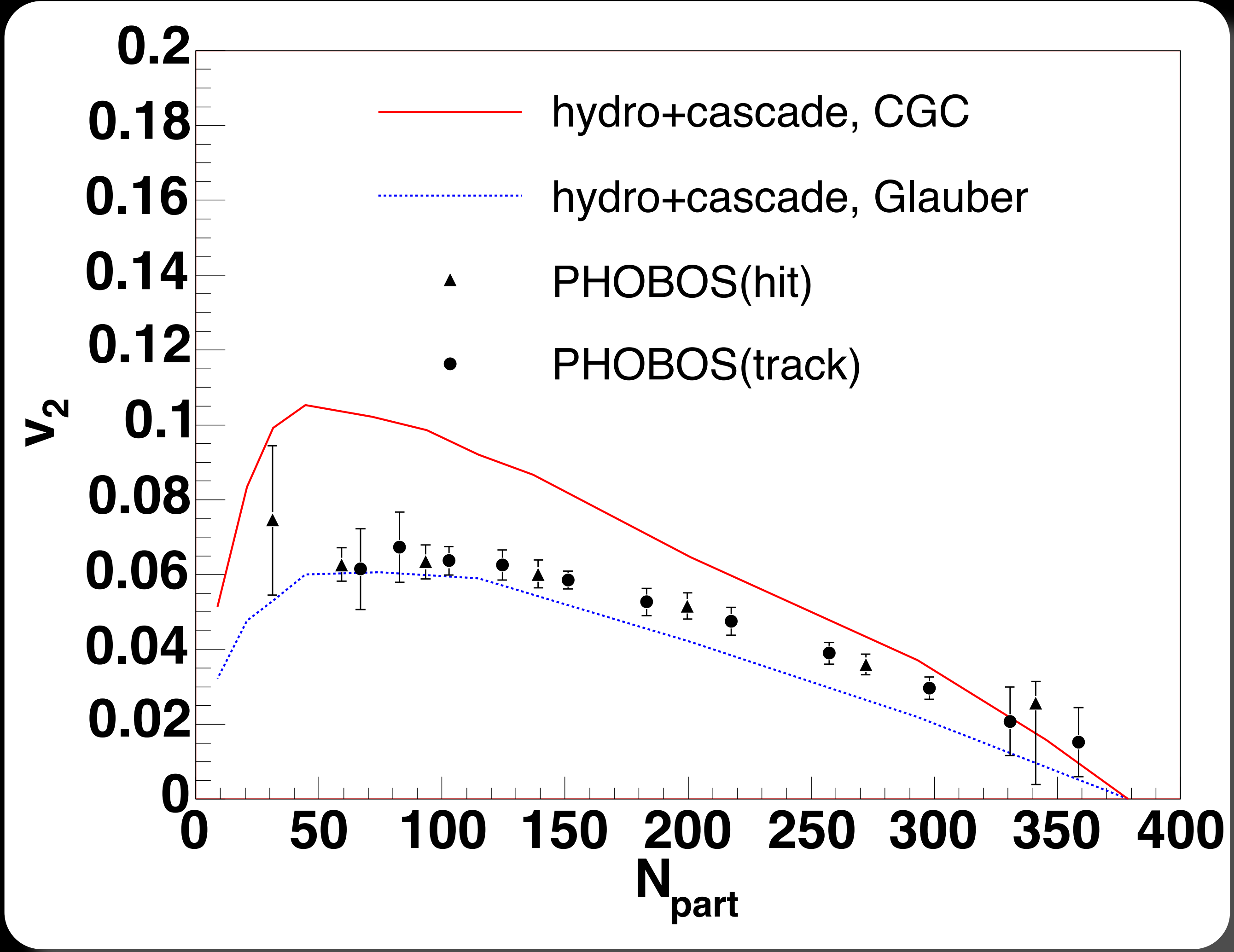
MC-KLN

gluons

MC-Glauber



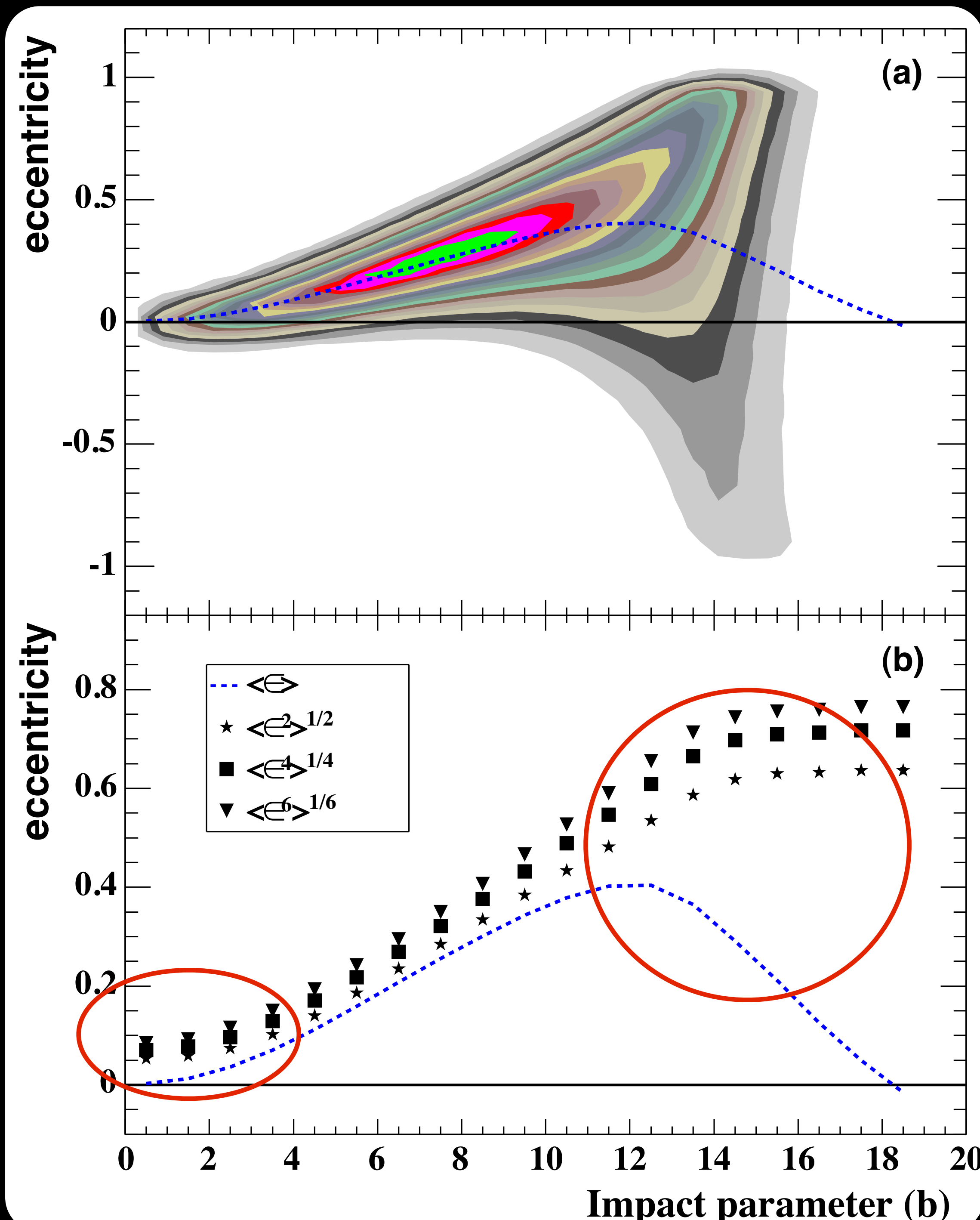
$$v_2 \propto \varepsilon$$



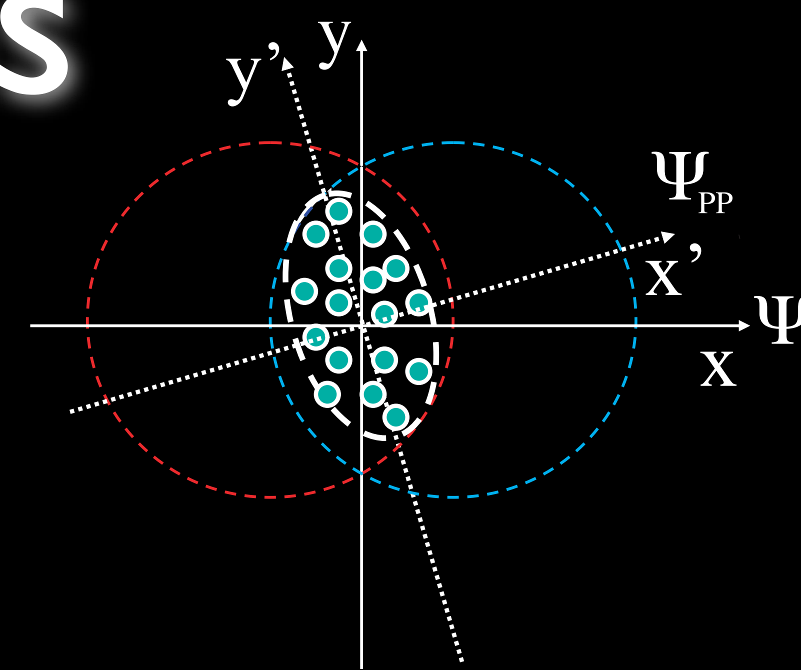
T. Hirano et al., Phys. Lett. B 636 299 (2006)
T. Hirano et al., J.Phys.G34:S879-882,2007

limit on how "non-ideal" the system is allowed to be depends on our understanding of the initial conditions!

v_2 fluctuations



M. Miller and RS, arXiv:nucl-ex/0312008



- measured: $v_2\{2\} = \sqrt{(\langle v_2\rangle^2 + \sigma_v^2 + \delta)}$
- using: $v_2 \propto \varepsilon$
- If the eccentricity fluctuates
  $$\langle \varepsilon^2\rangle - \langle \varepsilon\rangle^2 \neq 0$$
  $$\langle v_2\rangle \neq \sqrt{\langle (v_2)^2\rangle}$$
- fluctuations change v_2 estimate significantly!

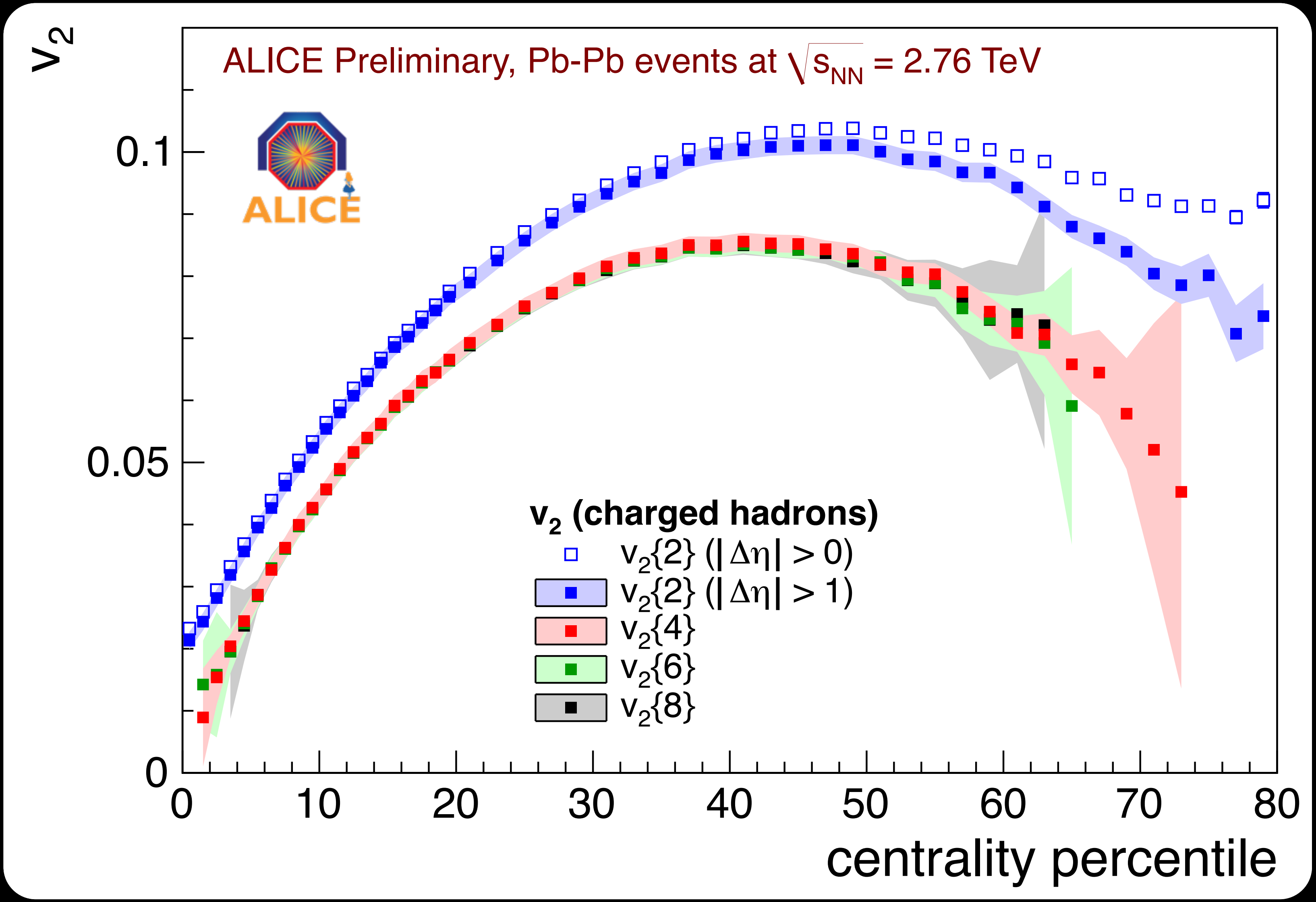
v_2 versus centrality in ALICE

$$v\{2\} = \langle v\rangle + \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$

$$v\{4\} = \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$

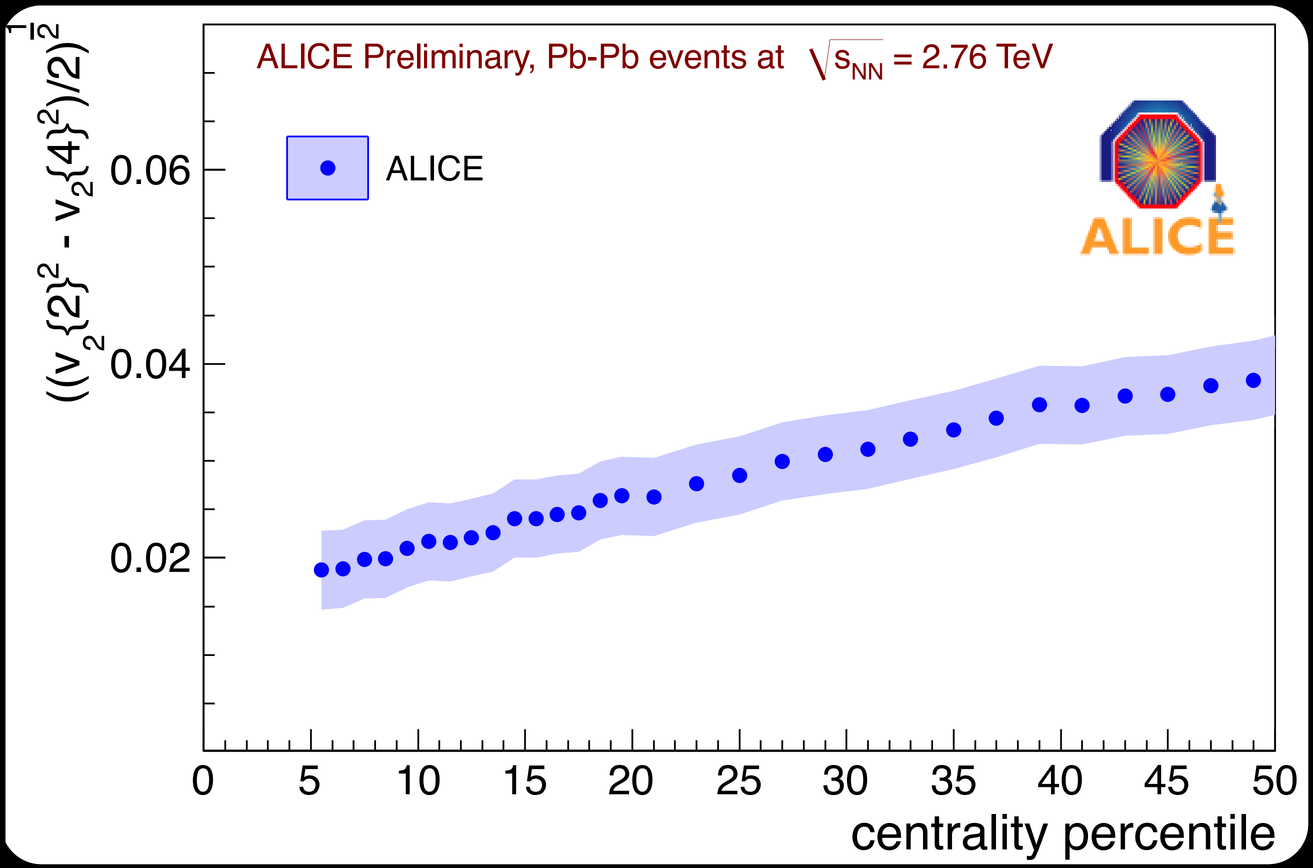
$$v\{6\} = \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$

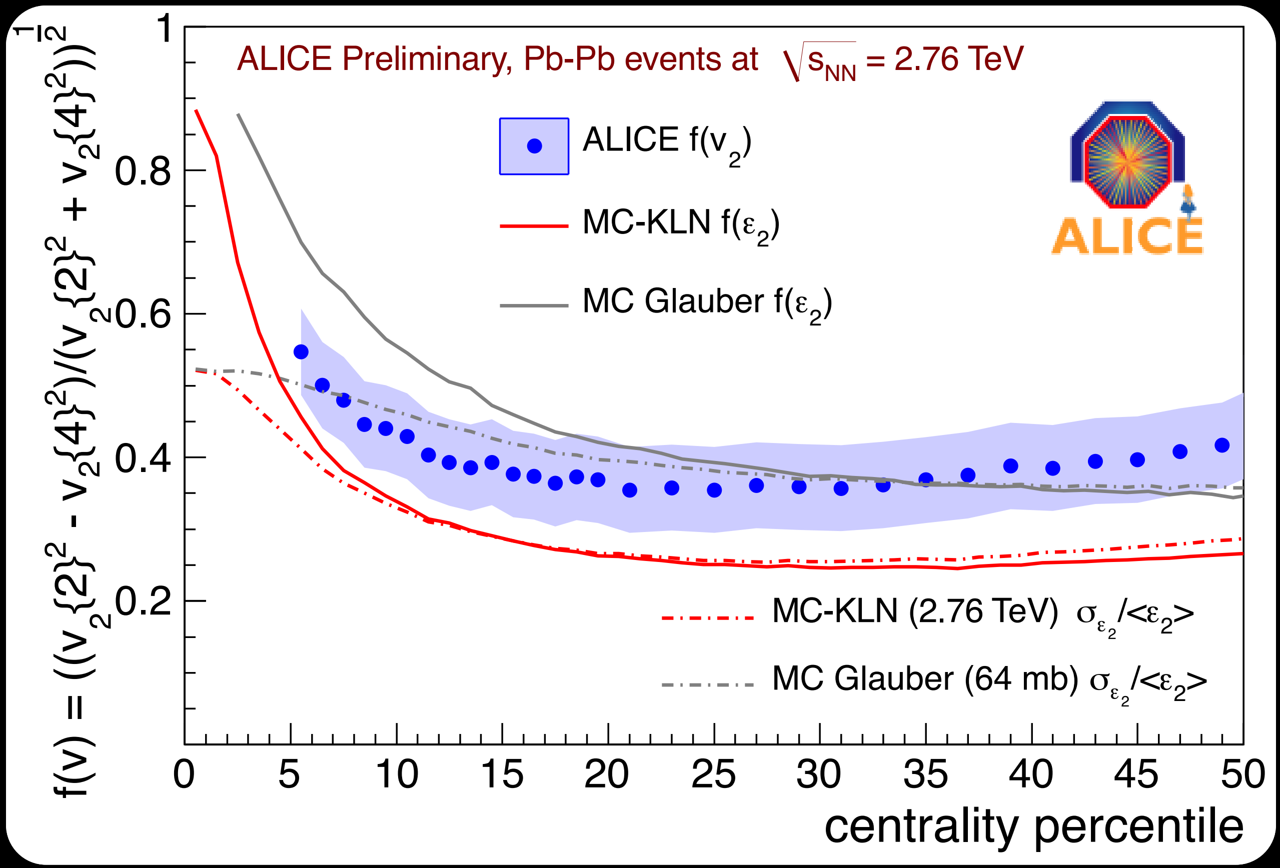
$$v\{8\} = \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$



Clear separation between $v_2\{2\}$ and higher order cumulants
Higher order cumulant v_2 estimates are consistent within uncertainties

v_2 fluctuations



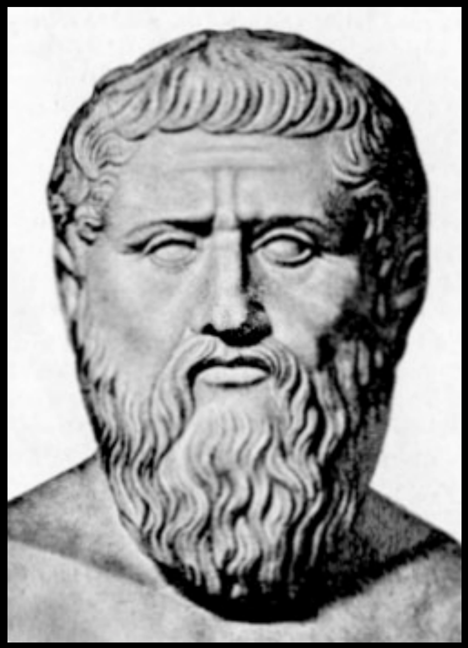


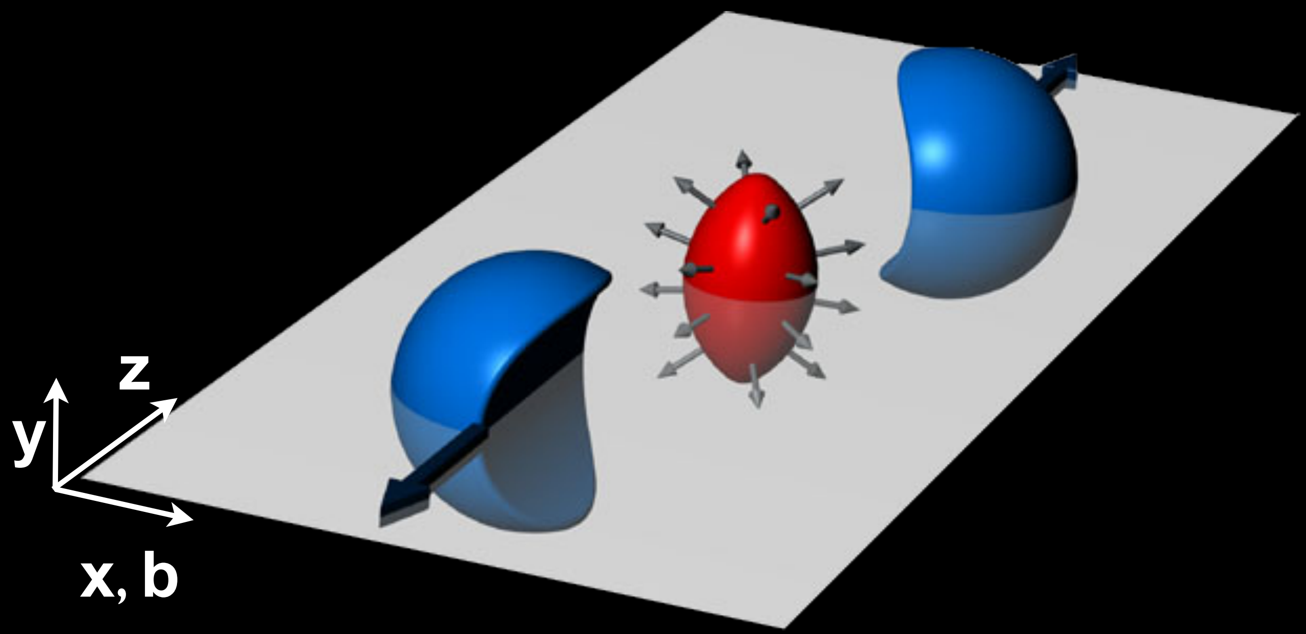
$$\sigma_{v_n} \simeq [\frac{1}{2}\left(v_n^2\{2\} - v_n^2\{4\}\right)]^{\frac{1}{2}}$$

$$\frac{\sigma_{v_n}}{v_n} \simeq \left(\frac{v_n^2\{2\} - v_n^2\{4\}}{v_n^2\{2\} + v_n^2\{4\}}\right)^{\frac{1}{2}}$$

For more central collisions the data is between MC Glauber and MC-KLN CGC

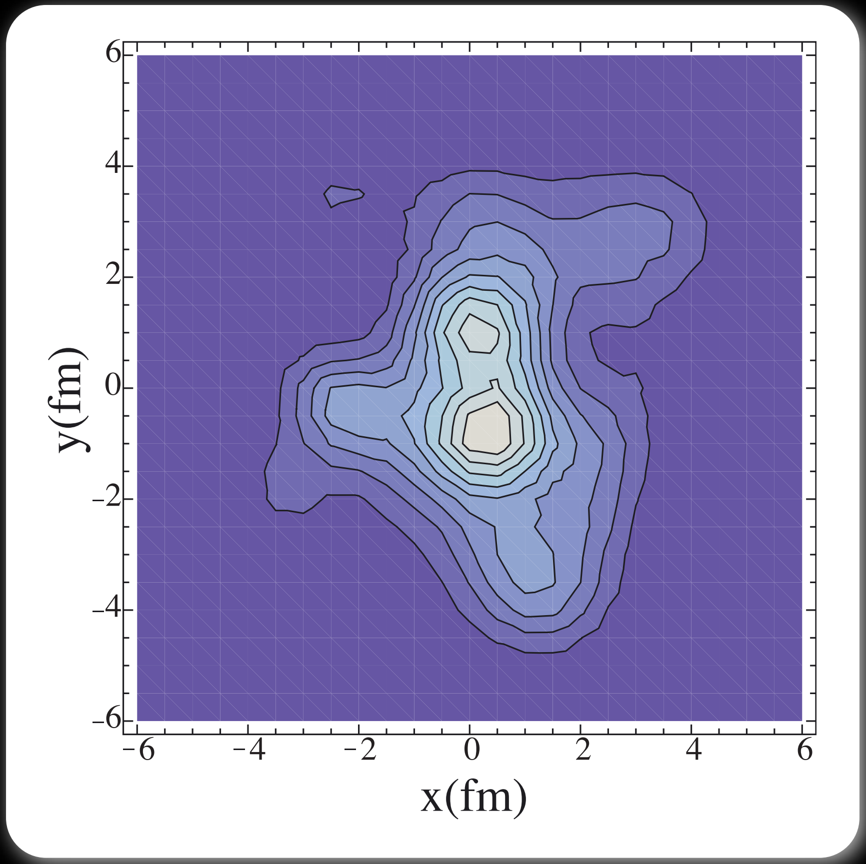
Ideal Shapes



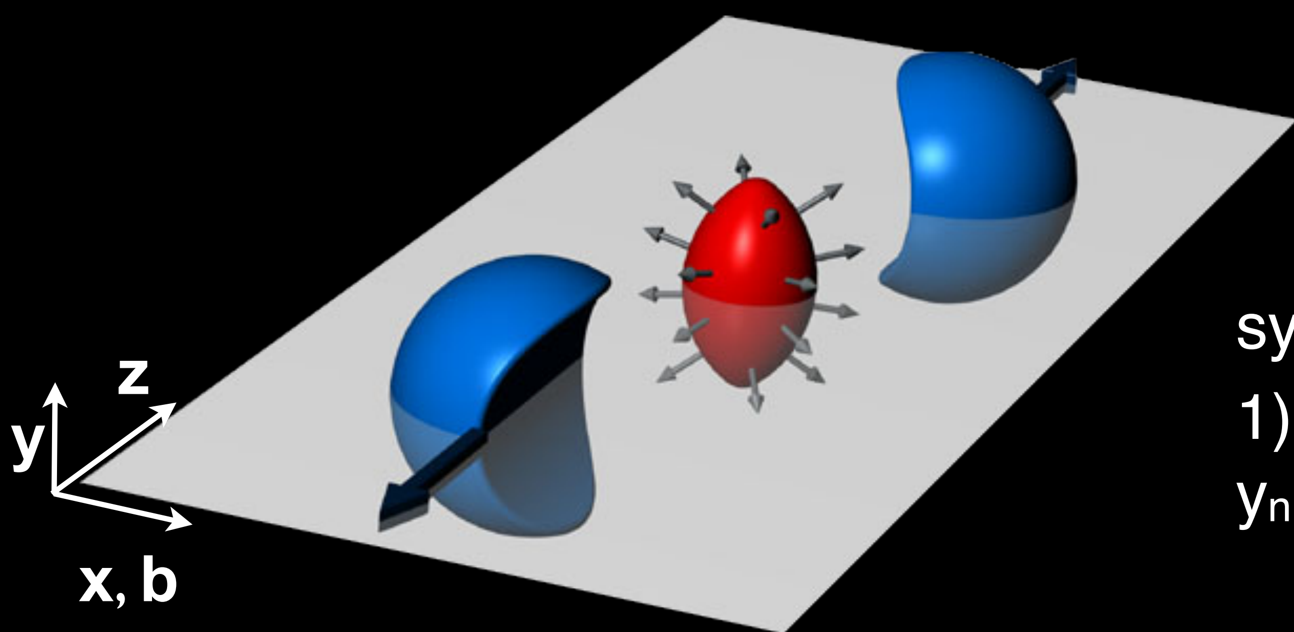


we are not in
Plato's ideal world

symmetries are not there in single collisions



Azimuthal distributions



$$r(\varphi) = \frac{x_0}{2\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty}[x_n \cos(n\varphi) + y_n \sin(n\varphi)]$$

symmetries reduce the number of parameters

1) particle yield at ϕ and -ϕ should be equal -> y_n = 0 (no sin terms)

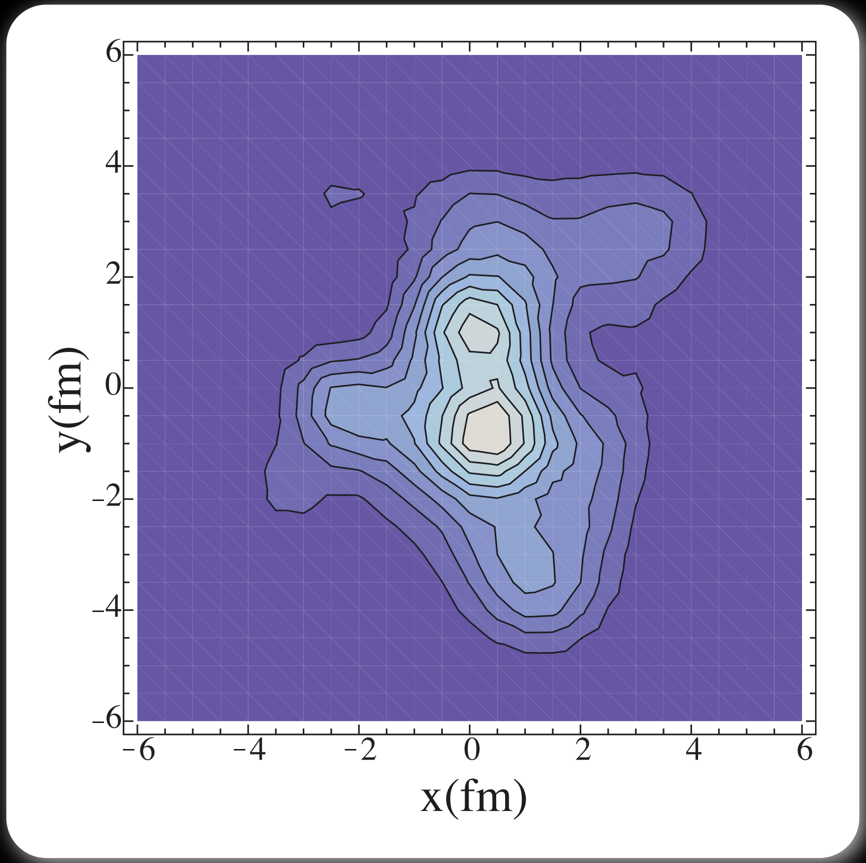
$$\frac{dN}{d\varphi} = \frac{1}{2\pi}\left[1 + \sum_{n=1}^{\infty} 2v_n \cos[n(\varphi - \Psi_R)]\right]$$

2) particle yield at ϕ and ϕ+π should be equal -> cos(nϕ) = 0 for odd n

only even harmonics at mid-rapidity, v_2, v_4, v_6, etc

$$v_n = \langle \cos[n(\varphi - \Psi_R)]\rangle$$

Azimuthal distributions



$$r(\varphi) = \frac{x_0}{2\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty}[x_n \cos(n\varphi) + y_n \sin(n\varphi)]$$

in general can be written as:

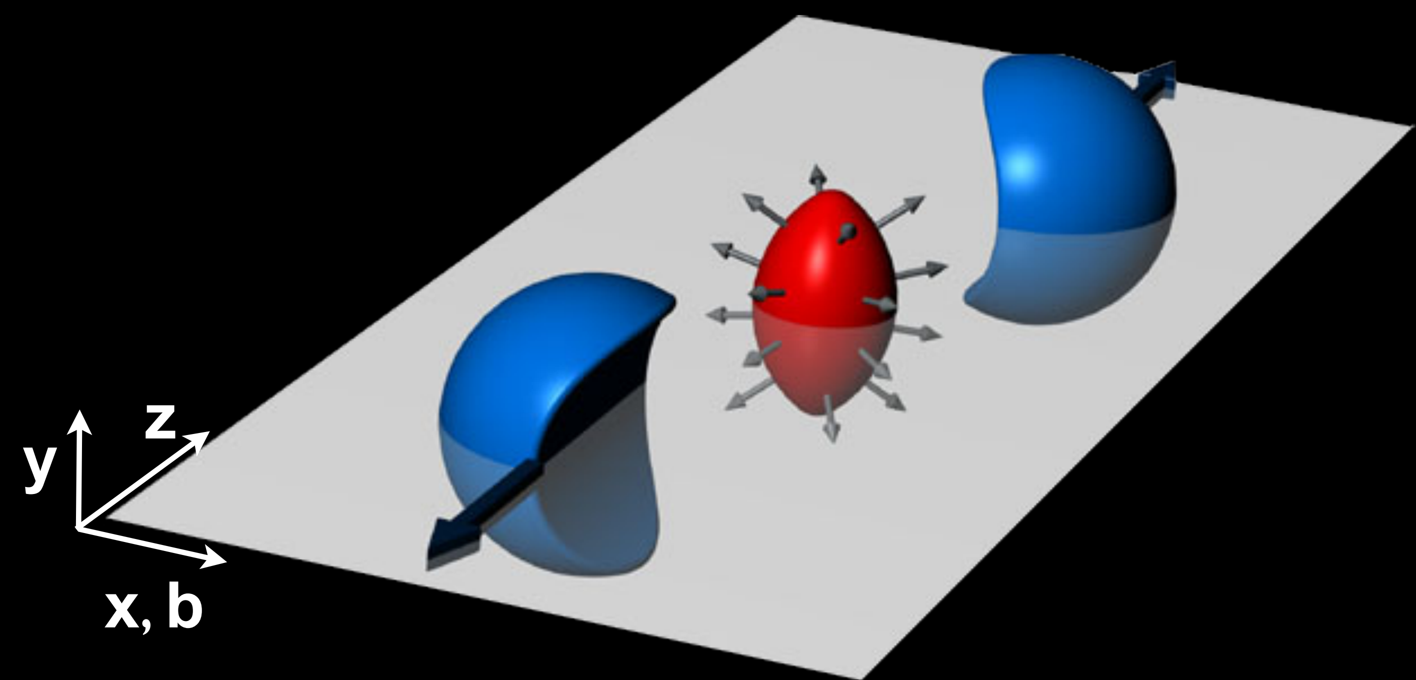
$$r(\varphi) = \frac{v_0}{2\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty}[v_n \cos(n\varphi - \Psi_n)]$$

all harmonics at mid-rapidity; v_1, v_2, v_3, v_4, v_5, v_6, etc

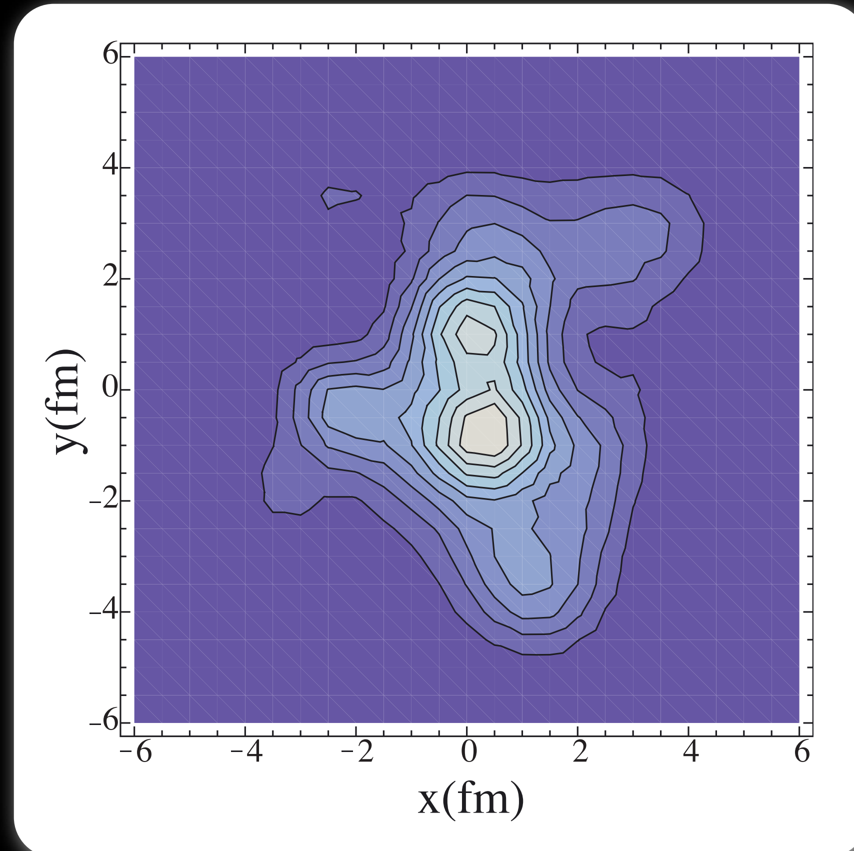
$$v_n = \langle \cos[n(\varphi - \Psi_n)] \rangle$$

in addition; ψ_1, ψ_2, ψ_3, ψ_4, ψ_5, ψ_6, etc

initial conditions and v_n



G. Qin, H. Petersen, S. Bass, and B. Muller

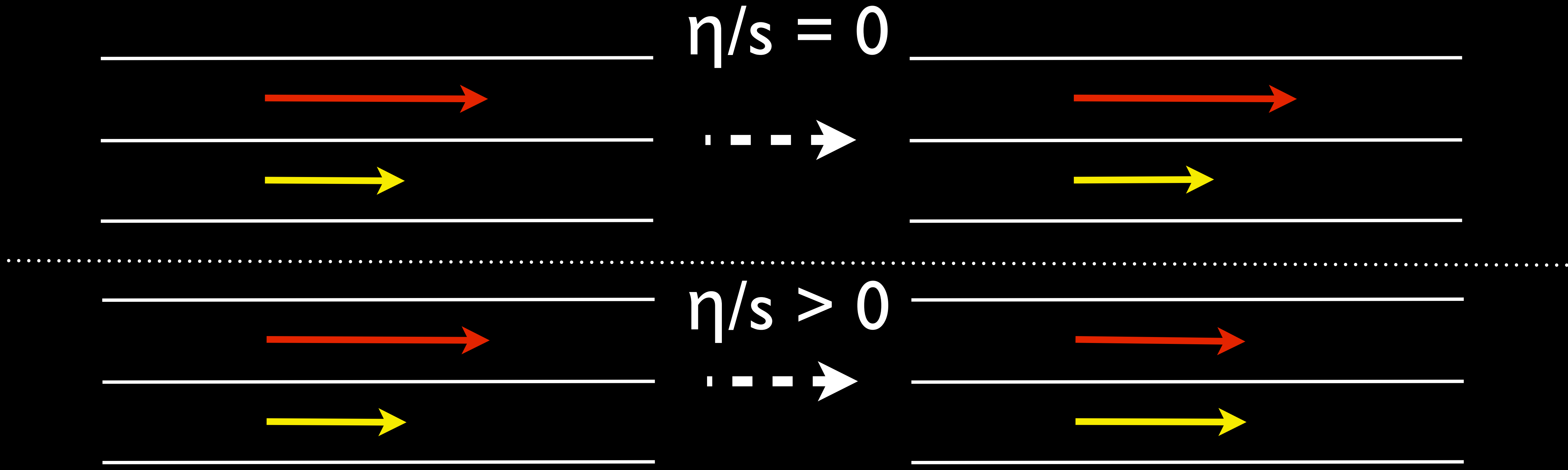


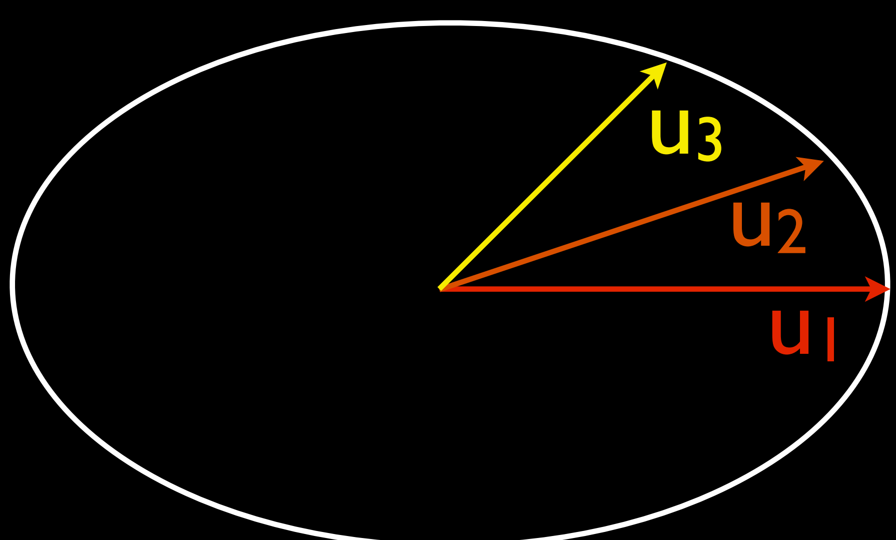
$$\frac{2\pi}{N}\frac{dN}{d\phi} = 1 + \sum_{n=2,4,6,\ldots}^{\infty} 2v_n \cos n(\phi - \Psi_R)$$

$$\frac{2\pi}{N}\frac{dN}{d\phi} = 1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \Psi_n)$$

initial spatial geometry not a smooth almond (for which all odd harmonics and sin $n(\Phi-\psi_R)$ are zero due to symmetry) and fluctuates event-by-event leads to higher odd harmonics and symmetry planes

Shear Viscosity

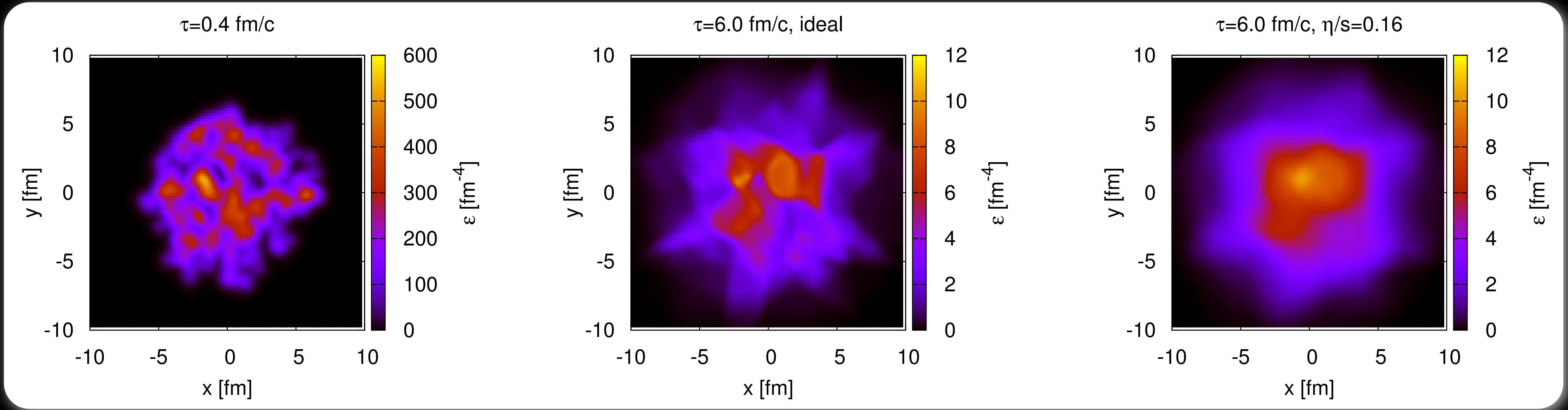




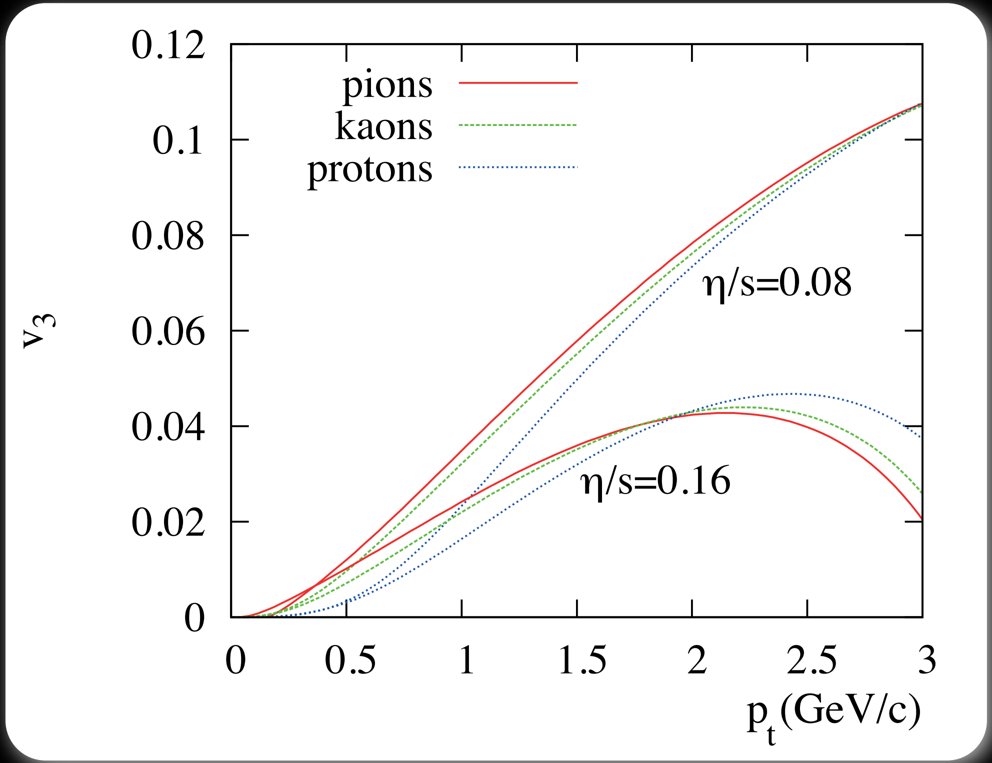
$u_1 > u_2 > u_3$ shear viscosity will make them equal and destroy the elliptic flow v_2 higher harmonics represent smaller differences which get destroyed more easily, and which, if measurable, makes them more sensitive probes to η/s

shear viscosity

Music, Sangyong Jeon



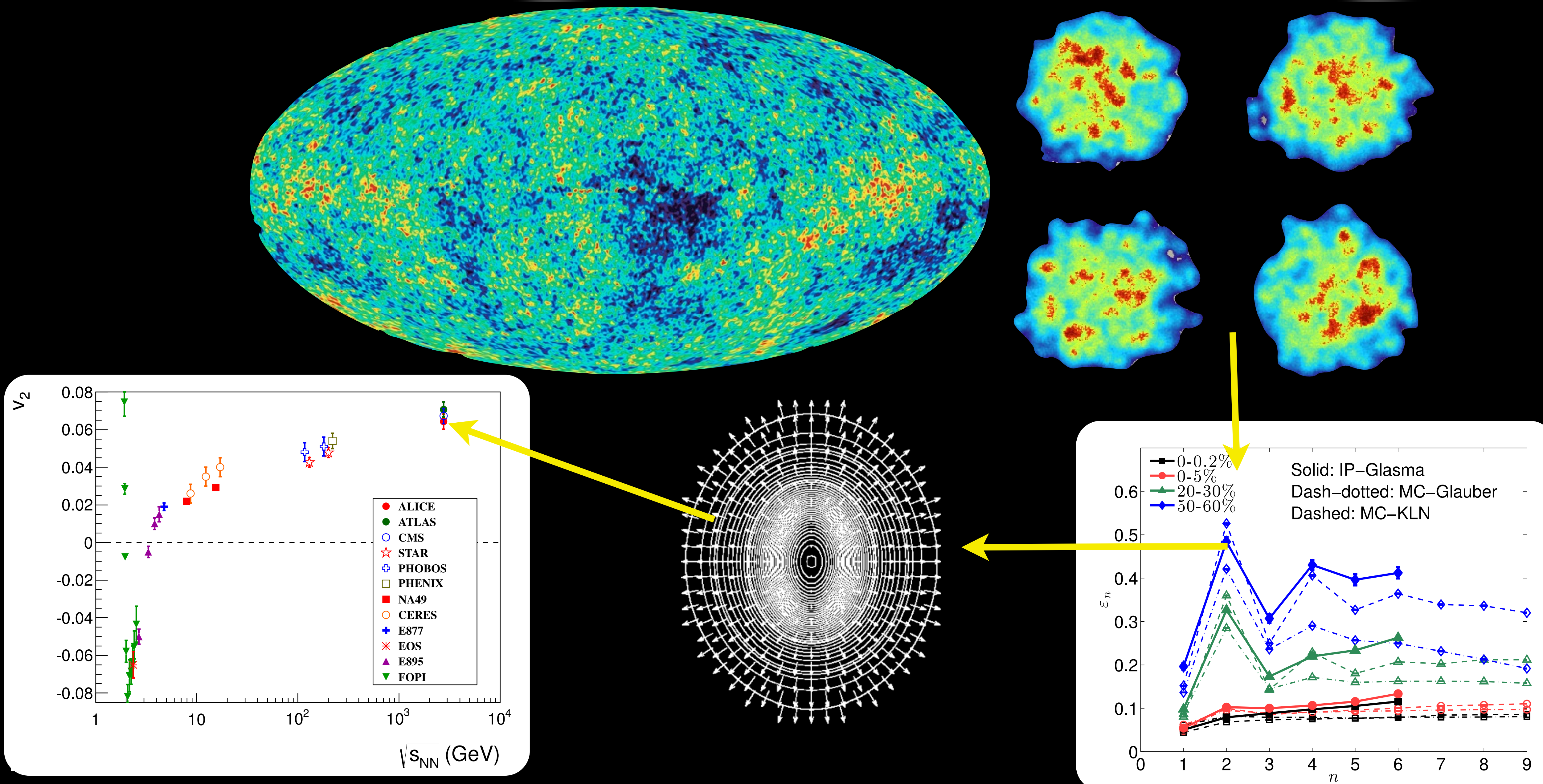
initial conditions | ideal hydro $\eta/s=0$ | viscous hydro $\eta/s=0.16$



larger η/s clearly smoothes the distributions and suppresses the higher harmonics (e.g. v_3)

Hydro: Alver, Gombeaud, Luzum & Ollitrault, Phys. Rev. C82 (2010)

The big and little bang

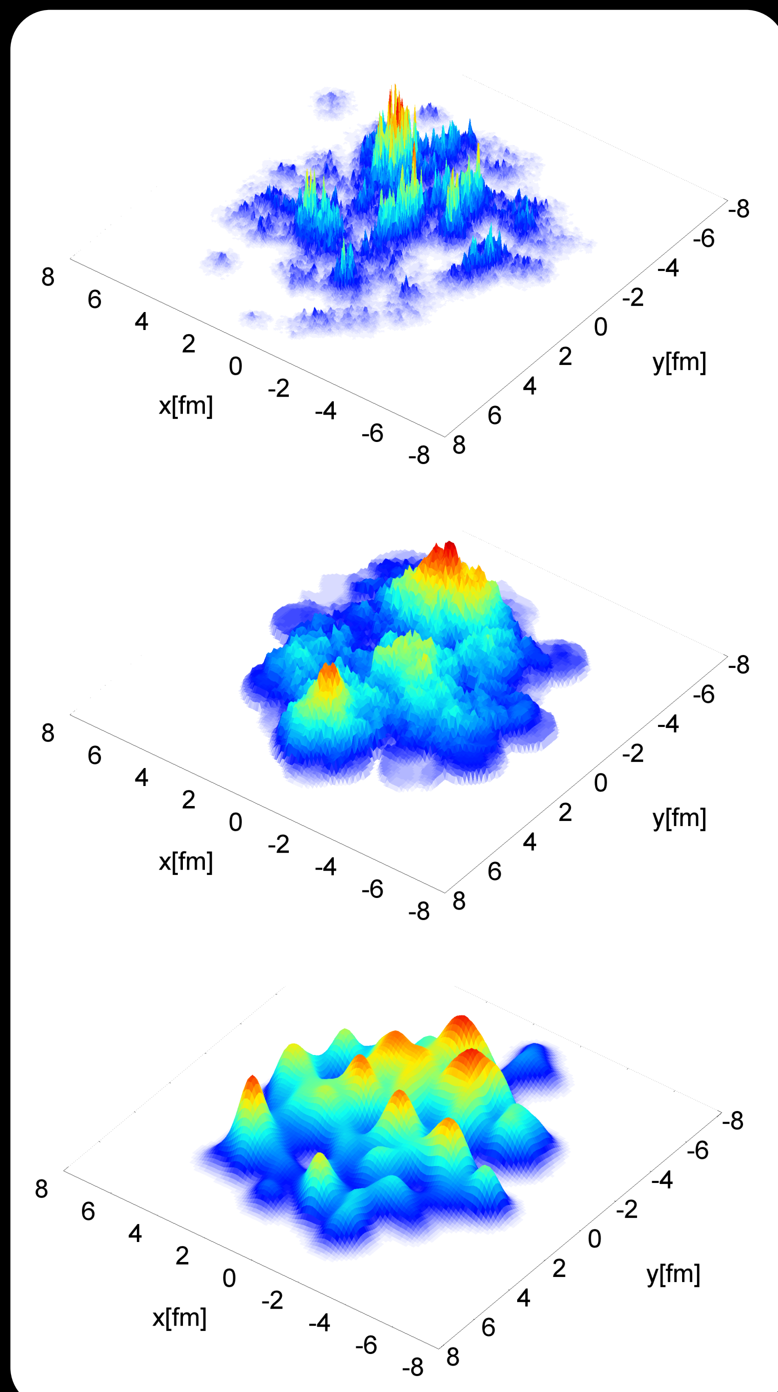


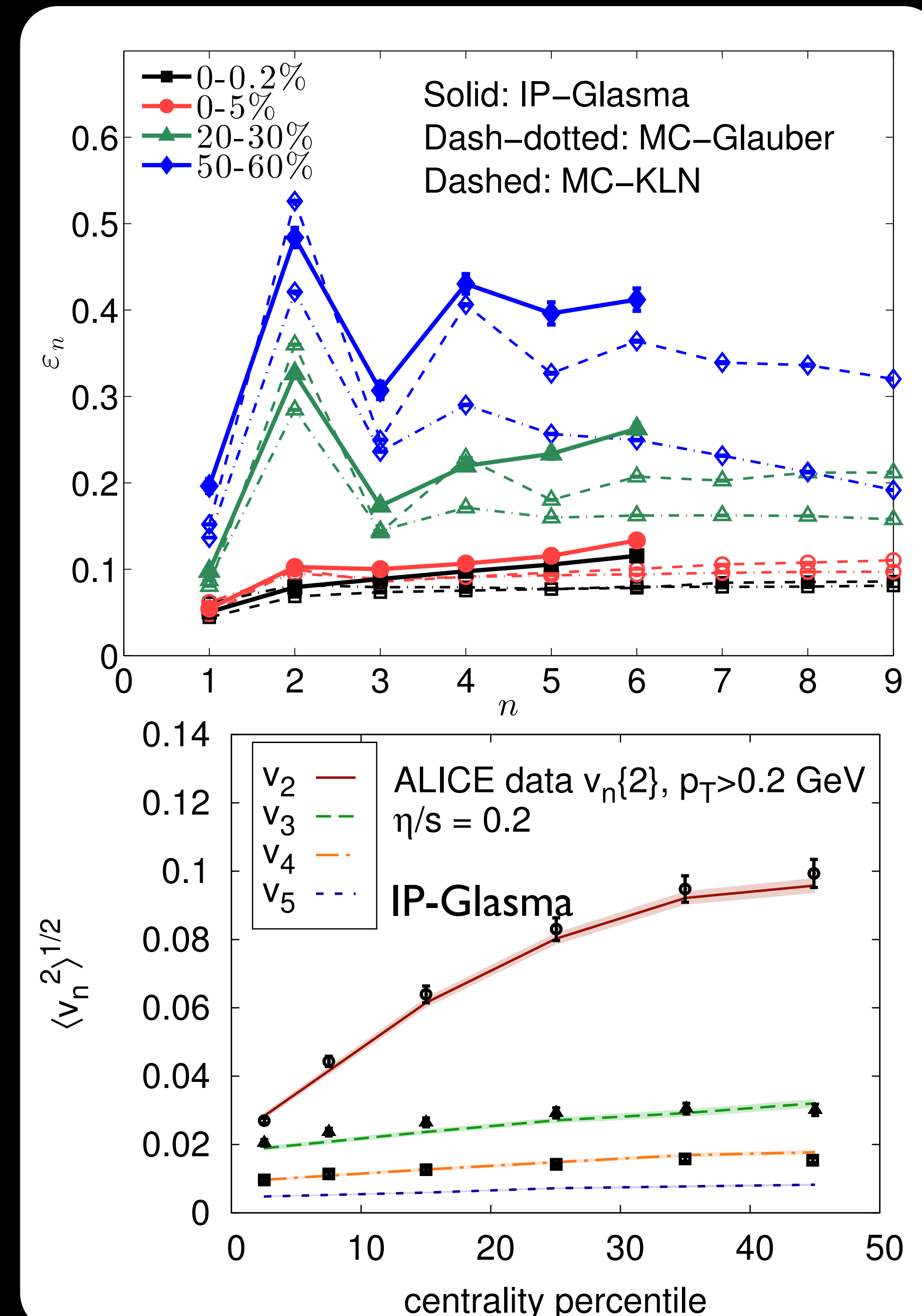
Fluctuating initial conditions generate power spectrum of harmonics which are different for different collision centralities.

Central collisions dominated by fluctuations resulting in a flat power spectrum peripheral collisions dominated by the 2[nd] order coefficient.

Measurements of the power spectrum allows us to disentangle the various initial state models

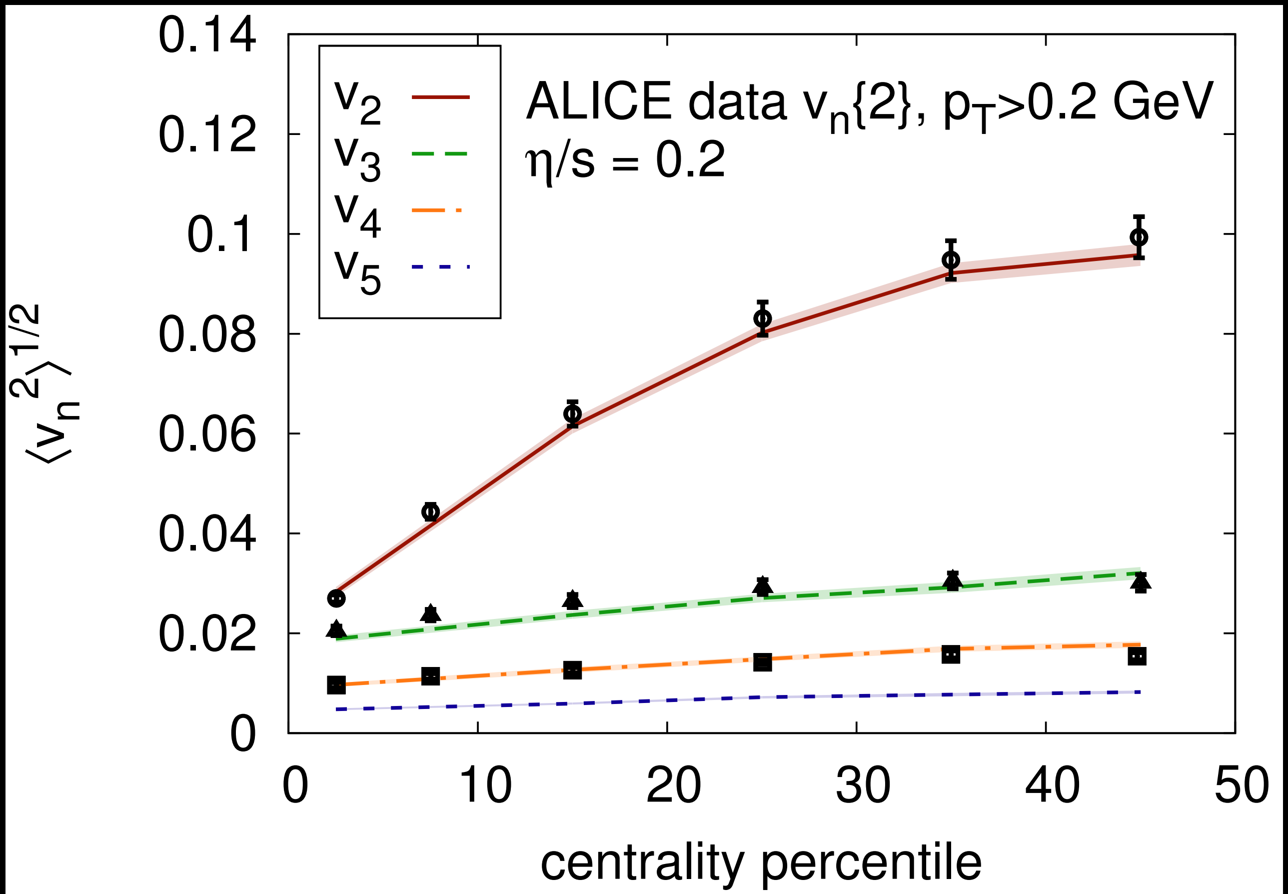
Power Spectrum

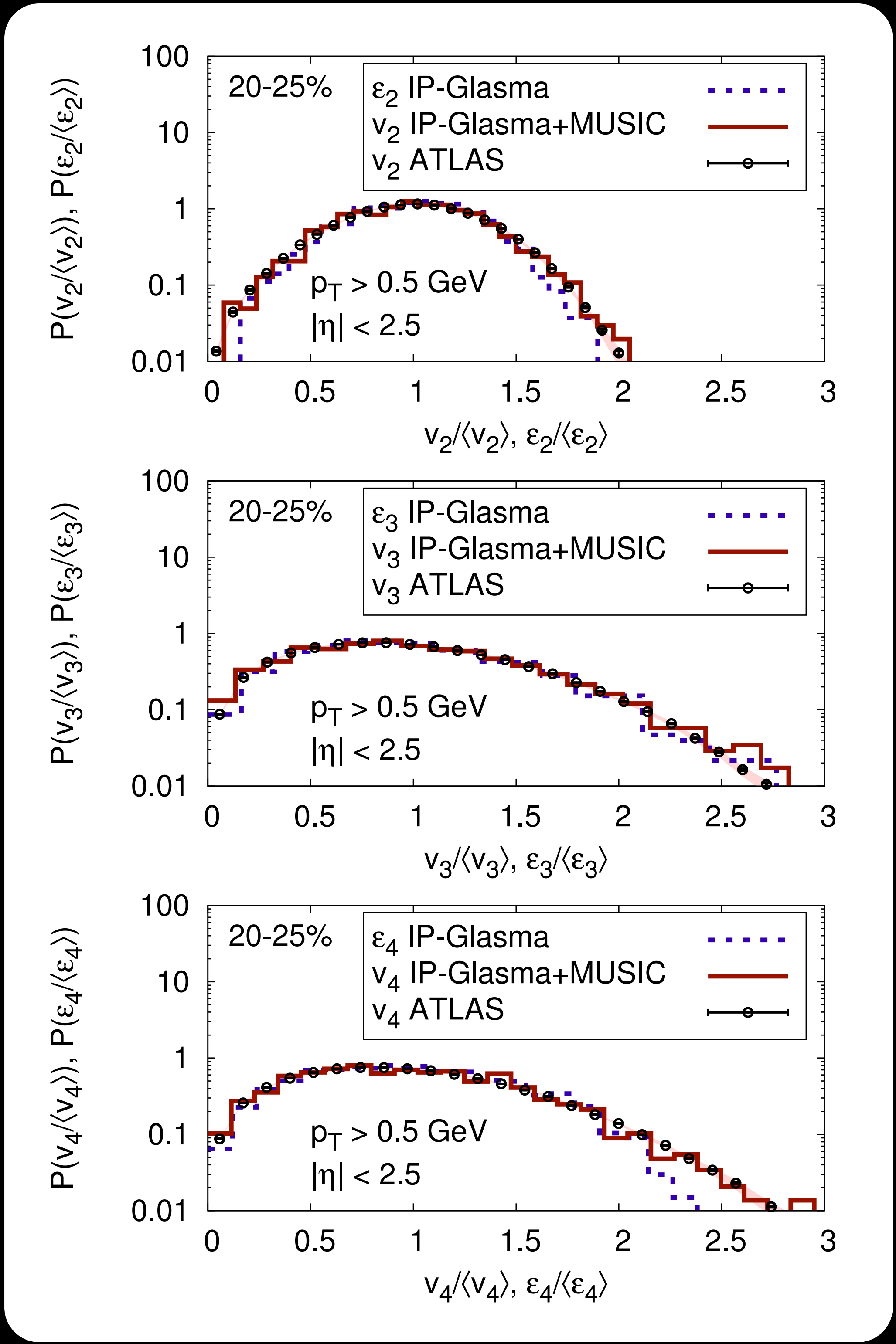




Currently only the IP-Glasma initial conditions provide a consistent description of the measurements

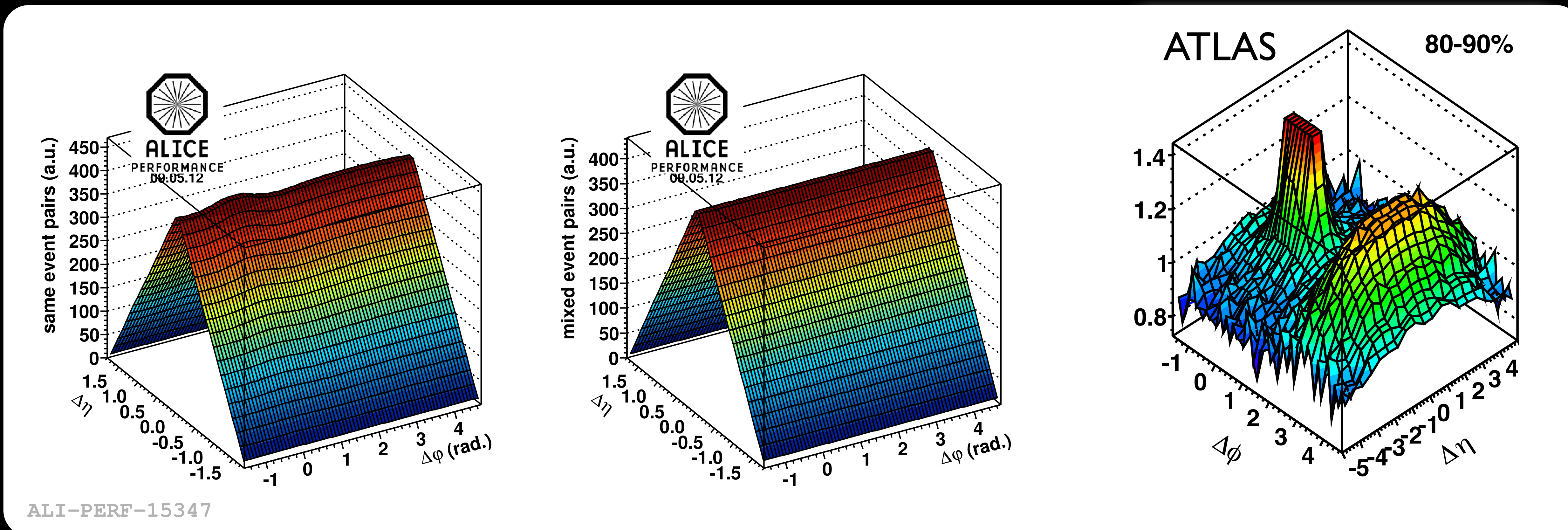
Compared to data





Almost perfect match between data and theory!

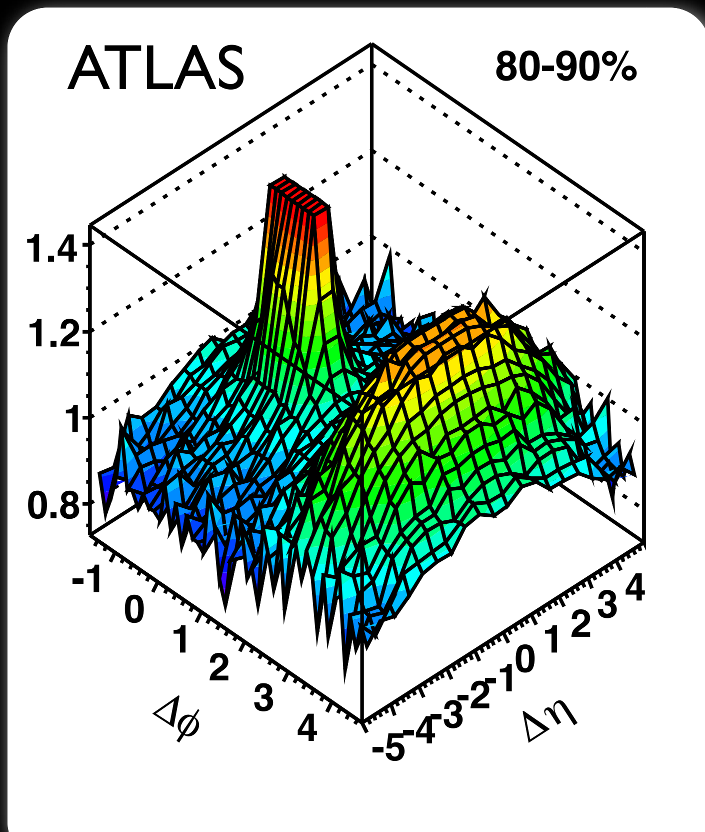
correlations

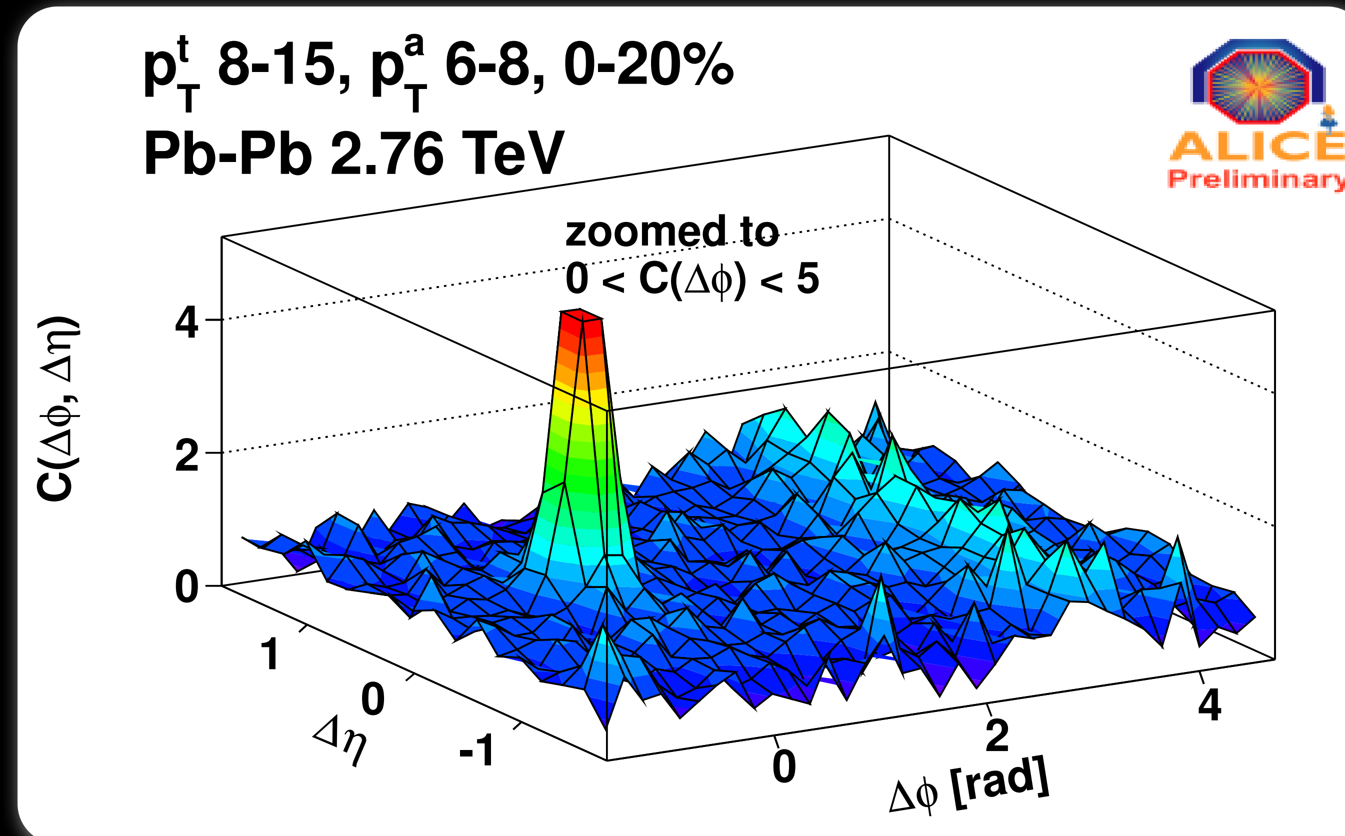


$$S(\Delta\eta, \Delta\varphi) = \frac{1}{N_{\rm trig}} \frac{d^2 N_{\rm same}}{d\Delta\eta d\Delta\varphi} \quad B(\Delta\eta, \Delta\varphi) = \frac{1}{N_{\rm trig}} \frac{d^2 N_{\rm mixed}}{d\Delta\eta d\Delta\varphi}$$

- often two-particle correlations are measured using the normalized number of pairs in an angular bin
- same distribution is measured using mixed events
- correlation is seen in the difference

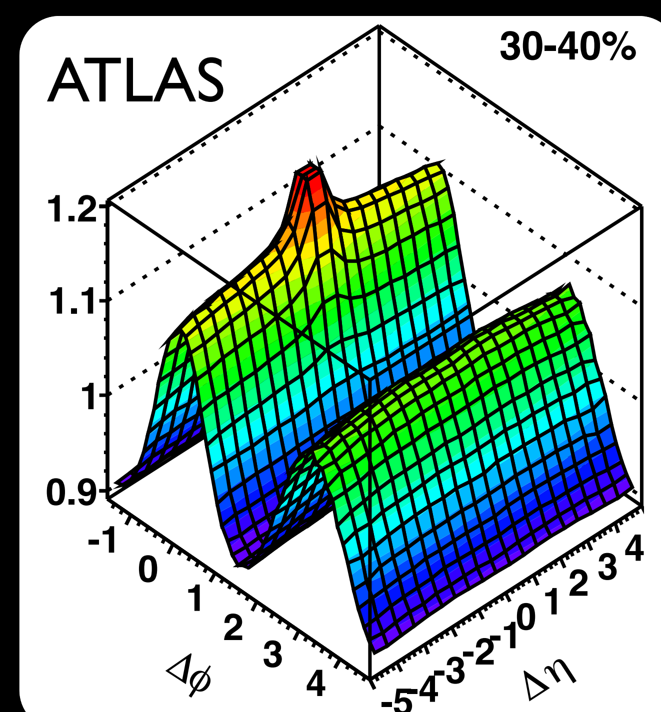
Angular Correlations

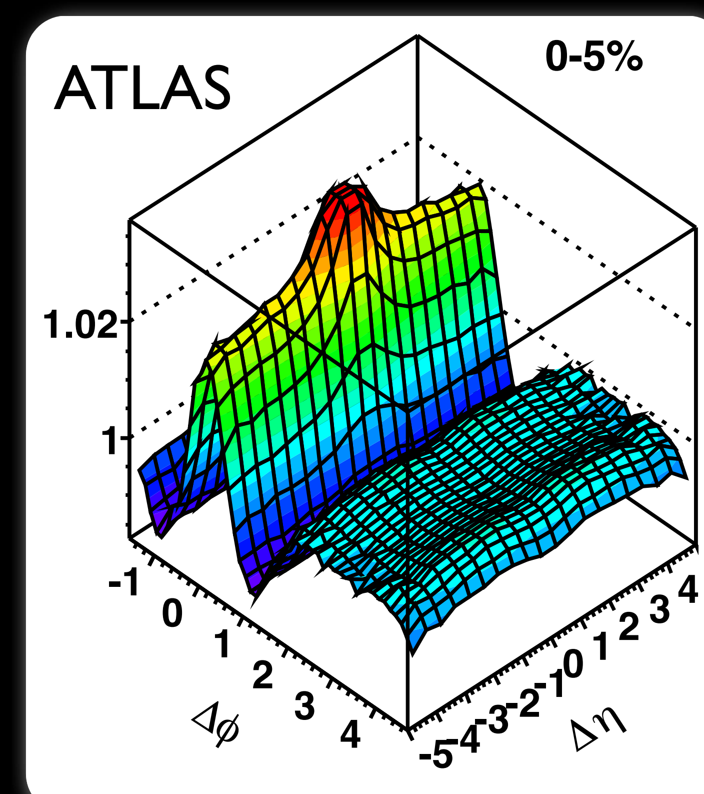




For very peripheral collisions or when triggered with a high-p_t charged particle the dominant contribution to two particle angular correlations is due to jet-correlations

More central heavy ion collisions look very very different!

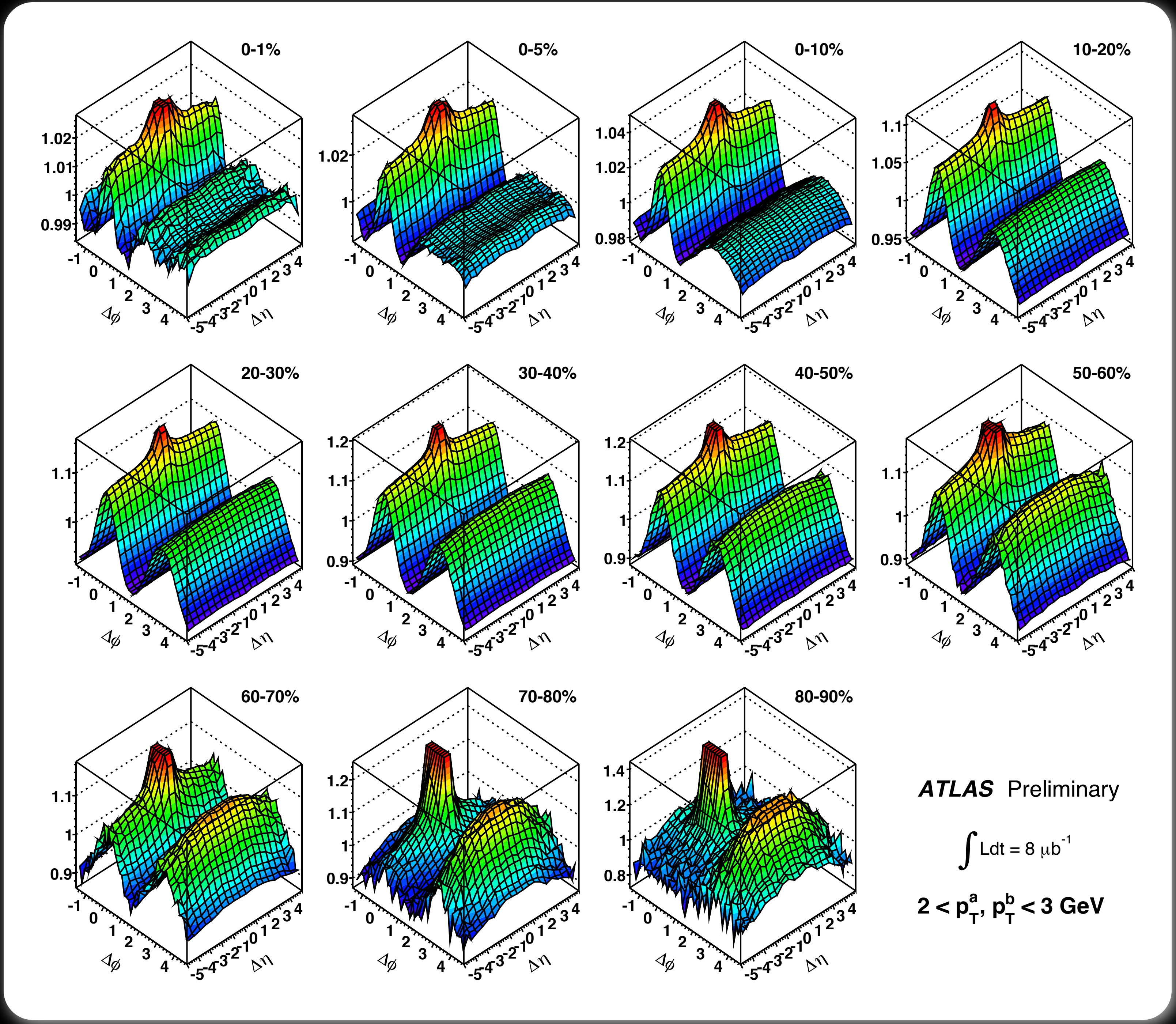




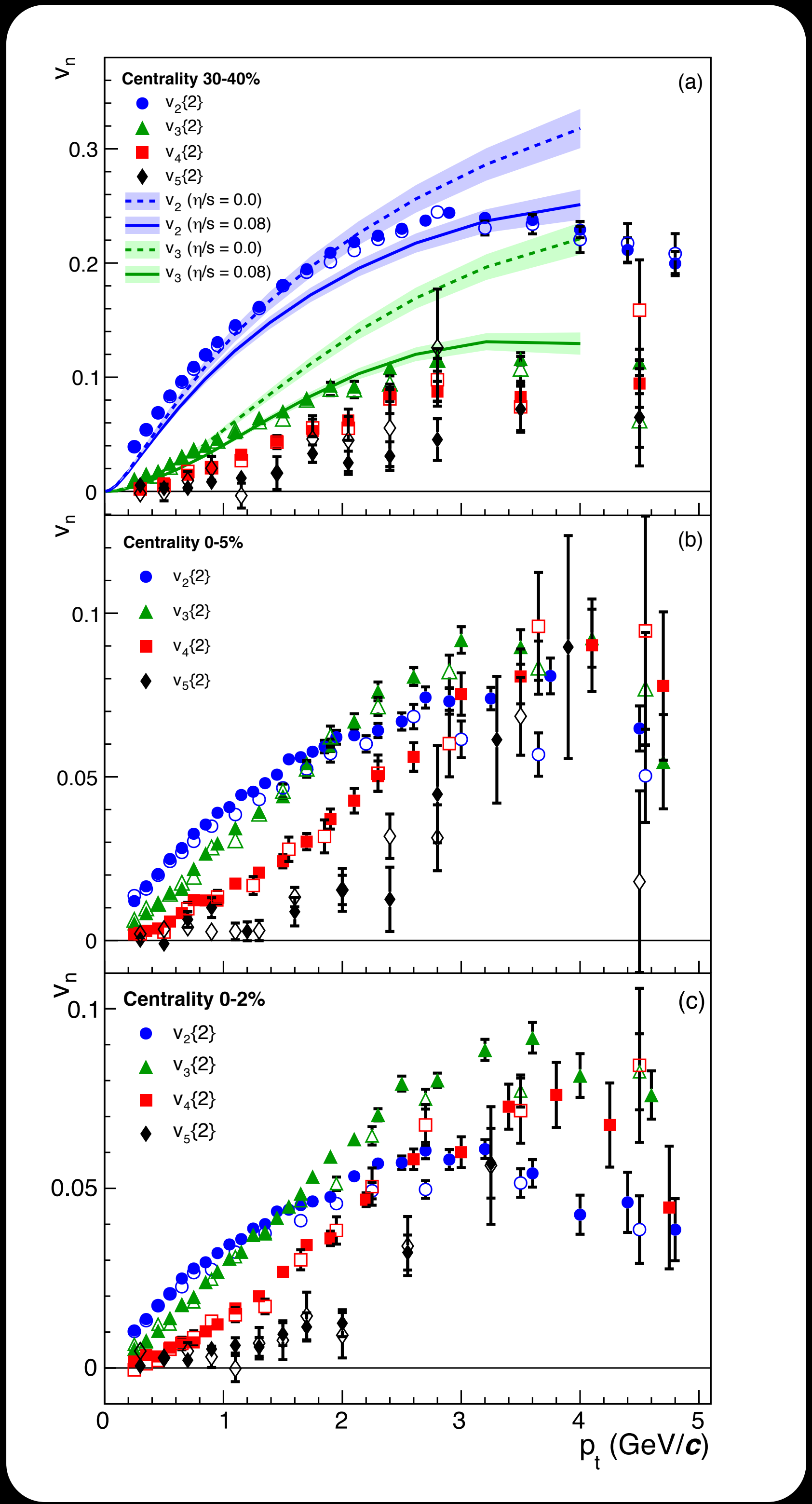
Angular Correlations at the LHC

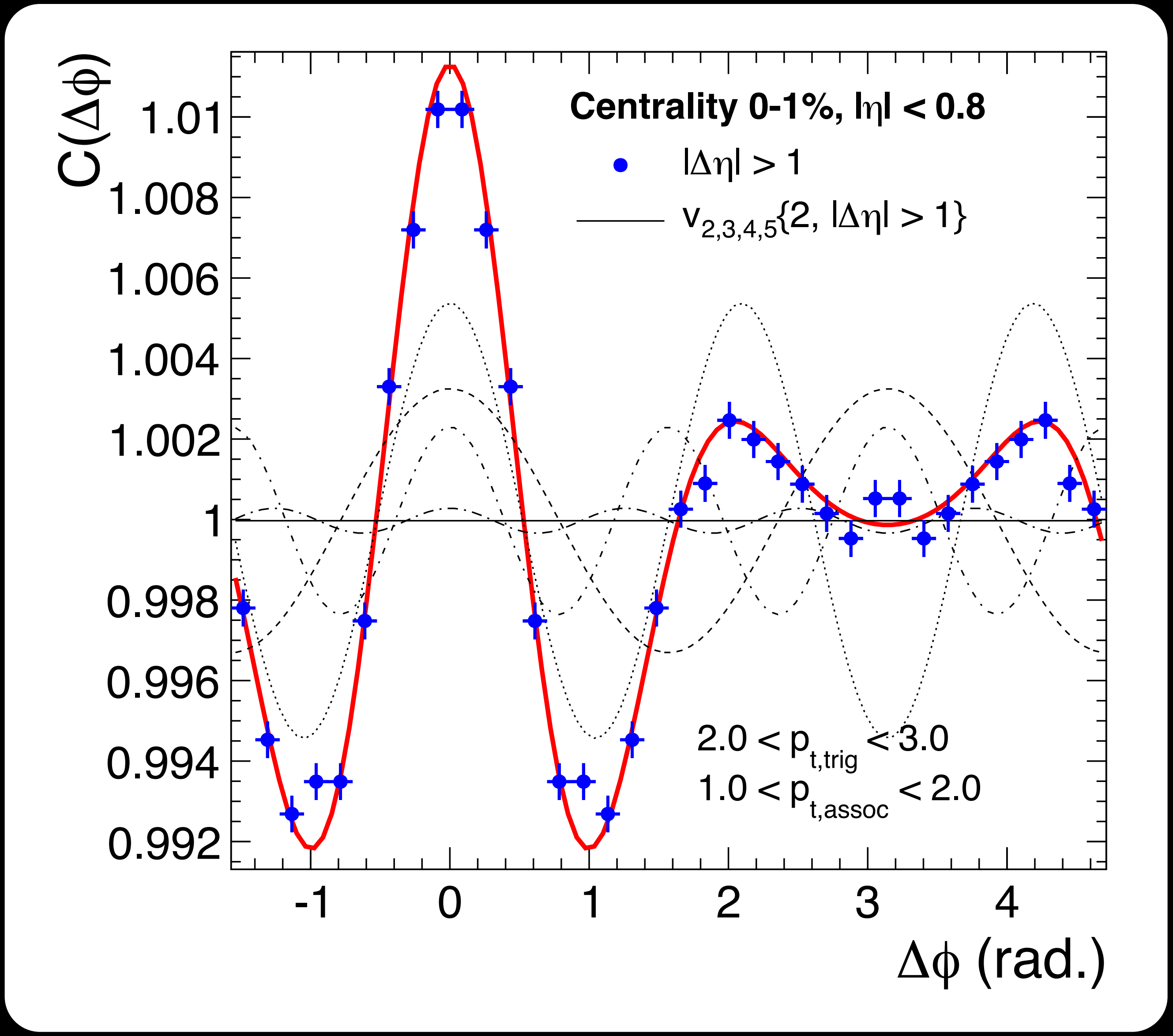
$$C(\Delta\phi\Delta\eta) \equiv \frac{N_{\mathrm{mixed}}}{N_{\mathrm{same}}} \frac{\mathrm{d}^2 N_{\mathrm{same}}/\mathrm{d}\Delta\phi \mathrm{d}\Delta\eta}{\mathrm{d}^2 N_{\mathrm{mixed}}/\mathrm{d}\Delta\phi \mathrm{d}\Delta\eta}$$

Contributions to the two-particle ΔΦ , Δη angular correlation come from anisotropic flow; v1, v2, v3, ..., Jets, resonances, HBT, etc



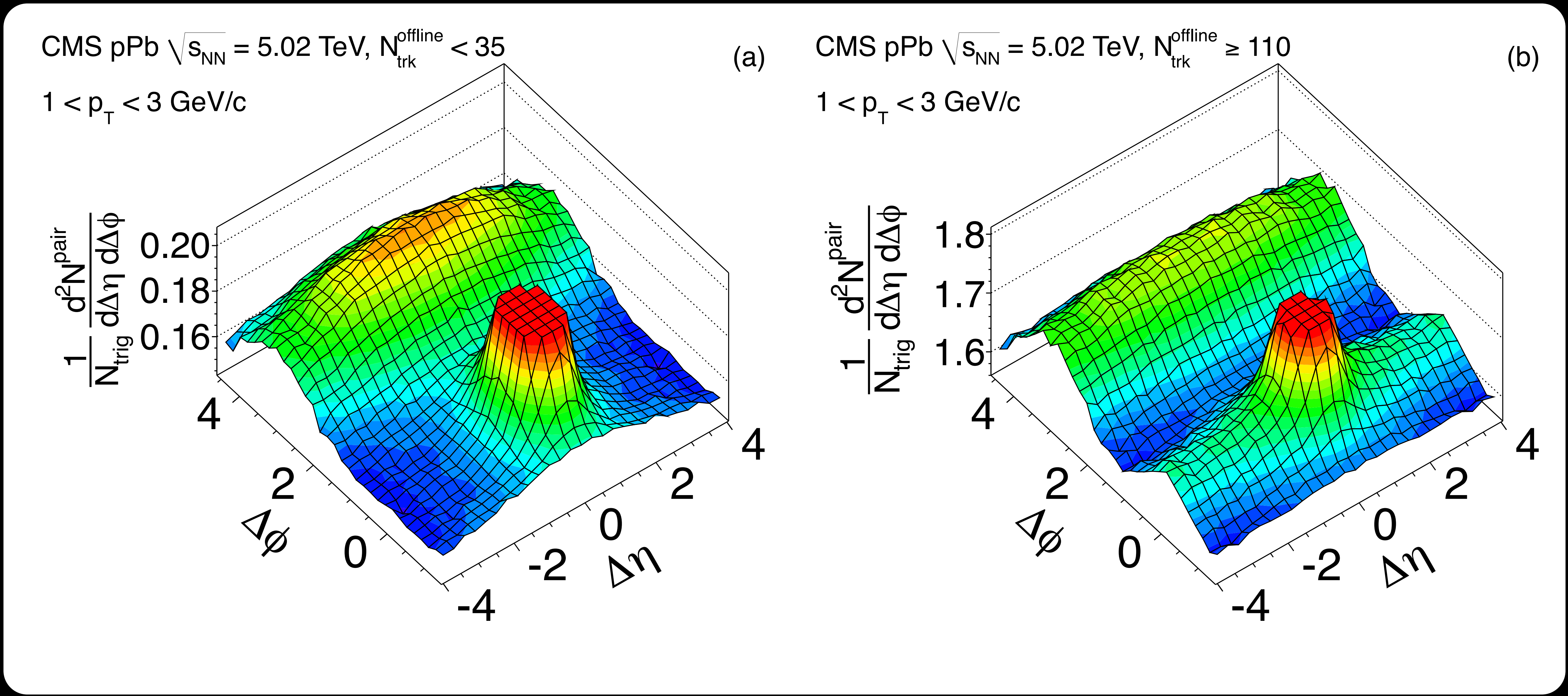
Angular Correlations





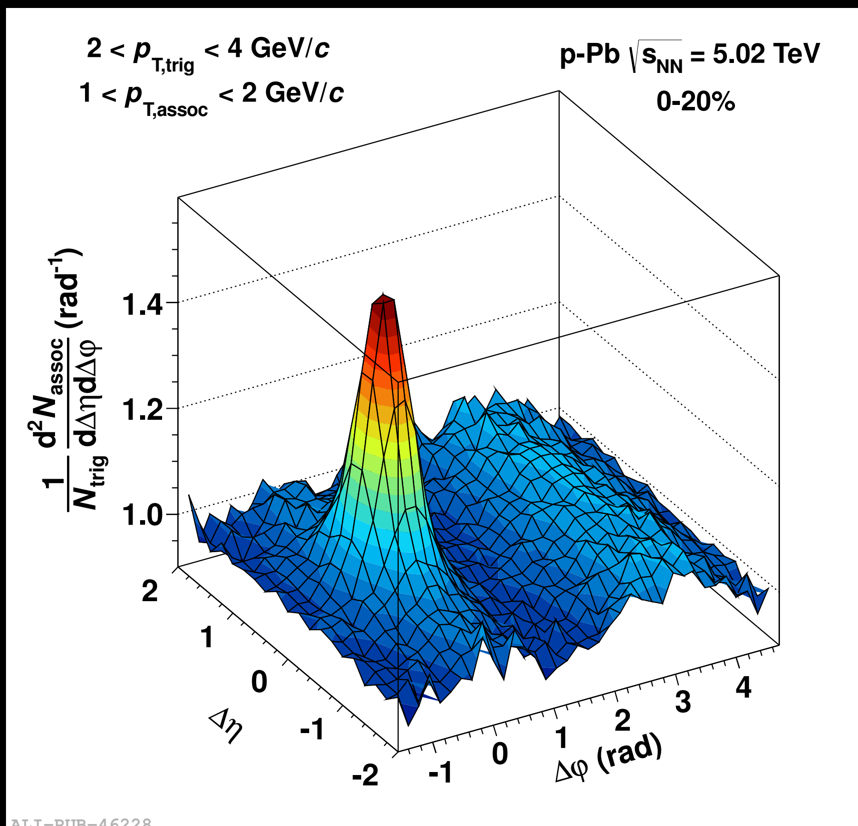
understood from initial eccentricities
followed by a hydrodynamical evolution

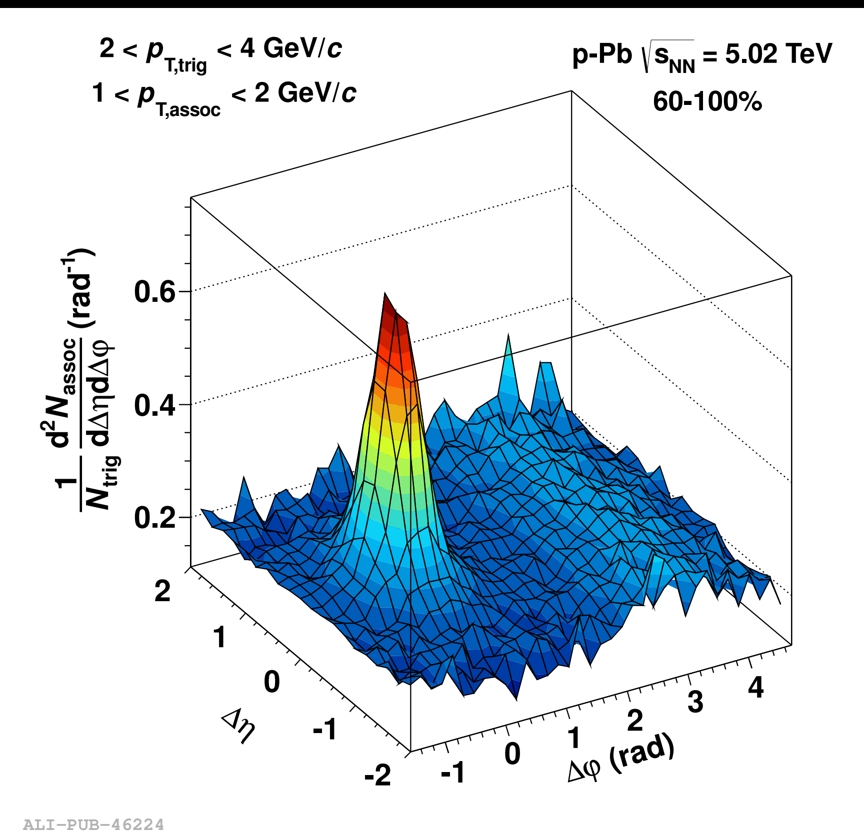
correlations pA

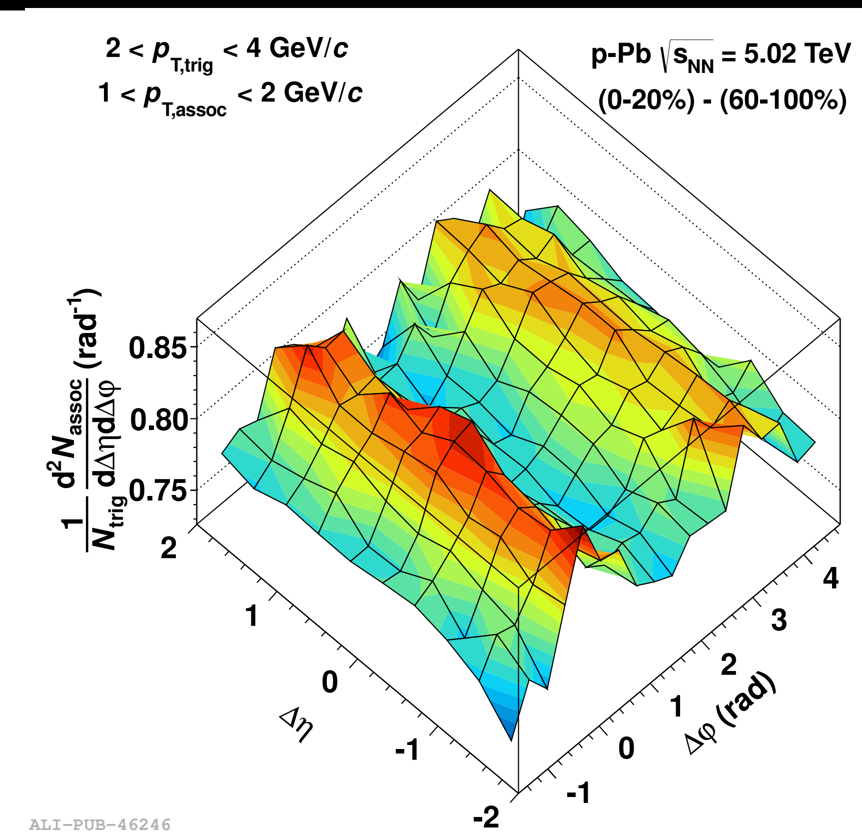


high-multiplicty pA collisions show in addtion to the jet correlations also a ridge structure
is this collective flow?

correlations



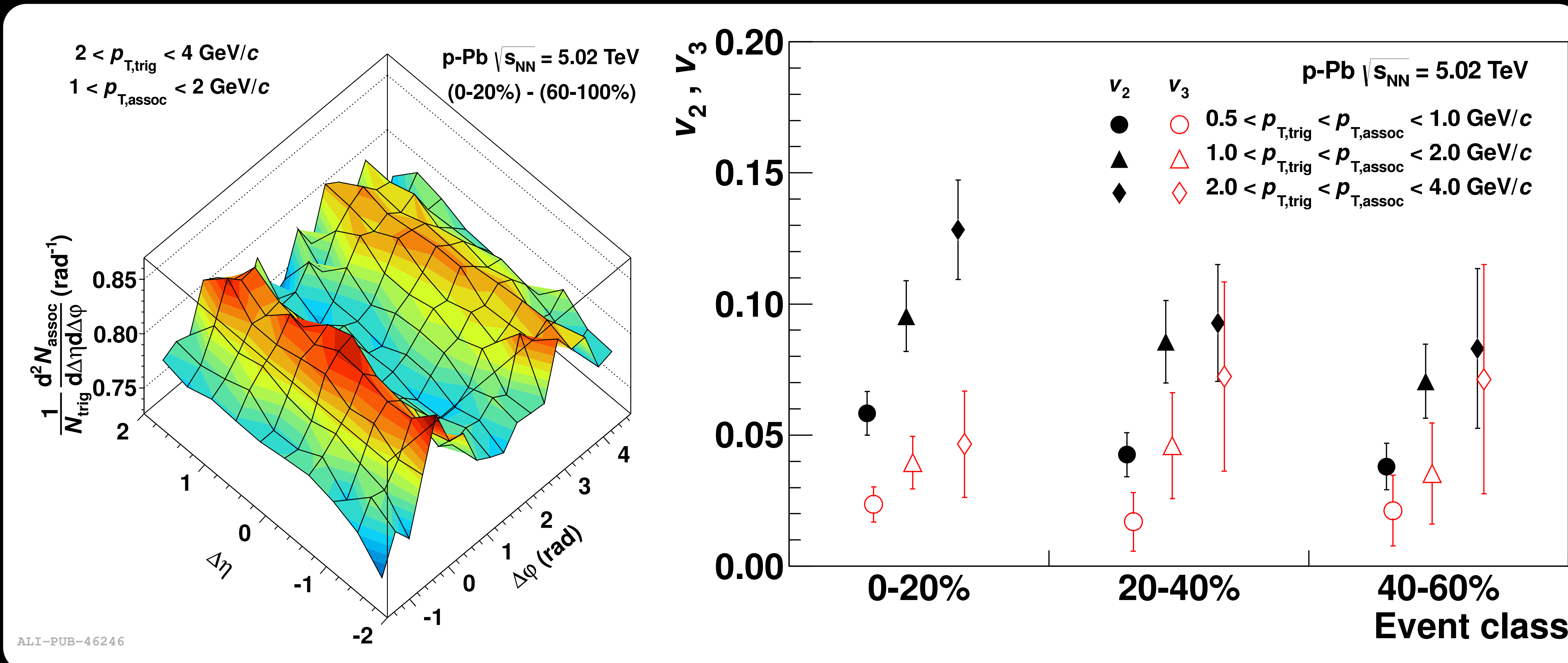




angular correlation in high-multiplicity events - angular correlation in low-multiplicity events = additional angular correlation

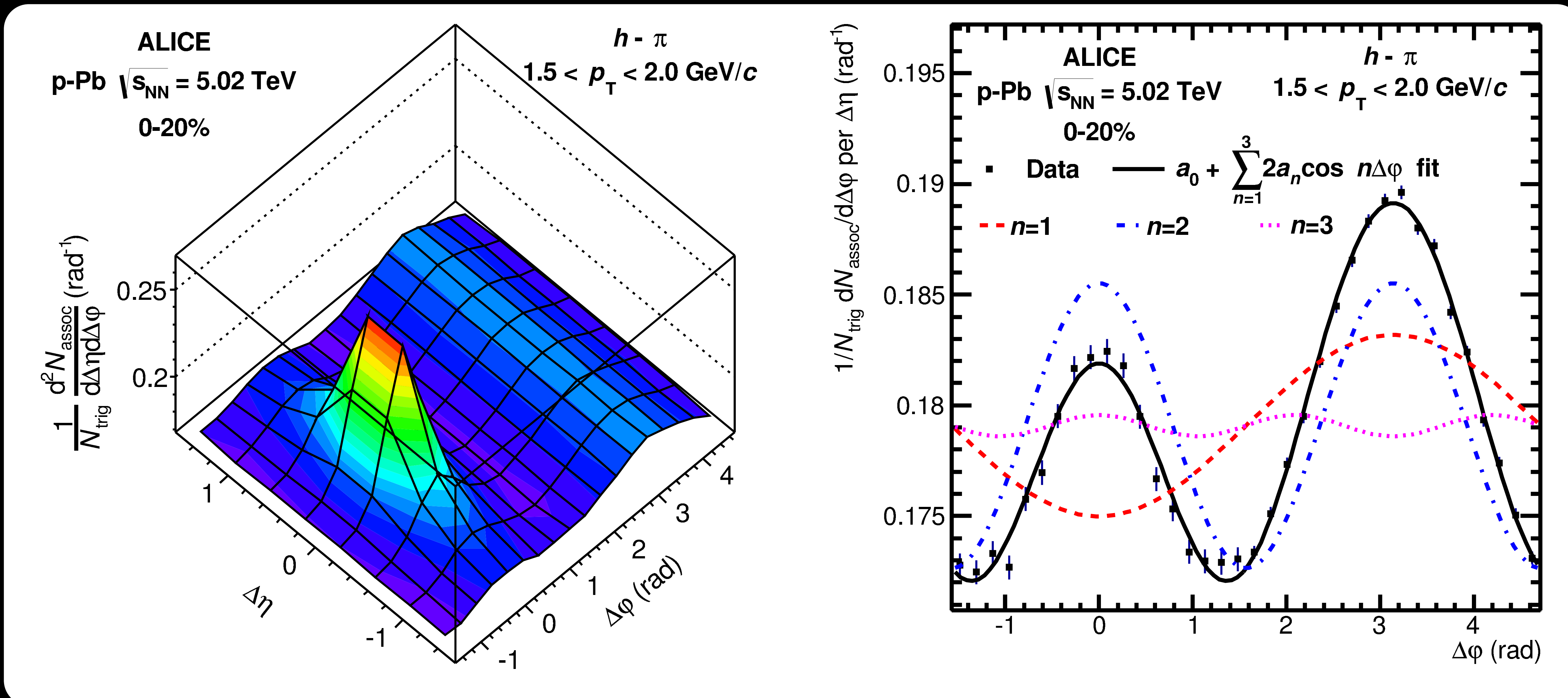
- suggestive picture, but should be interpreted with great care
- jets + additional structure, measured in an ensemble of events e.g. not clear structure in on event or different event types not clear various structures are connected
- additional structure reminds us of collective motion in heavy-ion collisions

correlations



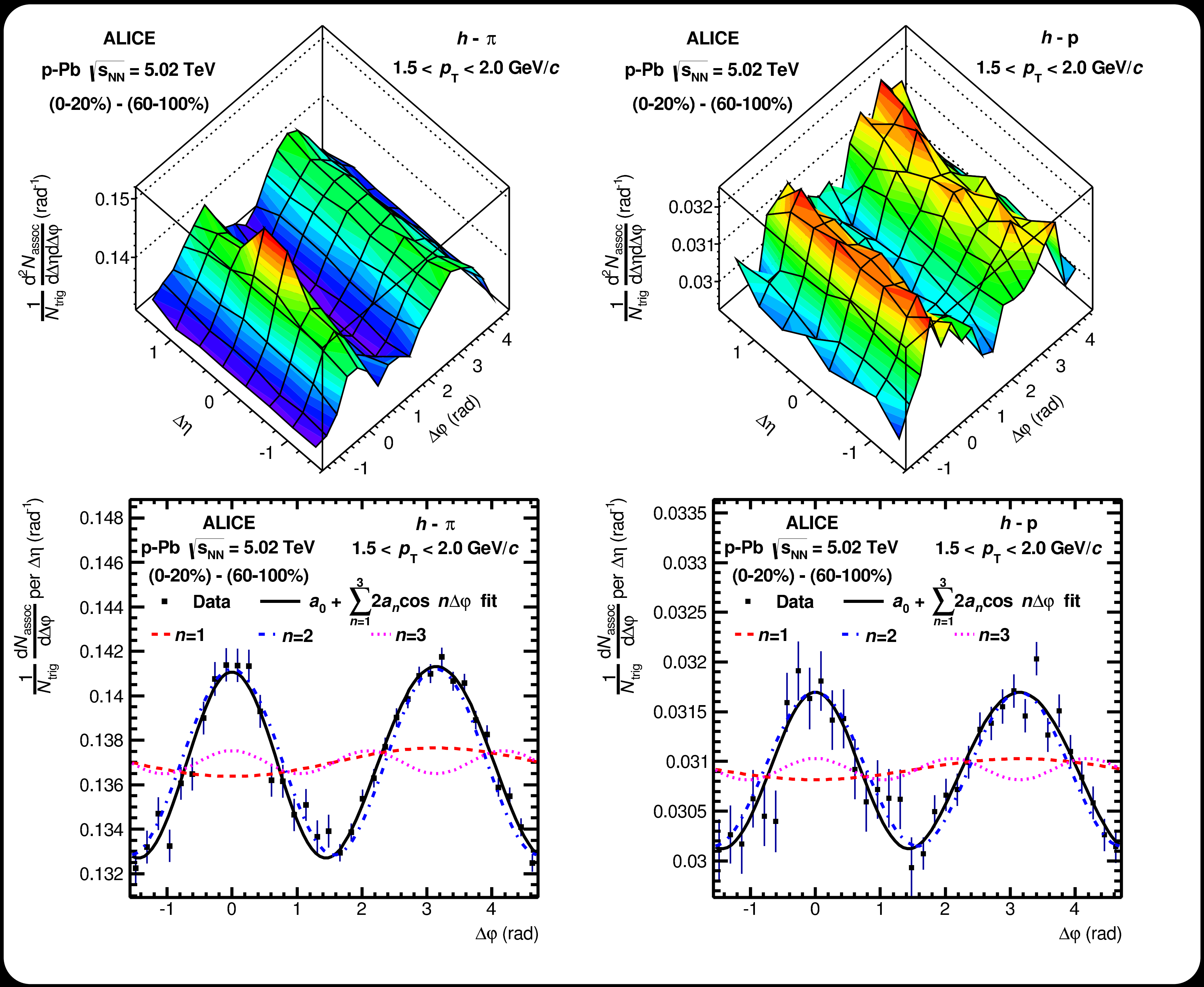
- large v_2 and v_3 components measured in pA collisions as well
- does it behave as collective flow?

Angular Correlations (pid)



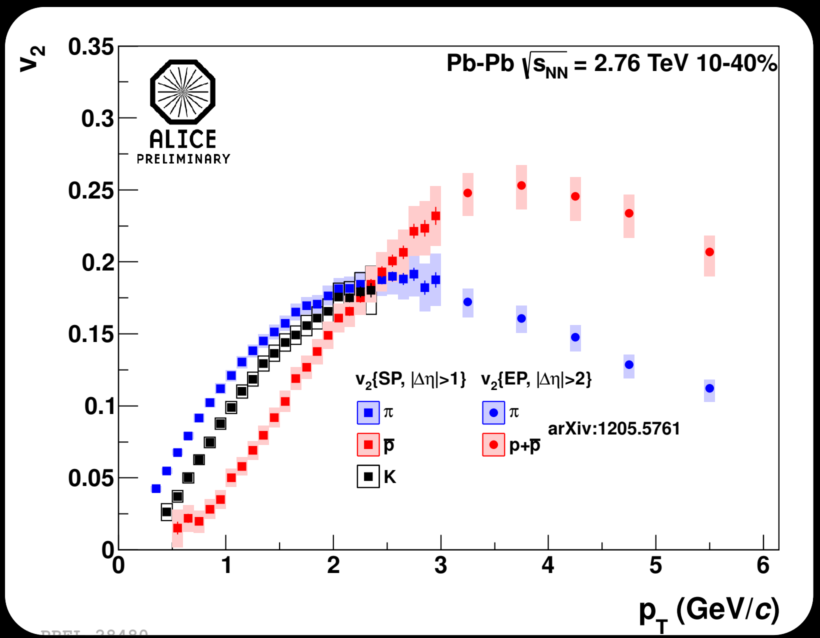
perform same analysis using particle identification

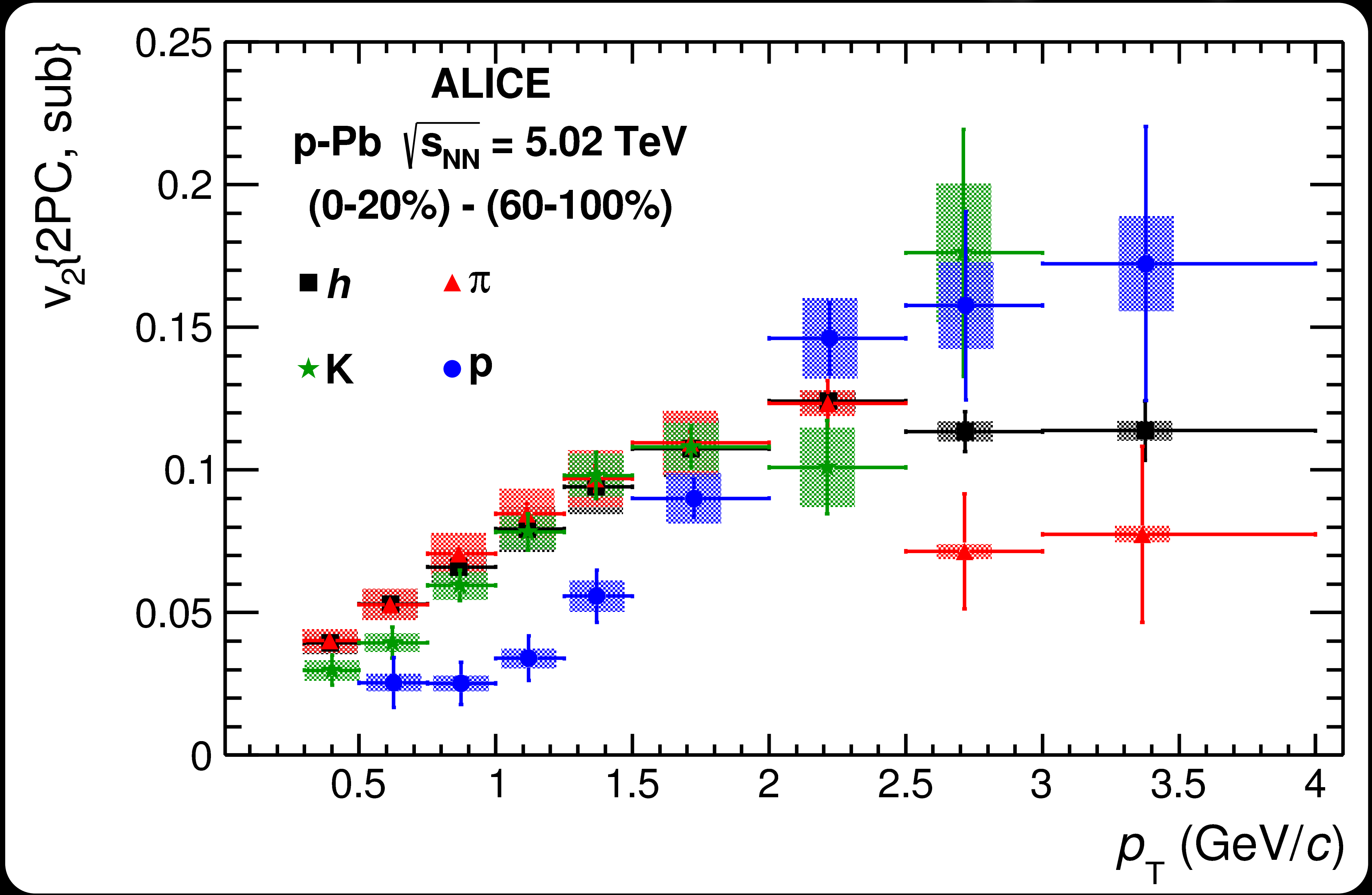
Angular Correlations (pid)



for pions and protons similar almost symmetric ridges are observed

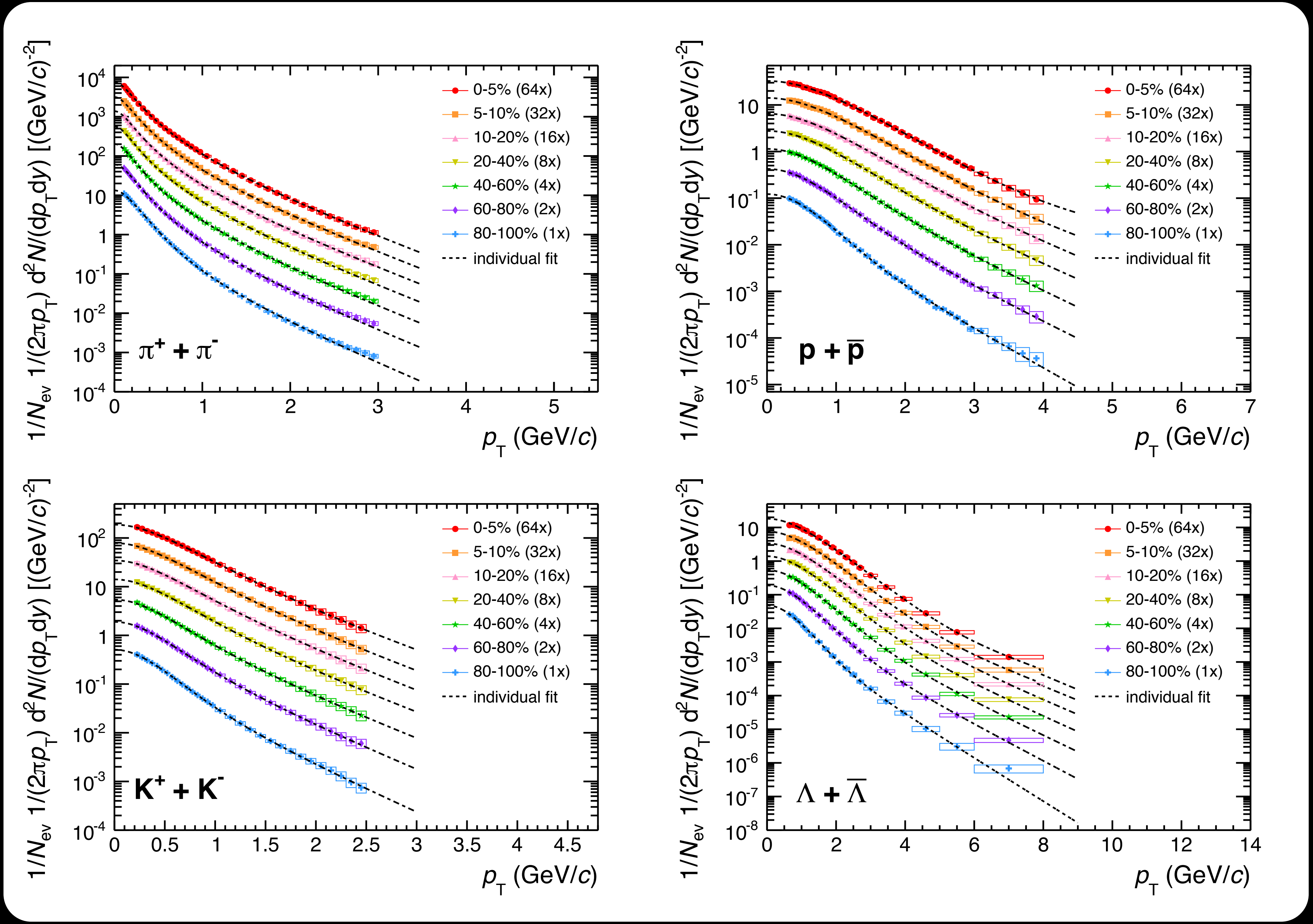
Angular Correlations (pid)





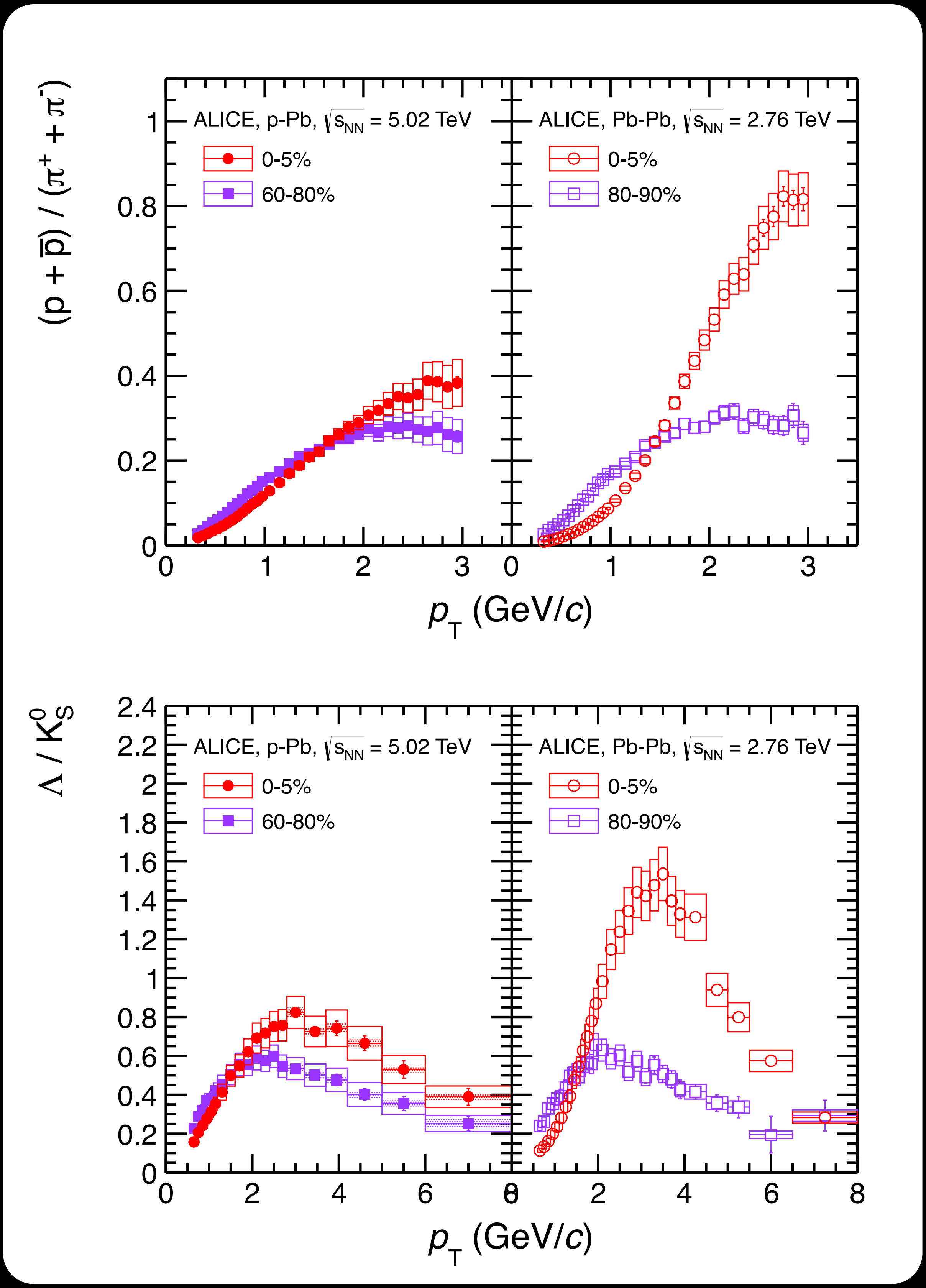
very similar trends as observed in AA collisions!

Identified Particle Spectra in pA



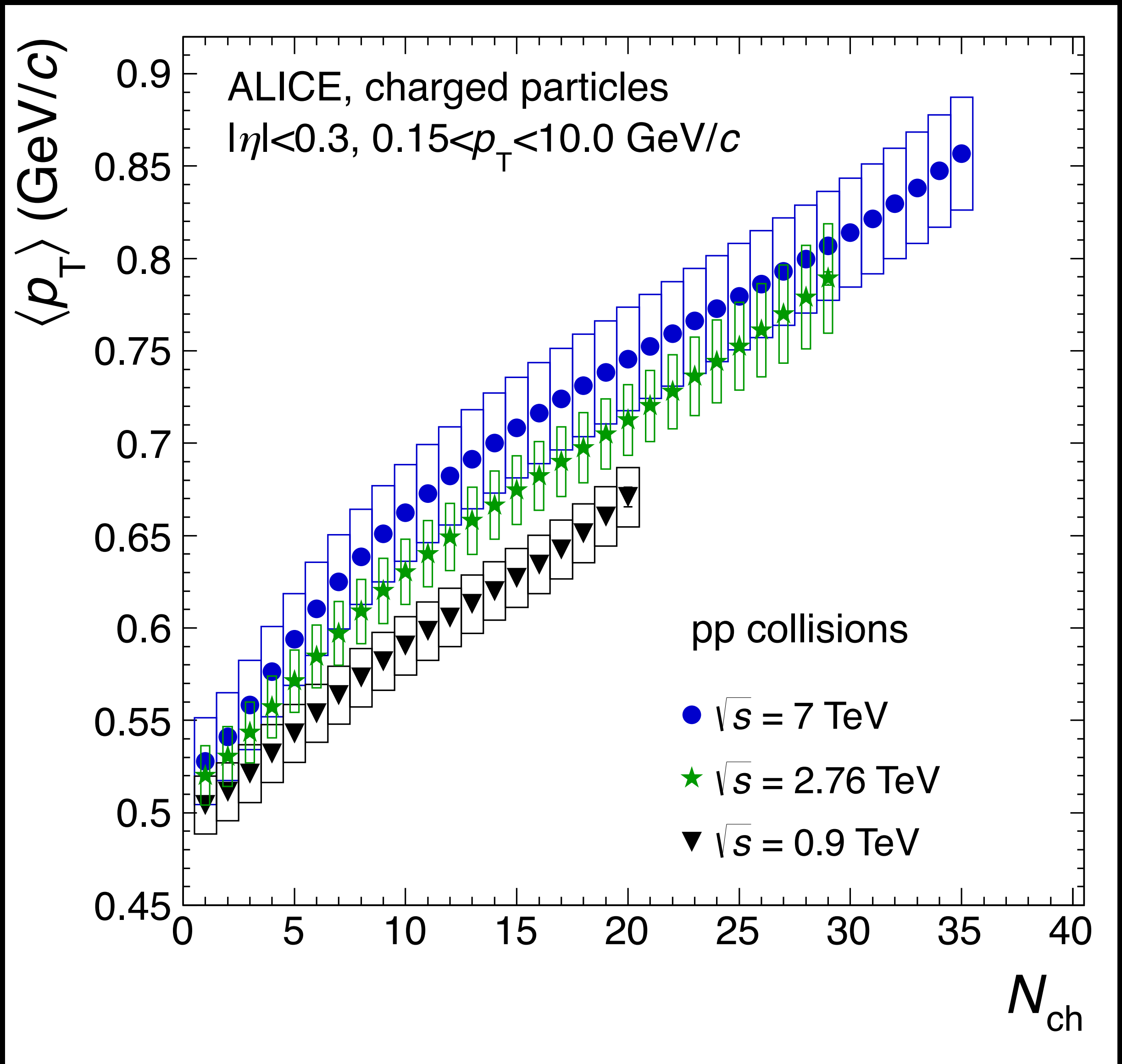
very similar trends as observed in AA collisions!

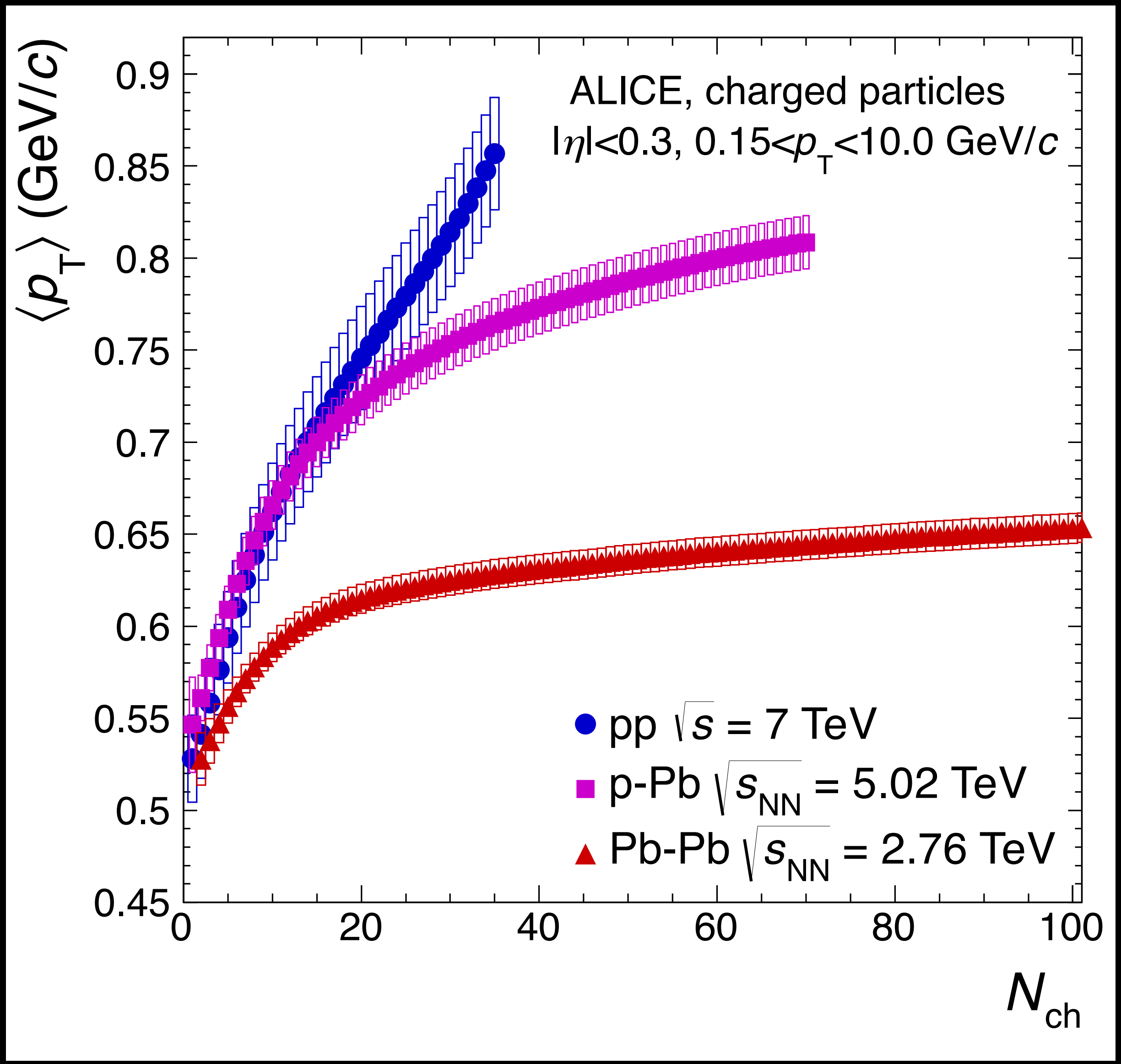
Identified Particle Spectra ratios



very similar trends as observed in AA collisions!

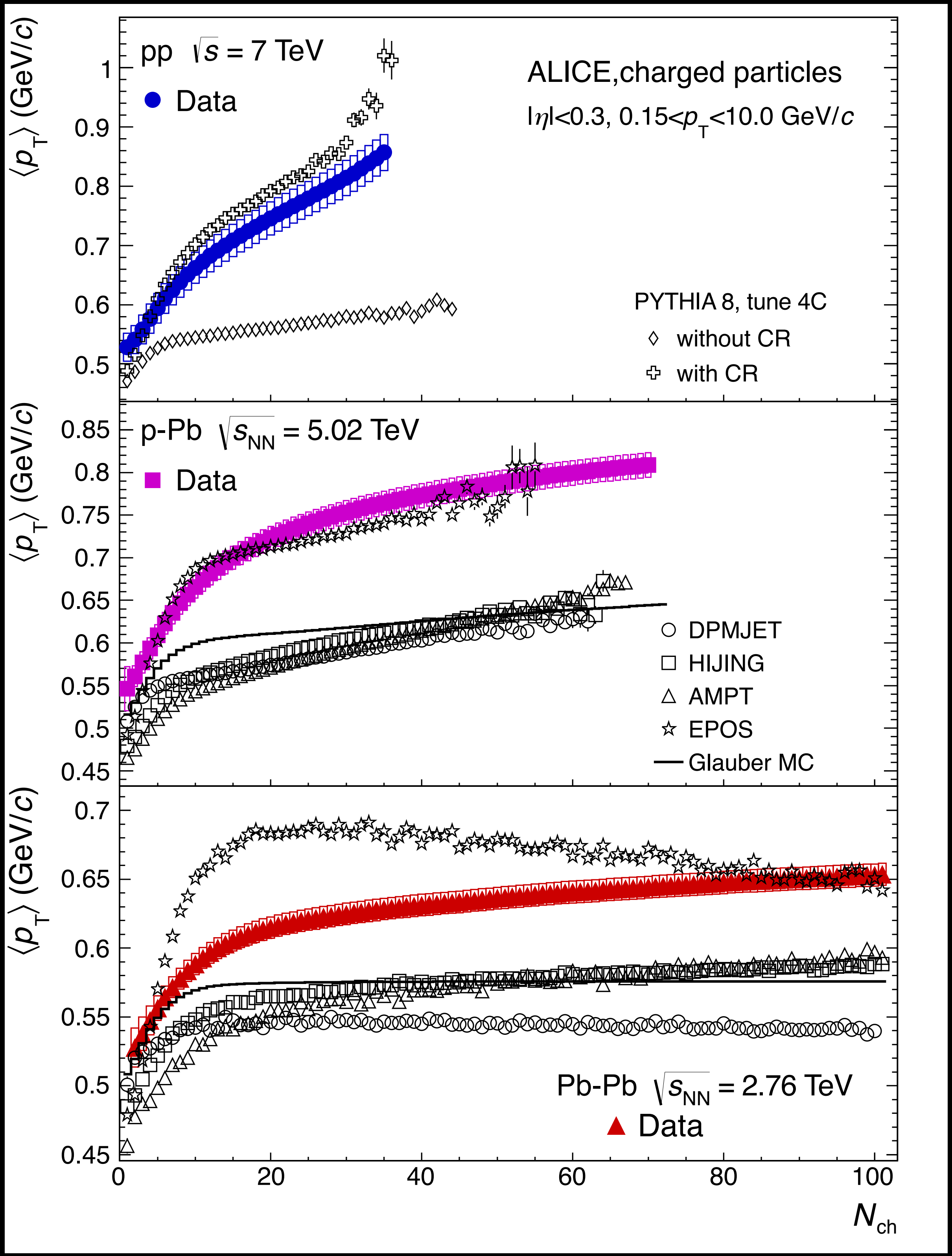
multiple interactions





proton proton collisions show a very strong increase of $<p_t>$ versus multiplicity (no strong energy dependence)
pA and AA look rather different though

multiple interactions



in proton proton collisions PYTHIA can describe the data rather well using color reconnections

in pA and AA rise much slower, not a incoherent sum of pp

pA combination of trends in pp and AA

using Glauber to check what happens for independent superposition of pp

Summary

- Produced in AA a system which for bulk observables behaves as a nearly ideal fluid
- soft probes (integrated yields, spectra, correlations) are relatively well understood in term of thermal boosted distributions
- clear connection to the geometry of the collision (and fluctuations in the initial geometry, strong constraints on initial state models)
- LHC energies allow us to do detailed studies of the hard probes
  - nevertheless much of our understanding of the produced system comes from soft probes
- in (rare) pp and pA collisions “surprising” similarities with AA observed
  - also final state effects? or initial state, e.g. CGC?
- pp and pA much more interesting than just “boring” reference for hard probes
  - geometrical picture in pp and pA theoretically rather unconstrained
- LHC ideal testing ground for understanding soft QCD not only in AA but also in pp and pA