Introduction to Lattice QCD
and
applications to nuclear and hadronic physics

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Abstract

This lecture aims to be an introductory course to the lattice techniques used to solve the Quantum Field Theory in a non perturbative regime, with special interest in the Quantum Chromodynamics (QCD). This activity started about 40 years ago when K. Wilson proposed a formulation of QCD in an euclidean discrete space-time lattice [1] as a statistical mechanics problem in a way that exactly preserves the main symmetry property of the theory: its gauge invariance. This branch of theoretical physics, denoted by Lattice QCD (LQCD), constitutes nowadays – even with some limitations – a unique tool to solve ab initio the theory of strong interactions.

The computation resources at that time were not large enough to obtain convincing results and one had to wait still for some years until Creutz [2] performed the first numerical simulation of a quantum field pure gauge theory on a – very modest 10_4 – lattice showing in particular the first numerical evidence of coexistence of confinement and asymptotic freedom. Since then, and despite the numeous ”infinite walls” to be jumped, the progress in the comprehension of the discrete theory as well as in algorithmic developments and in computer power, has been continuous and spectacular. We can safely say today that lattice QCD simulations have finally entered an era where a complete simulation of QCD with physical parameters is possible.

It is worth noticing that all the complexity of the subatomic world – at the scale of nuclear and hadronic physics – can be described by a very few number of parameters, essentially two. These are the bare quarks masses (m_u,m_d, . . .) plus an additional parameter _ depending on the unique bare coupling constant and controlling the lattice spacing.

These lectures have a duration of two hours and are adressed to PhD students and non specialists in the subject. Its ambition is to provide the audience with a road map of a standard lattice simulation and a key to decrypt the abondant literature in the subject.

The first part will be devoted to introduce the theoretical and numerical tools allowing to obtain the solution of a continuous theory formulated in a Minkowski space time on a discret euclidean 4-dimensional lattice. This challenging task has been possible using the Feynman path-integral formulation of the field theories, performing analytically the integral over the fermionic degrees of freedom and using the Monte-Carlo methods to evaluate the functional integral of the remaining bosonic fields in the configuration space.

We will first derive the Wilson action to account for the gluonic degrees of freedom (gauge fields) as well as some of the improvements commonly used in the present calculations. The basic concepts of plaquette, Wilson and Polyakov loops will be defined and their relation to the confining q¯q potential will be discussed.

The quark degrees of freedom will be then introduced. The ”naive” discretisation of the

fermions fields in the lattice produced a long series of failures in the theory – like spurious degrees of fredom and bad chiral properties – which were systematically considered as essential limitations of LQCD but that have been overcame step by step, giving raise to better and better discretisation schemes. The doublers problem and the Wilson solution, the Nielsen-Ninomiya no-go theorem, the Ginzparg-Wilson equation and the solution proposed by Neuberger will illustrate the difficulty to implement the chiral symmetry in a discrete world.

Finally we will describe the severals steps to be achieved in a practical realisation of a lattice QCD simulation. These are: (i) The generation of a statistical ensemble of gauge configurations, in particular when taking into account the $q\bar{q}$ loops in the gluon propagation (unquenched or dynamical calculations). This has been possible using different variants of the so-called Hybrid Monte Carlo algorithms, based on the Quantum Molecular Dynamics developed long ago in theoretical chemistry. (ii) The computation of quark propagator, which according to Wick theorem are the basic ingredient of any further calculation of hadronic observables. (iii) The evaluation of statistical erros which are assosciated to any computed observable in a Monte Carlo approach.

The second part will be focused on some practical applications of interest in nuclear and hadronic physics. We will select among:

- The obtention of ground state hadron spectrum. Starting with the simplest and very clean case of the meson masses, we will increase one degre in the complexity by considering baryons, and see the kind of problems one is faced to, due to the worsen of signal to noise ratio. We will show how the procedure can be ”easily” generalized up to computing the energies of the simplests nuclei – deuteron, ^{3}He and ^{4}He – from the very few LQCD parameters alone, as well as the limits of this progression. The results of the computation of excited states are much less satisfactories, even in the simplest cases, and will be presented.
- The computation of a hadron-hadron low energy parameters (scattering length and effective range). We will show how, despite the Wick rotation in the time component and some ”no-go” theorems, it is possible to obtain low energy scattering results following a method proposed by Luscher [3]. This allows to obtain reliable predictions in systems, like hyperon-hyperon, where the experimental results will always be missing.
- The application of the preceding results – plus some some ad hoc hypothesis – to obtain ”a” Nucleon-Nucleon potential from QCD. The reliability of this procedure will be discussed.
- The computation of baryon form factors. We will present the method allowing to obtain the simplest structure function of the baryon (form factors, PDF, first moments of GPD) and summarize the results obtained until now.
- We will show how one can introduce the temperature in the quark gluon system and obtain some interesting thermodynamical properties like its equation of state.

Despite presenting a minimal content for a rough understanding of LQCD, this program is maybe too ambitious to be covered in the allowed time and will be reoriented according to the interest of the audience.

An abondant literature is nowadays available. We start by recommending some excellent books, ordered by dates of publication:

- C. Rebbi: Lattice Gauge Theories andMonte Carlo Simulations; World Scientific:Singapor (1983)
- M. Creutz, Quarks, Gluons and Lattices, Cambridge Monographs onMathematical Physics

(1983)

- H. J. Rothe, Lattice gauge theories. An Introduction,World Scientific Publishing Co.Pte.Ltd. (1992)
- I. Montvay and G. Munster, Quantum Fields on a Lattice, Cambridge University Press, Cambridge (1994)
- J. Smit, Introduction to Quantum Fields on a Lattice, Cambridge University Press, Cambridge (2002)
- Th. DeGrand and C. De Tar, Lattice Methods for Quantum Chromodynamics (2007)
- C. Gattringer and C. B. Lang, Quantum Chromodynamics on the Lattice, Springer: Berlin Heidelberg, New York (2009)

This can be completed by the lecture given by O. P`ene in the Ecole Joliot Curie 2005 [4] and some review articles accessible in the web [5, 6, 7]. We can also recommend PhD mansucripts devoted to LQCD which are available in [8].

The reader interested in the state of the art can take benefit of consulting the proceedings of the last yearly Lattice conference [9, 10, 11] as well as the recent Summer School ”Modern Perspectives in Lattice QCD” at Les Houches in 2009 [12]. To put some salt in this bibliography and a limit to any hagiographic attempt it could be of some interest to read the historical notes writen by one of the founders of LQCD [13].

References

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http://pos.sissa.it/cgi-bin/reader/conf.cgi?confid=105
[10] Proceedings of Science, http://pos.sissa.it/cgi-bin/reader/conf.cgi?confid=139, https://latt11.llnl.gov/
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[12] Ecole de Physique des Houches, Modern Perspectives in Lattice QCD, Session XCIII, august 2009, Edit by L. Lellouch et al, Oxford University Press 2011
[13] K. G. Wilson, The Origins of lattice gauge theory, Nucl.Phys.Proc.Suppl. 140 (2005) 3–19.