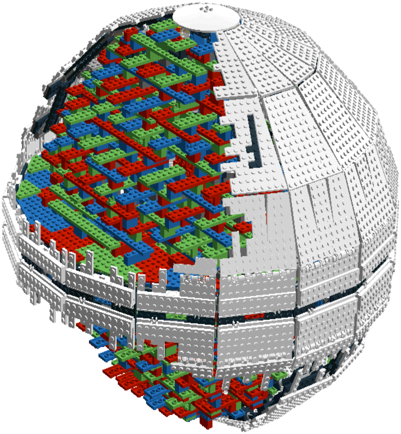
Nucleon Reverse Engineering

Structuring the nucleon with quarks and gluons

Hervé MOUTARDE

Irfu/SPhN, CEA-Saclay



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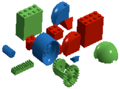

Motivation.

Study the nucleon structure to shed new light on non-perturbative QCD.

Reverse engineering

Reverse engineering is the process of discovering the technological principles of a device, object, or system through analysis of its structure , function, and operation.

Eilam and Chikofsky, *Reversing: secrets of reverse engineering*, John Wiley & Sons, 2007.

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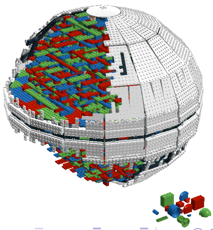
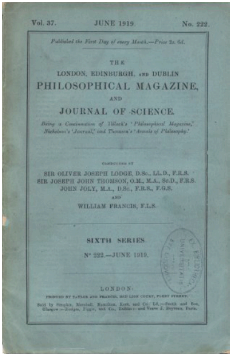
- Interplay between **perturbative** and **non-perturbative** QCD.
- Interacting colored degrees of freedom confined in **colorless** hadrons.
- **Emergence** of hadron characteristics from **fundamental building blocks**.



Proton.

Identified by Rutherford in 1919.

- *"We must conclude that the nitrogen atom is disintegrated under the intense forces developed in a close collision with a swift alpha particle, and that the hydrogen atom which is liberated formed a constituent part of the nitrogen nucleus."*



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Rutherford, Phil. Mag. **37**, 537 (1919)

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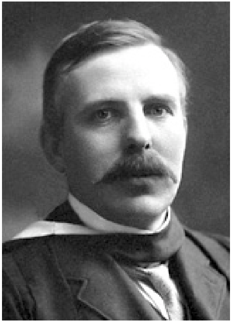

Rutherford, NP 1908

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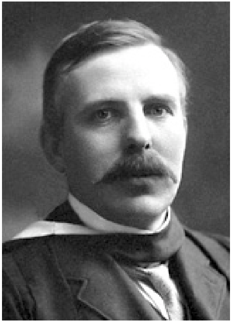

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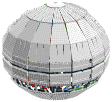


$M = 938.272046\ (21)$ MeV

Beringer *et al.* (Particle Data Group), Phys. Rev. **D86**, 010001 (2012)

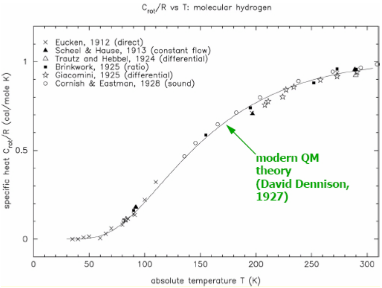
Proton.

The discovery of nuclear spin.

- Proton spin obtained from the measurement of the **specific heat** of molecular hydrogen!

$$E = E_{\mathrm{elec}} + E_{\mathrm{vib}} + E_{\mathrm{rot}} + E_{\mathrm{trans}}$$

Dennison, Proc. Roy. Soc. **A115**, 483 (1927)



- Two varieties of molecular hydrogen from wave function symmetry considerations.
- Slow transition between the two varieties.

Fig. from Gearhart, HQ2, 2008

Proton.

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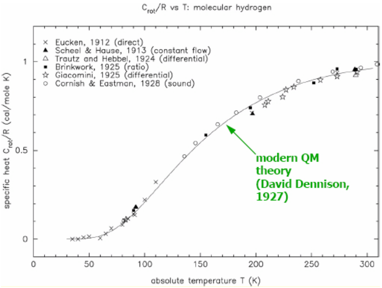
- Proton spin obtained from the measurement of the **specific heat** of molecular hydrogen!

$$E = \underbrace{E_{\mathrm{elec}}}_{\mathcal{O}(10000\ \mathrm{K})} + \underbrace{E_{\mathrm{vib}}}_{\mathcal{O}(1000\ \mathrm{K})} + \underbrace{E_{\mathrm{rot}}}_{\mathcal{O}(10\ \mathrm{K})} + \underbrace{E_{\mathrm{trans}}}_{\frac{3}{2}R}$$

Dennison, Proc. Roy. Soc. **A115**, 483 (1927)



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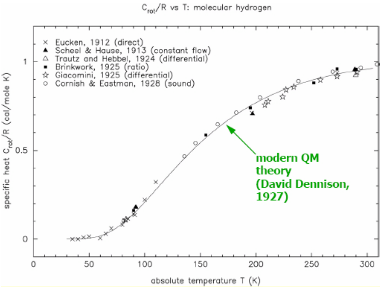

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CEA - Saclay

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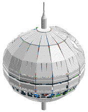

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$J = \frac{1}{2}$

Beringer *et al.* (Particle Data Group), Phys. Rev. **D86**, 010001 (2012)

See more.

Neutron.

Starting point of modern nuclear physics.

- 1932: **Discovery**.

  Chadwick, Nature **A129**, 312 (1932)

- 1934: **Mass measurement** (γ rays on deuteron yielding protons and neutrons).

  Chadwick and Goldhaber, Nature **A134**, 237 (1934)

- 1934: **Spin measurement** (from deuteron spin).

  Murphy and Johnston, Phys. Rev. **46**, 95 (1934)

Chadwick, NP 1935

Nucleon and isospin.

Starting point of modern nuclear physics.

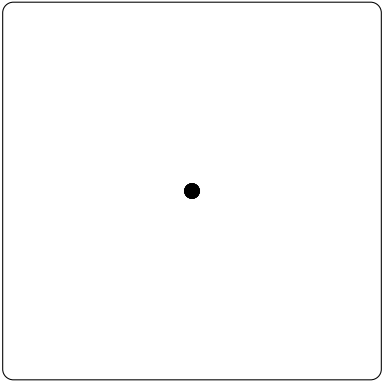
- 1932: **Nucleon** and **Isospin**.
  - Nuclei are made of protons and neutrons.
  - Strong interaction = exchange force described by isospin.

Heisenberg, Z. Phys. **77**, 1 (1932),
Z. Phys. **78**, 156 (1932),
Z. Phys. **80**, 587 (1932)

Heisenberg, NP 1932

Nucleon magnetic moment.

First evidence for a non trivial nucleon structure.



- Before 1933: elementary proton.
- Pauli : *"Don't you know the Dirac theory ? It is obvious from Dirac's equation that the moment must be $|e|/2M$."*

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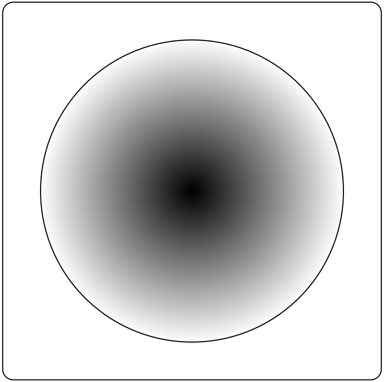

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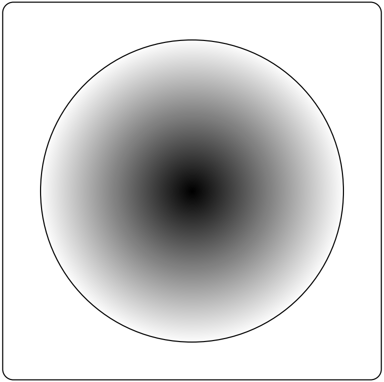


- From $\simeq$ 1933 to $\simeq$ 1960: composite nucleon with unkown structure.
- Proton mag. moment: 1933

  Estermann and Stern
  Z. Phys. **85**, 7 (1933)
- Neutron mag. moment: 1934

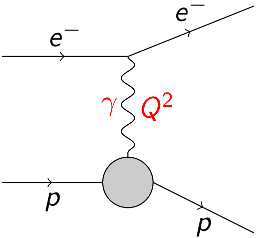
  Rabi *et al.*
  Phys. Rev. **46**, 163 (1934)

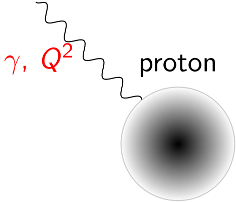
Stern, NP 1943

Rabi, NP 1944

Nucleon form factors and charge radius.

Elastic scattering.





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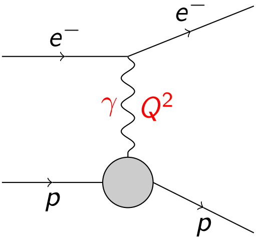

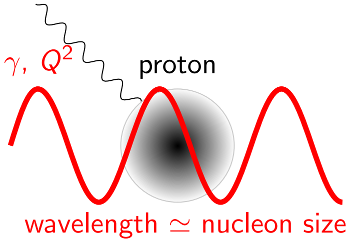

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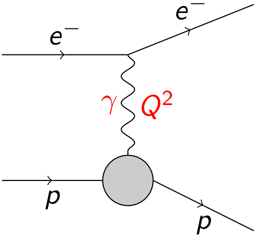
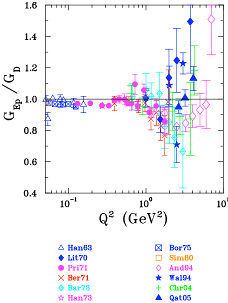
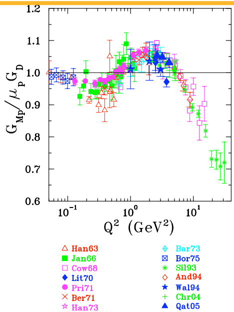






Perdrisat *et al.* , Prog. Part. Nucl. Phys. **59**, 694 (2007)

- **Elastic scattering** electron / proton described by means of **form factors** depending on Q^2 :

- G_E electric charge distribution,
- G_M magnetic moment distribution.

Hofstadter, NP 1961

Some properties of the nucleon.

Summary.

- **proton**
  - $M_p = 938.27203\ (8)$ GeV
  - $J = \frac{1}{2}$
  - $I = \frac{1}{2}$
  - $\tau \gtrsim 10^{32}$ years
  - $r_p = 0.875\ (7)$ fm
  - $\mu_p = 2.792847351\ (28)\ \mu_N$
  - $d < 0.54.10^{-23}$ e cm
- **neutron**
  - $M_n = 939.56536\ (8)$ GeV
  - $J = \frac{1}{2}$
  - $I = \frac{1}{2}$
  - Lifetime $=$ 885.7 (8) s
  - $< r_n^2 > =$ -0.1161 (22) fm^2
  - $\mu_p =$ -1.9130427 (5) μ_N
  - $d < 0.29.10^{-25}$ e cm

Numerous indications of a non-trivial structure.

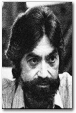
Quarks.

Existence in spite of unobserved free quarks.

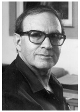
- Success of the **quark model** with symmetry $SU(3)$.

Gell-Mann, Ne'eman and Zweig (1964)

Zweig



Ne'eman



Gell-Mann, NP 1969

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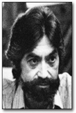

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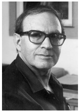

Gell-Mann, NP 1969

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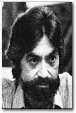
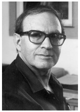
Zweig  Ne'eman  Gell-Mann, NP 1969 

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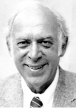
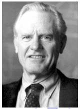
- Experimental evidence for ponctual charged spin-1/2 particles inside the nucleon.
  Friedman and Kendall, Ann. Rev. Nucl. Part. Sci. **A22**, 203 (1972)

Friedman, NP 1990  Kendall, NP 1990  Taylor, NP 1990 

Quarks.

Six quark flavors have been observed so far.

- Quarks and their **current masses**:
  - Light flavors:

saveur	u	d	s
masse (MeV)	[1.5, 3.3]	[3.5, 6.]	105^{+25}_{-35}
charge (e)	$+\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$

  - Heavy flavors:

saveur	c	b	t
masse (GeV)	$1.27^{+0.17}_{-0.11}$	$4.20^{+0.17}_{-0.07}$	$171.3^{+1.6}_{-1.6}$
charge (e)	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$

- Masses spread out over 5 orders of magnitude.
- In the following focus on light flavors to discuss nucleon structure.

Gluons.

Existence in spite of unobserved free gluons.

- Indirect observation in a **3-jet event**.

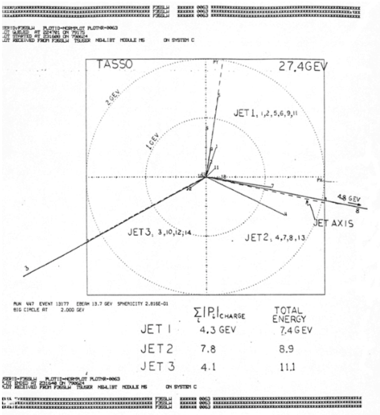
  Wiik, *Neutrino '79*, Bergen

- Spin-1 confirmed by analysis of 3-jet events.

  Brandelik *et al.* , Phys. Lett. **B86**, 243 (1979)

- Gluon mass $\lesssim$ a few MeVs.

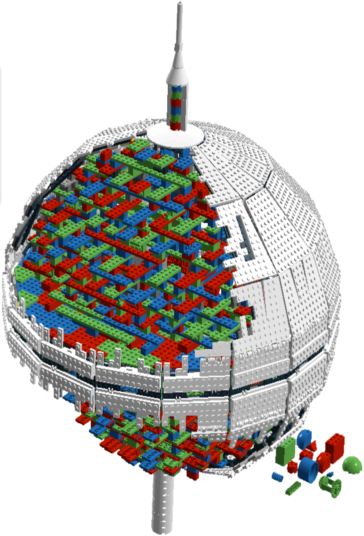
  Yndurain, Phys. Lett. **B345**, 524 (1995)



Reverse engineering.

The nucleon as a quantum relativistic system of confined particles.

How can we recover the well-known characteristics of the nucleon form the properties of its building blocks?



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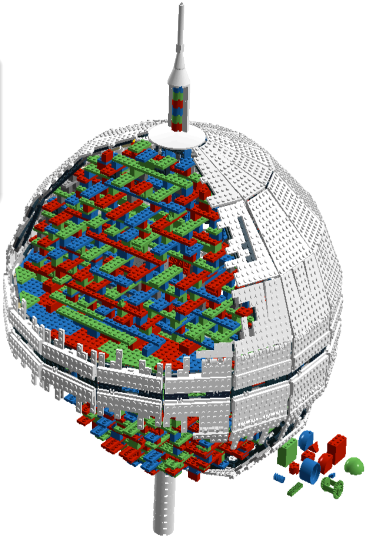

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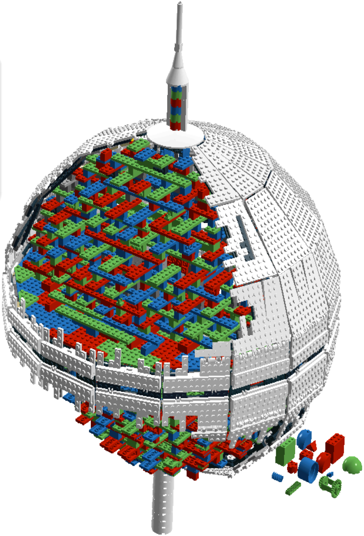
Spin?



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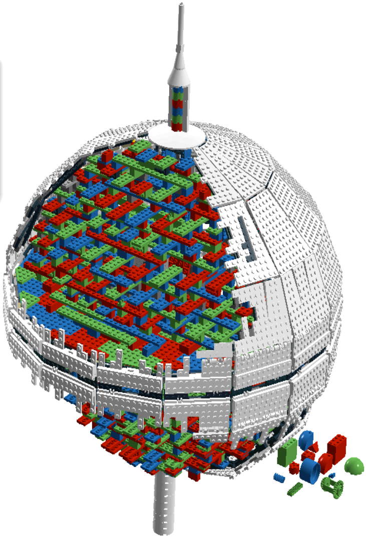

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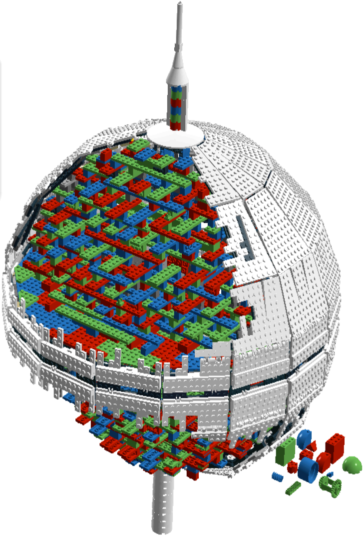

Reverse engineering.

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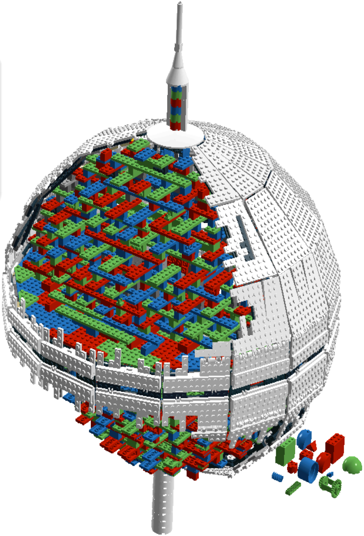
First need to give a well defined (**quantitative**) formulation of the problem!



General outline.

1. **Anatomy of the nucleon**
   General physical context. What would be desirable? What is achievable?

   Go to Part I.

2. **Hard probes and partonic content**
   Technical. All the words you have already heard but which were maybe left undefined.

   Go to Part II.

3. **3D imaging and beyond**
   Phenomenological status of 3D imaging in the most advanced case.

   Go to Part III.

Part. I
Anatomy of the nucleon

Tuesday 1 Oct. 2013
8H30 - 10H

General physical context. What would be desirable? What is achievable?

Phase space distribution function.

Microscopic description of an assembly of particles.

- Massive particles (mass m, particle density N).
- Orders of magnitude:

$$\begin{array}{c}\text{de Broglie}\\ \text{wavelength } \lambda\end{array} \ll \begin{array}{c}\text{Average}\\ \text{distance } d_0\end{array} \ll \begin{array}{c}\text{Typical length}\\ \text{scales } L\end{array}$$

  e.g. hydrogen in stellar atmosphere at $T \simeq 10^4\ \mathrm{K}$:
  - $N \simeq 10^{16}\ \mathrm{cm}^{-3}$,
  - $d_0 = (4\pi/3N)^{-1/3} \simeq 3 \times 10^{-6}\ \mathrm{cm}$,
  - $L \simeq 100\ \mathrm{km}$,
  - $\lambda = h/\sqrt{3mk_B T} \simeq 2 \times 10^{-9}\ \mathrm{cm}$.
- Approx.: **continuous distribution of classical particles**.

Distribution function $f(\vec{r}, \vec{v}, t)$

$f(\vec{r}, \vec{v}, t)\, \mathrm{d}^3\vec{r}\mathrm{d}^3\vec{v}$ is the average number of particles contained, at time t, in a volume element $\mathrm{d}^3\vec{r}$ about $\vec{r}$ and velocity-space element $\mathrm{d}^3\vec{v}$ about $\vec{v}$.

Phase space distribution function.

Macroscopic properties of an assembly of particles.

- **Macroscopic properties** are computed from the distribution function, *e.g.* :
  - Particle density:

$$N(\vec{r}, t) = \int \mathrm{d}^3\vec{v}\, f(\vec{r}, \vec{v}, t)$$

  - Mass density ρ (atomic weight A):

$$\rho(\vec{r}, t) = A m_H N(\vec{r}, t)$$

  - Average velocity $\langle \vec{v} \rangle$:

$$\langle \vec{v} \rangle (\vec{r}, t) = \int \mathrm{d}^3\vec{v}\, \vec{v} f(\vec{r}, \vec{v}, t)$$

- f is a **1-particle distribution function**: the probability of finding a particle at a given point in phase space us independent of the coordinates of all other particles.
- By construction $f(\vec{r}, \vec{v}, t)$ is **positive**.

Wigner quasiprobability distribution.
Including quantum effects.

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- Must modify definition of phase space distribution $f(\vec{r}, \vec{v}, t)$ to satisfy **Heisenberg uncertainty principle**.

Wigner quasiprobability distribution.

Including quantum effects.

- Must modify definition of phase space distribution $f(\vec{r}, \vec{v}, t)$ to satisfy **Heisenberg uncertainty principle**.
- Change **kinetic momentum** $\vec{p} = m\vec{v}$ to **canonical momentum** $\vec{p} = \partial\mathcal{L}/\partial\vec{v}$.

Wigner distribution $\mathcal{W}$ (pure state)

Let ψ be the **wavefunction** of the considered system. The **Wigner distribution** $\mathcal{W}(\vec{r}, \vec{p})$ is:

$$\mathcal{W}(\vec{r}, \vec{p}, t) = \int \frac{\mathrm{d}^3\vec{s}}{(2\pi)^3}\, \psi^* \left(\vec{r} - \frac{1}{2}\vec{s}, t\right) \psi \left(\vec{r} + \frac{1}{2}\vec{s}, t\right) e^{i\vec{p}\cdot\vec{s}}$$

Wigner, Phys. Rev. **40**, 749 (1932)

Wigner quasiprobability distribution.

Including quantum effects.

- Must modify definition of phase space distribution $f(\vec{r}, \vec{v}, t)$ to satisfy **Heisenberg uncertainty principle**.
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Wigner, Phys. Rev. **40**, 749 (1932)

- By construction $\mathcal{W}(\vec{r}, \vec{p}, t)$ is **real** but **not necessarily positive**.

Wigner quasiprobability distribution (pure state).
Properties.

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- Recover $\vec{r}$ and $\vec{p}$ **probability densities**:

$$\int \mathrm{d}^3\vec{p}\,\mathcal{W}(\vec{r},\vec{p}) \quad = \quad \int \frac{\mathrm{d}^3\,\vec{s}}{(2\pi)^3}\psi^*\left(\vec{r}-\frac{\vec{s}}{2}\right)\psi\left(\vec{r}+\frac{\vec{s}}{2}\right)\int \mathrm{d}^3\vec{p}e^{i\vec{p}\cdot\vec{s}}$$

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$$= \int \mathrm{d}^3\vec{s}\psi^*\left(\vec{r}-\frac{\vec{s}}{2}\right)\psi\left(\vec{r}+\frac{\vec{s}}{2}\right)\delta^{(3)}(\vec{s})$$

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Wigner quasiprobability distribution (pure state).

Properties.

- Recover $\vec{r}$ and $\vec{p}$ **probability densities**:

$$\begin{aligned}
\int \mathrm{d}^3\vec{p}\,\mathcal{W}(\vec{r},\vec{p}) &= \int \frac{\mathrm{d}^3\,\vec{s}}{(2\pi)^3}\psi^*\left(\vec{r}-\frac{\vec{s}}{2}\right)\psi\left(\vec{r}+\frac{\vec{s}}{2}\right)\int \mathrm{d}^3\vec{p}e^{i\vec{p}\cdot\vec{s}} \\
&= \int \mathrm{d}^3\vec{s}\psi^*\left(\vec{r}-\frac{\vec{s}}{2}\right)\psi\left(\vec{r}+\frac{\vec{s}}{2}\right)\delta^{(3)}(\vec{s}) \\
&= \psi^*(\vec{r})\psi(\vec{r})
\end{aligned}$$

Wigner quasiprobability distribution (pure state).

Properties.

- Recover $\vec{r}$ and $\vec{p}$ **probability densities**:

$$\int \mathrm{d}^3\vec{p}\,\mathcal{W}(\vec{r},\vec{p}) \quad = \quad |\psi(\vec{r})|^2$$

Wigner quasiprobability distribution (pure state).
Properties.

- Recover $\vec{r}$ and $\vec{p}$ **probability densities**:

$$\int d^3\vec{p}\,\mathcal{W}(\vec{r},\vec{p}) = |\psi(\vec{r})|^2$$

$$\int d^3\vec{r}\,\mathcal{W}(\vec{r},\vec{p}) = \frac{1}{(2\pi)^3}|\psi(\vec{p})|^2$$

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- For an observable A associated to a function $a(\vec{r},\vec{p})$ of **phase-space coordinates**:

$$\langle A \rangle = \int \mathrm{d}^3\vec{r}\mathrm{d}^3\vec{p}\, a(\vec{r},\vec{p})\mathcal{W}(\vec{r},\vec{p})$$

Moyal, Proc. Cam. Phil. Soc. **45**, 99 (1949)

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Moyal, Proc. Cam. Phil. Soc. **45**, 99 (1949)

- **Quantum mechanical generalization** of distribution function $f(\vec{r},\vec{p})$.

Wigner quasiprobability distribution (pure state).

Properties.

- Recover $\vec{r}$ and $\vec{p}$ **probability densities**:

$$\int \mathrm{d}^3\vec{p}\,\mathcal{W}(\vec{r},\vec{p}) = |\psi(\vec{r})|^2$$

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Moyal, Proc. Cam. Phil. Soc. **45**, 99 (1949)

- **Quantum mechanical generalization** of distribution function $f(\vec{r},\vec{p})$.
- Need to consider **mixed states** *e.g.* to take spin into account.

Density matrices.

Putting mixed states in Wigner distributions.

- Consider a system $|\psi\rangle$ which is in state $|k\rangle$ with probability p_k ($1 \leq k \leq K$ and $\sum_1^K p_k = 1$).
- Choose a complete set of (orthonormal) states $|u_n\rangle$:

$$|k\rangle = \sum_n c_n^{(k)} |u_n\rangle \quad \text{for } 1 \leq k \leq n$$

- Compute average value of observable A in state $|k\rangle$:

$$\langle k |A| k\rangle = \sum_{n,m} c_n^{(k)*} c_m^{(k)} A_{nm} \quad \text{with } A_{nm} = \langle u_n |A| u_m\rangle$$

- **Define** operator ρ by matrix element:

$$\rho_{nm} = \langle u_n |\rho| u_m\rangle = \sum_{k=1}^{K} p_k \, c_n^{(k)*} c_m^{(k)}$$

- By construction:

$$\langle \psi |A| \psi\rangle = \sum_{n,m} \rho_{nm} A_{nm} = \mathrm{Tr}\, \rho A$$

Density matrices.

Formal definition.

Density operator ρ

Every state can be represented by an **density operator** ρ with the following properties:

1. ρ is hermitian.
2. $\mathrm{Tr}\,\rho = 1$.
3. ρ is positive:
$$\langle \psi \,|\rho|\, \psi \rangle \geq 0 \quad \text{for all states } \psi$$
4. The state is pure if and only if $\rho^2 = \rho$.

Density matrices.

Formal definition.

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Every state can be represented by an **density operator** ρ with the following properties:

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   The average value of the identity is 1.

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   The average value of $B = AA^\dagger$ is positive.

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Formal definition.

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   The average value of $B = AA^\dagger$ is positive.

4. The state is pure if and only if $\rho^2 = \rho$.

   ρ is a projection operator.

Wigner quasiprobability distribution.

Nonrelativistic quantum mechanical definition (mixed state).

- Reminder: definition for a pure state.

$$\mathcal{W}_{\mathrm{pure}}(\vec{r}, \vec{p}, t) = \int \frac{\mathrm{d}^3\vec{s}}{(2\pi)^3} \psi^* \left(\vec{r} - \frac{1}{2}\vec{s}, t \right) \psi \left(\vec{r} + \frac{1}{2}\vec{s}, t \right) e^{i\vec{p}\cdot\vec{s}}$$

Wigner distribution $\mathcal{W}$ (mixed state)

Let ρ be the **density operator** of the considered system. The **Wigner distribution** $\mathcal{W}(\vec{r}, \vec{p})$ is:

$$\mathcal{W}(\vec{r}, \vec{p}) = \int \frac{\mathrm{d}^3\vec{s}}{(2\pi)^3} \left\langle \vec{r} - \frac{1}{2}\vec{s} \middle| \rho \middle| \vec{r} + \frac{1}{2}\vec{s} \right\rangle e^{i\vec{p}\cdot\vec{s}}$$

- Need extensions to describe:
  - Quark fields.
  - Color gauge invariance.
  - Lorentz invariance.

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Wigner quasiprobability distribution.

Nonrelativistic Wigner distribution for quarks in QCD (field theory).

- (Trial) Wigner distribution operator $\hat{\mathcal{W}}$:

$$\hat{\mathcal{W}}_\Gamma\Big((t,\vec{r}),p\Big) = \int \mathrm{d}^4 s\, \bar{\psi}\left(\vec{r}-\frac{1}{2}\vec{s}\right)\Gamma\psi\left(\vec{r}+\frac{1}{2}\vec{s}\right)e^{ip\cdot s}$$

where $\Gamma = 1$, γ_μ, $\gamma_\mu\gamma_5$ or γ_5.

- Choose a **constant 4-vector** n^μ and a non-singular gauge (gauge potentials vanish at spacetime infinity).
- Connect quark fields at $r \pm s/2$ with a **Wilson line** $\mathcal{L}$ *via* intermediate points at $n\infty$ to **ensure gauge invariance**.
- Sandwich between nucleon states with relativistic normalization:

$$\mathcal{W}_\Gamma\Big((t,\vec{r}),p\Big) = \frac{1}{2M}\int\frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\left\langle N,\frac{\vec{q}}{2}\right|\hat{\mathcal{W}}_\Gamma\Big((t,\vec{r}),p\Big)\left|N,-\frac{\vec{q}}{2}\right\rangle$$

Ji, Phys. Rev. Lett. **91**, 062001 (2003)
Ji *et al.* , Phys. Rev. **D69**, 074014 (2004)

Wigner quasiprobability distribution.

Nonrelativistic Wigner distribution for quarks in QCD (field theory).

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- Relativistic normalization of 1-particle states:

$$\langle N, p \,|\, N, k \rangle = (2\pi)^3 2E_{\vec{p}} \delta^{(3)}(\vec{p} - \vec{k})$$

- Use translation operator $\mathbb{P}$: $\phi(x + a) = e^{+i\mathbb{P}\cdot a}\phi(x)e^{-i\mathbb{P}\cdot a}$

$$\mathcal{W}_\Gamma\Big((t, \vec{r}), p\Big) = \frac{1}{2M} \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} \left\langle N, \frac{\vec{q}}{2} \middle| \hat{\mathcal{W}}_\Gamma\Big((t, \vec{0}), p\Big) \middle| N, -\frac{\vec{q}}{2} \right\rangle$$

- To get a **non-trivial** phase-space dependence on $\vec{r}$, take initial and final hadrons with **different** center-of-mass momenta.

Exercise I.1

Recover the nonrelativistic quantum mechanical definition.

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Nonrelativistic Wigner distribution for quarks in QCD.

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Ji, Phys. Rev. Lett. **91**, 062001 (2003)
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- Is it measurable?

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Ji, Phys. Rev. Lett. **91**, 062001 (2003)
Ji *et al.* , Phys. Rev. **D69**, 074014 (2004)

- Is it measurable? Not clear!

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Nonrelativistic Wigner distribution for quarks in QCD (field theory).

Nonrelativistic Wigner distribution for quarks in QCD.

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Ji, Phys. Rev. Lett. **91**, 062001 (2003)
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- Is it measurable? Not clear!
- It is familiar?

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- Is it measurable? Not clear!
- It is familiar? Try with $\Gamma = \gamma_\mu$.

$$\hat{\mathcal{W}}_{\gamma_\mu}\Big((t,\vec{r}),p\Big) = \int \mathrm{d}^4 s\, \bar{\psi}\left(\vec{r}-\frac{1}{2}\vec{s}\right)\gamma_\mu\psi\left(\vec{r}+\frac{1}{2}\vec{s}\right)e^{ip\cdot s}$$

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Nonrelativistic Wigner distribution for quarks in QCD.

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$$\hat{\mathcal{W}}_{\gamma_\mu}\Big((t,\vec{r}),p\Big) = \int \mathrm{d}^4 s\,\bar{\psi}\left(\vec{r}-\frac{1}{2}\vec{s}\right)\gamma_\mu\psi\left(\vec{r}+\frac{1}{2}\vec{s}\right)e^{ip\cdot s}$$

$$\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\hat{\mathcal{W}}_{\gamma_\mu}\Big((t,\vec{r}),p\Big) = \bar{\psi}\Big(t,\vec{r}\Big)\gamma_\mu\psi\Big((t,\vec{r})\Big)$$

Wigner quasiprobability distribution.

Nonrelativistic Wigner distribution for quarks in QCD (field theory).

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$$\mathcal{W}_\Gamma\Big((t,\vec{r}),p\Big) = \frac{1}{2M}\int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\left\langle N,\frac{\vec{q}}{2}\right|\hat{\mathcal{W}}_\Gamma\Big((t,\vec{r}),p\Big)\left|N,-\frac{\vec{q}}{2}\right\rangle$$

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- Is it measurable? Not clear!
- It is familiar? Yes!

$$\hat{\mathcal{W}}_{\gamma_\mu}\Big((t,\vec{r}),p\Big) = \int \mathrm{d}^4 s\,\bar{\psi}\left(\vec{r}-\frac{1}{2}\vec{s}\right)\gamma_\mu\psi\left(\vec{r}+\frac{1}{2}\vec{s}\right)e^{ip\cdot s}$$

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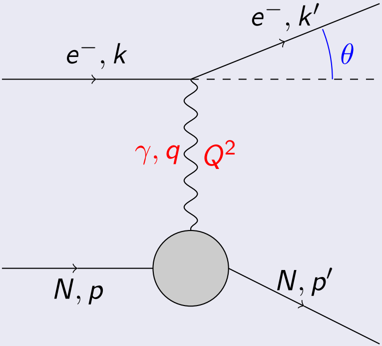
- Matrix element of the **electromagnetic current**!

Elastic scattering.

Kinematics and standard notations.

Kinematics of elastic scattering on the nucleon



$$
\begin{aligned}
q &\equiv k - k', \\
Q^2 &\equiv -q^2, \\
\nu &\equiv \frac{p \cdot q}{M}, \\
x_B &\equiv \frac{Q^2}{2p \cdot q}, \\
W^2 &\equiv (p+q)^2, \\
s &\equiv (p+k)^2.
\end{aligned}
$$

In the target rest frame $\theta \in [0, \pi]$ and:

$$p \equiv (M, \vec{0}), \quad k \equiv (E, \vec{k}), \quad k' \equiv (E', \vec{k'}).$$

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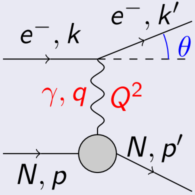
Elastic scattering.

Kinematics, standard notations, orders of magnitude, variable ranges.

Kinematics of elastic scattering on the nucleon

$$\begin{aligned} q &\equiv k - k', \\ Q^2 &\equiv -q^2, \\ \nu &\equiv \frac{p \cdot q}{M}, \end{aligned}$$



$$\begin{aligned} x_B &\equiv \frac{Q^2}{2p \cdot q}, \\ W^2 &\equiv (p+q)^2, \\ s &\equiv (p+k)^2. \end{aligned}$$

Exercise I.2

Give the typical energy range to probe the nucleon structure with electromagnetic elastic scattering. Justify the neglect of the electron mass and show that $q^2 \simeq -4EE' \sin^2 \theta/2$ and $Q^2 > 0$. What is the value of x_B?

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Elastic scattering.

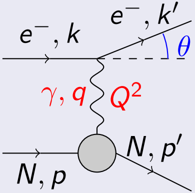
Amplitude at Born order.

- Electromagnetic current:

$$J_\mu^{\mathrm{em}}(y) = \sum_{q=u,d,s,\ldots} e_q \bar{q}(y)\gamma_\mu q(y)$$

- From invariance under translations, take J^{em} at 0.

Kinematics of elastic scattering on the nucleon



- Amplitude $\mathcal{M}(eN \to eN)$ at **Born order**:

$$\mathcal{M}(eN \to eN) = \bar{u}(k', \lambda')\gamma^\mu u(k, \lambda)\frac{e^2}{q^2}\left\langle N, p', h' \middle| J_\mu^{\mathrm{em}}(0) \middle| N, p, h \right\rangle$$

Unpolarized elastic scattering at Born order.

Parameterization of the hadronic matrix element: spin-0 case.

- Most general **Lorentz structure** ($q = p' - p$):

$$\left\langle \pi, p' \left| J^{\mathrm{em}}_{\mu}(0) \right| \pi, p \right\rangle = a_1 p_\mu + a_2 p'_\mu + a_3 q_\mu$$

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- Most general **Lorentz structure** ($q = p' - p$):

$$\left\langle \pi, p' \left| J_\mu^{\mathrm{em}}(0) \right| \pi, p \right\rangle = a_1 p_\mu + a_2 p'_\mu + a_3 q_\mu$$

- Use **4-momentum conservation**:

$$\left\langle \pi, p' \left| J_\mu^{\mathrm{em}}(0) \right| \pi, p \right\rangle = (a_1 + a_2) \frac{(p + p')_\mu}{2} + \left(a_3 - \frac{a_1}{2} + \frac{a_2}{2} \right) q_\mu$$

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- F **depends on** q^2 **only** (elastic scattering: $-q^2 = 2p \cdot q$).

Unpolarized elastic scattering at Born order.

Parameterization of the hadronic matrix element: spin-1/2 case (1/2).

Anatomy of the nucleon

Phase space distributions
- Aside on kinetic theory
- Wigner distribution

Nucleon spatial structure
- Elastic scattering
- Interpretation
- Nucleon charge radius

Quark Wigner distributions
- Relativistic treatment
- Light-cone physics
- 5-dimensional Wigner distribution

- Most general **Lorentz structure** ($q = p' - p$):

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- Expand Γ_μ in **16 matrices** 1, γ_ρ, $[\gamma_\rho, \gamma_\sigma]$, $\gamma_5\gamma_\rho$ and γ_5:

$$\begin{array}{lcl} 1 & : & p_\mu, p'_\mu \\ \gamma_\rho & : & \gamma_\mu, p_\mu \not{p}, p'_\mu \not{p}, p_\mu \not{p}', p'_\mu \not{p}' \\ [\gamma_\rho, \gamma_\sigma] & : & [\gamma_\mu, \not{p}], [\gamma_\mu, \not{p}'], [\not{p}, \not{p}'] p_\mu, [\not{p}, \not{p}'] p'_\mu \\ \gamma_5\gamma_\rho & : & \gamma_5\gamma_\rho \epsilon^{\rho\mu\nu\sigma} p_\nu p'_\sigma \\ \gamma_5 & : & \emptyset \end{array}$$

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- Use **Dirac equations** for u and $\bar{u}$:

$$\bar{u}(p')(\not{p}' - m) = 0 \text{ and } (\not{p} - m)u(p) = 0$$

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where $\bar{u}(p')(\not{p}' - \not{p})u(p) = 0$ (Dirac equation) and $\sigma_{\mu\nu} q^\nu q^\mu = 0$ (symmetry).

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Unpolarized elastic scattering at Born order.

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- **Hermiticity**: b is real and c is purely imaginary.
- b and c **depend on** q^2 **only** ($-q^2 = 2p \cdot q$ for elastic scattering).

$$\left\langle N \left| J_\mu^{\mathrm{em}}(0) \right| N \right\rangle = \bar{u}(p') \left(F_1(Q^2) \gamma_\mu + F_2(Q^2) \frac{i}{2M} \sigma_{\mu\nu} q^\nu \right) u(p)$$

Nucleon form factors.

Pauli-Dirac and Sachs parameterizations.

Pauli-Dirac parameterization

$$\langle N \left| J^{\mathrm{em}}_\mu(0) \right| N \rangle = \bar{u}(p') \left(F_1(Q^2)\gamma_\mu + F_2(Q^2)\frac{i}{2M}\sigma_{\mu\nu}q^\nu \right) u(p)$$

Sachs parameterization

$$\begin{aligned}\langle N \left| J^{\mathrm{em}}_\mu(0) \right| N \rangle &= \bar{u}(p') \left(\frac{G_E(Q^2) - \tau GM(Q^2)}{1-\tau}\frac{P_\mu}{M} \right. \\ &\left. + G_M(Q^2)\frac{i}{2M}\sigma_{\mu\nu}q^\nu \right) u(p)\end{aligned}$$

with $\tau = Q^2/(4M^2)$ and:

$$\begin{aligned}G_E(Q^2) &= F_1(Q^2) + \frac{Q^2}{4M^2}F_2(Q^2), \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2).\end{aligned}$$

Nucleon form factors and elastic scattering.

Expression of the cross section in terms of form factors.

Mott cross section

Scattering of a relativistic electron on a point-like spinless particle:

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} = \frac{Q^2\alpha^2\cos^2\frac{\theta}{2}}{4E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}$$

Rosenbluth cross section

Scattering of a relativistic electron on a spin-1/2 composite target:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left.\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}}\left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau}+2\tau G_M^2(Q^2)\tan^2\frac{\theta}{2}\right)$$

with: $\tau \equiv Q^2/(4M^2)$

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with: $\tau \equiv Q^2/(4M^2)$

Exercise I.3

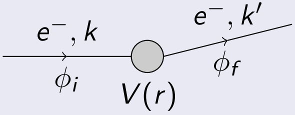
Establish the relation between the energies E and E' of the incoming and outgoing electrons and the scattering angle θ. Comment on the number of independent kinematic variables.

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}$$

Interpretation of form factors.

Nonrelativistic scattering of a scalar particle on a spherically symmetric potential.

Nonrelativistic scattering

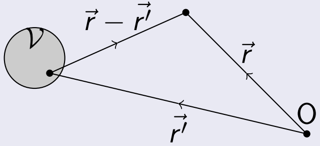


$$\frac{\mathrm{d}\sigma}{\mathrm{d}k'\mathrm{d}\Omega'} \propto |\langle f|V|i\rangle|^2$$

$$\langle f|V|i\rangle = \int \mathrm{d}^3\vec{r} e^{-i\vec{k}'\cdot\vec{r}} V(r) e^{i\vec{k}\cdot\vec{r}}$$

$$\vec{q} = \vec{k} - \vec{k}'$$

Spherically symmetric charge distribution



$$V(r) = \frac{Ze^2}{4\pi} \int_{\mathcal{V}} \mathrm{d}^3\vec{r}' \frac{\rho(r')}{|\vec{r} - \vec{r}'|}$$

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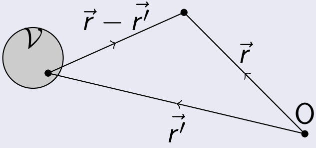

$$V(r) = \frac{Ze^2}{4\pi} \int_{\mathcal{V}} \mathrm{d}^3\vec{r'} \frac{\rho(r')}{|\vec{r} - \vec{r'}|}$$

$$\langle f\,|V|\,i\rangle = \int \mathrm{d}^3\vec{r} e^{-i\vec{q}\cdot\vec{r}} V(r)$$

- Compute in spherical coordinates:

$$\langle f\,|V|\,i\rangle = Ze^2 \int_{\mathcal{V}} \mathrm{d}^3\vec{r'}\, e^{i\vec{q}\cdot\vec{r'}} \rho(r') \int_0^{+\infty} \mathrm{d}R\, R \frac{\sin qR}{qR}$$

Interpretation of form factors.

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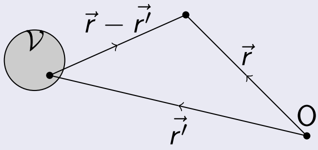
Spherically symmetric charge distribution



$$V(r) = \frac{Ze^2}{4\pi} \int_{\mathcal{V}} \mathrm{d}^3\vec{r}' \frac{\rho(r')}{|\vec{r} - \vec{r}'|}$$

$$\langle f \,|V|\, i \rangle = \int \mathrm{d}^3\vec{r} e^{-i\vec{q}\cdot\vec{r}} V(r)$$

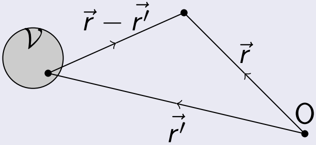
- Compute in spherical coordinates: Diverge!

$$\langle f \,|V|\, i \rangle = Ze^2 \int_{\mathcal{V}} \mathrm{d}^3\vec{r}' \, e^{i\vec{q}\cdot\vec{r}'} \rho(r') \int_0^{+\infty} \mathrm{d}R \, R \frac{\sin qR}{qR}$$

Interpretation of form factors.

Nonrelativistic scattering of a scalar particle on a spherically symmetric potential.

Spherically symmetric charge distribution



$$V(r) = \frac{Ze^2}{4\pi} \int_{\mathcal{V}} \mathrm{d}^3\vec{r'} \frac{\rho(r') e^{-\frac{|\vec{r}-\vec{r'}|}{a}}}{|\vec{r}-\vec{r'}|}$$

$$\langle f \,|V|\, i \rangle = \int \mathrm{d}^3\vec{r} e^{-i\vec{q}\cdot\vec{r}} V(r)$$

- Compute in spherical coordinates:

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- Regularize: Yukawa screening ($a \simeq 10^{-10}$ $m \simeq 0.5$ keV^{-1})

$$\langle f \,|V|\, i \rangle = \frac{Ze^2}{q^2 + \frac{1}{a^2}} F(Q^2) \quad \text{with } F(Q^2) = \int_{\mathcal{V}} \mathrm{d}^3\vec{r} \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}}$$

Interpretation of form factors.

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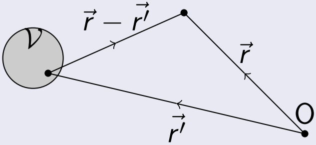
Spherically symmetric charge distribution



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$$\simeq \frac{Ze^2}{q^2} F(Q^2) \quad \text{for } Q \simeq 1.\ \text{GeV}$$

Interpretation of form factors.

Rutherford scattering.

Rutherford cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{Z\alpha}{2E}\right)^2 \frac{1}{\sin^2\frac{\theta}{4}} |F(Q^2)|^2$$

where F is the **3D Fourier transform of the target charge distribution**.

Exercise I.4

Compute the form factor F for the charge distributions :

- A single structureless charge located at $\vec{r}_0$.
- A uniform spherical distribution with a *sharp cut-off a*:

$$\begin{aligned} \rho(r) &= \rho_0 \quad \text{if } r \leq a, \\ \rho(r) &= 0 \quad \text{if } r > a. \end{aligned}$$

Comment on the zero-recoil approximation.

Interpretation of form factors.

Rutherford scattering.

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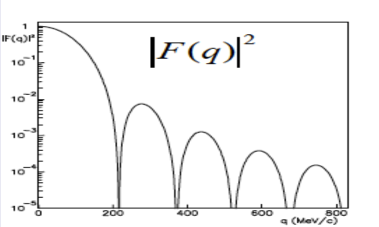
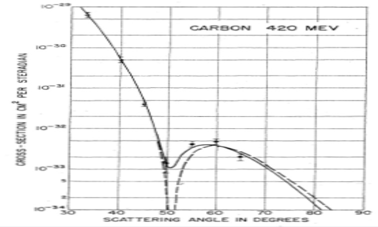
Sharp cut-off distribution



Lead form factor



Interpretation of form factors.

Normalization of nucleon form factors.

- Take **proton state** with momentum k: $|p, k\rangle$.
- Consider **charge operator**: $\mathbb{Q}\,|p, k\rangle = +\,|p, k\rangle$

$$\mathbb{Q} = \int \mathrm{d}^3\vec{r}\, J_0^{\mathrm{e.m.}}(\vec{r})$$

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- Then $\langle p, k' \,|\mathbb{Q}|\, p, k\rangle = \langle k' \,|k\,\rangle = 2E_{\vec{k}}(2\pi)^3\delta^{(3)}(\vec{k'} - \vec{k})$ and

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$$\mathbb{Q} = \int \mathrm{d}^3\vec{r}\, J_0^{\mathrm{e.m.}}(\vec{r}) = e^{i\mathbb{P}\cdot(t,\vec{r})} J_0^{\mathrm{e.m.}}(0) e^{-i\mathbb{P}\cdot(t,\vec{r})}$$

- Then $\langle p, k' \,|\mathbb{Q}|\, p, k\rangle = \langle k' \,|k\,\rangle = 2E_{\vec{k}}(2\pi)^3\delta^{(3)}(\vec{k'} - \vec{k})$ and

$$\langle p, k' \,|\mathbb{Q}|\, p, k\rangle = \int \mathrm{d}^3\vec{r} e^{i(\vec{k'}-\vec{k})\cdot\vec{r}} e^{i(E_{\vec{k'}}-E_{\vec{k}})t} \langle p, k' \,|J_0^{\mathrm{e.m.}}(0)|\, p, k\rangle$$

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- Take **proton state** with momentum k: $|p, k\rangle$.
- Consider **charge operator**: $\mathbb{Q}\,|p, k\rangle = +\,|p, k\rangle$

$$\mathbb{Q} = \int \mathrm{d}^3\vec{r}\, J_0^{\mathrm{e.m.}}(\vec{r}) = e^{i\mathbb{P}\cdot(t,\vec{r})} J_0^{\mathrm{e.m.}}(0) e^{-i\mathbb{P}\cdot(t,\vec{r})}$$

- Then $\langle p, k' \,|\mathbb{Q}|\, p, k\rangle = \langle k' \,|\, k\rangle = 2E_{\vec{k}}(2\pi)^3\delta^{(3)}(\vec{k'} - \vec{k})$ and

$$\begin{aligned}\langle p, k' \,|\mathbb{Q}|\, p, k\rangle &= \int \mathrm{d}^3\vec{r} e^{i(\vec{k'}-\vec{k})\cdot\vec{r}} e^{i(E_{\vec{k'}}-E_{\vec{k}})t} \langle p, k' \,| J_0^{\mathrm{e.m.}}(0) |\, p, k\rangle \\ &= (2\pi)^3\delta^{(3)}(\vec{k'} - \vec{k})\bar{u}(k)\gamma_0 F_1(0) u(k)\end{aligned}$$

Interpretation of form factors.

Normalization of nucleon form factors.

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&= (2\pi)^3\delta^{(3)}(\vec{k'} - \vec{k})\bar{u}(k)\gamma_0 F_1(0) u(k) \\
&= 2E_{\vec{k}} F_1(0)(2\pi)^3\delta^{(3)}(\vec{k'} - \vec{k})
\end{aligned}$$

Interpretation of form factors.

Normalization of nucleon form factors.

Anatomy of the nucleon

Phase space distributions
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Wigner distribution

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Elastic scattering
Interpretation
Nucleon charge radius

Quark Wigner distributions
Relativistic treatment
Light-cone physics
5-dimensional Wigner distribution

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&= 2E_{\vec{k}} F_1(0)(2\pi)^3\delta^{(3)}(\vec{k'} - \vec{k})
\end{aligned}$$

- The form factor F_1 at zero momentum transfer is the **electric charge**.

Interpretation of form factors.

Normalization of nucleon form factors.

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- Wigner distribution

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- Elastic scattering
- Interpretation
- Nucleon charge radius

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&= (2\pi)^3\delta^{(3)}(\vec{k'} - \vec{k})\bar{u}(k)\gamma_0 F_1(0) u(k) \\
&= 2E_{\vec{k}} F_1(0)(2\pi)^3\delta^{(3)}(\vec{k'} - \vec{k})
\end{aligned}$$

- The form factor F_1 at zero momentum transfer is the **electric charge**.
- Similarly, the form factor F_2 is normalized to the **anomalous magnetic moment**.

Interpretation of form factors.

Nucleon form factors in the Breit frame.

Breit frame

$$\xrightarrow{\vec{p}\,' = -\vec{p}}$$

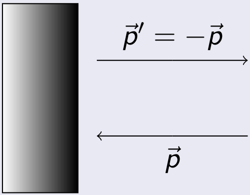
$$\underset{\vec{p}}{\longleftarrow}$$

Frame in which the **outgoing** nucleon has a 3-momentum opposite to that of the **incoming** nucleon.

Interpretation of form factors.

Nucleon form factors in the Breit frame.

Breit frame



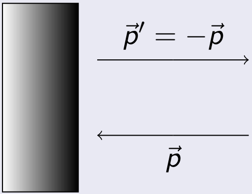
Frame in which the **outgoing** nucleon has a 3-momentum opposite to that of the **incoming** nucleon.

"Brick wall condition"

Interpretation of form factors.

Nucleon form factors in the Breit frame.

Breit frame



Frame in which the **outgoing** nucleon has a 3-momentum opposite to that of the **incoming** nucleon.

"Brick wall condition"

- Evaluate matrix element of $J^{\mathrm{e.m.}}$ in the Breit frame:

$$\langle N(-\vec{p})\,|J_0^{\mathrm{e.m.}}|\,N(\vec{p})\rangle = \bar{u}(p')\left(F_1\gamma_0 + F_2\frac{i}{2M}\sigma_{0\nu}q^\nu\right)u(p)$$

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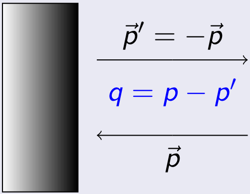
Breit frame



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"Brick wall condition"

- Evaluate matrix element of $J^{\text{e.m.}}$ in the Breit frame:

$$\begin{aligned}\langle N(-\vec{p})\,|J_0^{\text{e.m.}}|\,N(\vec{p})\rangle &= \bar{u}(p')\left(F_1\gamma_0 + F_2\frac{i}{2M}\sigma_{0\nu}q^\nu\right)u(p)\\ \text{(Gordon id.)} &= \bar{u}(p')\left((F_1+F_2)\gamma_0 - F_2\frac{(p+p')_0}{2M}\right)u(p)\end{aligned}$$

CEA - Saclay

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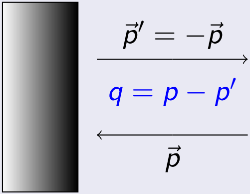
Interpretation of form factors.

Nucleon form factors in the Breit frame.

Breit frame



$\vec{p}' = -\vec{p}$

$q = p - p'$

$\vec{p}$

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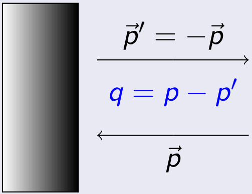
Interpretation of form factors.

Nucleon form factors in the Breit frame.

Breit frame



Frame in which the **outgoing** nucleon has a 3-momentum opposite to that of the **incoming** nucleon.

"Brick wall condition"

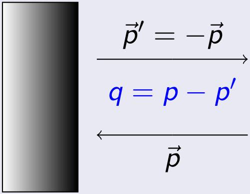
- Evaluate matrix element of $J^{\text{e.m.}}$ in the Breit frame:

$$
\begin{aligned}
\langle N(-\vec{p}) \, | J_0^{\text{e.m.}} | \, N(\vec{p}) \rangle &= \bar{u}(p') \left(F_1 \gamma_0 + F_2 \frac{i}{2M} \sigma_{0\nu} q^\nu \right) u(p) \\
&= \bar{u}(p') \left((F_1 + F_2) \gamma_0 - F_2 \frac{(p+p')_0}{2M} \right) u(p) \\
\text{(Breit fr.)} \quad &= 2M \delta_{hh'} \left[F_1 + F_2 \left(1 - \frac{E_p^2}{M^2} \right) \right]
\end{aligned}
$$

Interpretation of form factors.

Nucleon form factors in the Breit frame.

Breit frame



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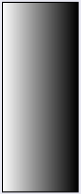
Interpretation of form factors.

Nucleon form factors in the Breit frame.

Breit frame



$$\vec{p}' = -\vec{p} \longrightarrow$$

$$q = p - p'$$

$$\longleftarrow \vec{p}$$

Frame in which the **outgoing** nucleon has a 3-momentum opposite to that of the **incoming** nucleon.

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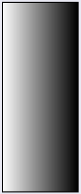
$$\begin{aligned} \langle N(-\vec{p})\,|J_0^{\mathrm{e.m.}}|\,N(\vec{p})\rangle &= \bar{u}(p')\left(F_1\gamma_0 + F_2\frac{i}{2M}\sigma_{0\nu}q^\nu\right)u(p) \\ &= \bar{u}(p')\left((F_1+F_2)\gamma_0 - F_2\frac{(p+p')_0}{2M}\right)u(p) \\ (q^2 = -4|\vec{p}|^2) \quad &= 2M\delta_{hh'}\left[F_1 + F_2\left(1 - \frac{E_p^2}{M^2}\right)\right] \end{aligned}$$

Interpretation of form factors.

Nucleon form factors in the Breit frame.

Breit frame



$$\vec{p}\,' = -\vec{p} \quad \longrightarrow$$

$$q = p - p'$$

$$\longleftarrow \quad \vec{p}$$

Frame in which the **outgoing** nucleon has a 3-momentum opposite to that of the **incoming** nucleon.

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- Evaluate matrix element of $J^{\text{e.m.}}$ in the Breit frame:

$$\begin{aligned}
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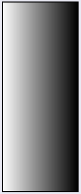
&= \bar{u}(p')\left((F_1+F_2)\gamma_0 - F_2\frac{(p+p')_0}{2M}\right)u(p) \\
&= 2M\delta_{hh'}G_E
\end{aligned}$$

Interpretation of form factors.

Nucleon form factors in the Breit frame.

Breit frame



$$\vec{p}' = -\vec{p}$$

$$q = p - p'$$

$$\vec{p}$$

Frame in which the **outgoing** nucleon has a 3-momentum opposite to that of the **incoming** nucleon.

"Brick wall condition"

Nucleon form factors in the Breit frame

- G_E is the 3D Fourier transform of the **charge density**.
- G_M is the 3D Fourier transform of the **magnetization density**.

Interpretation of form factors.

Quark contributions to form factors.

- Assume **isospin symmetry**.
- Note $G^{(p)}$ and $G^{(n)}$ the Sachs form factors of the proton and neutron.
- Define $G^{(u)}$ and $G^{(d)}$ such that:

$$\begin{aligned} G^{(p)} &\equiv \frac{2}{3}G^{(u)} - \frac{1}{3}G^{(d)} \\ G^{(n)} &\equiv \frac{2}{3}G^{(d)} - \frac{1}{3}G^{(u)} \end{aligned}$$

- Get non-trivial **spatial information** on the **quark structure** of the nucleon!

Nucleon charge radius.

Evaluation from elastic scattering in the Breit frame.

- Form factors are **3D Fourier transforms** of distributions in the Breit frame.
- For a **spherically symmetric** charge distribution ρ:

$$
\begin{aligned}
F(Q^2) &= \int_0^{+\infty} dr\,\rho(r) 4\pi r^2 \frac{\sin qr}{qr} \\
&= \int_0^{+\infty} dr\,\rho(r) 4\pi r \frac{1}{q} \left(qr - \frac{q^3 r^3}{6} + \ldots \right) \\
&\simeq \int_0^{+\infty} dr\, 4\pi r^2 \rho(r) - \frac{q^2}{6} \int_0^{+\infty} dr\, 4\pi r^2 r^2 \rho(r) + \ldots \\
&= 1 - \frac{q^2}{6} \langle r^2 \rangle + \ldots
\end{aligned}
$$

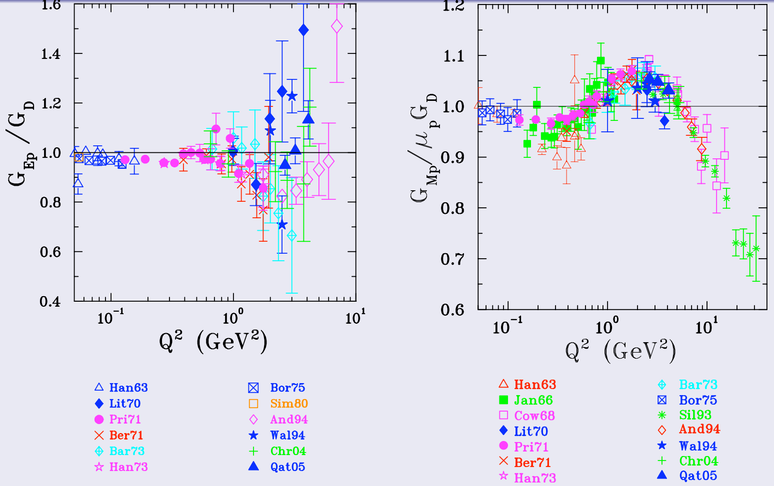
- **Define a charge radius** by:

$$
\langle r^2 \rangle \equiv -6 \left. \frac{dF}{dq^2} \right|_{q^2=0}
$$

Nucleon charge radius.

Extraction of the charge radius from elastic scattering measurements.

Measurements



Perdrisat *et al.* , Prog. Part. Nucl. Phys. **59**, 694 (2007)

Nucleon charge radius.

Extraction of the charge radius from elastic scattering measurements.

Exercise I.5

Consider $\rho(r) = Ce^{-mr}$ where $m > 0$ and C is such that the total charge is normalized to 1. Compute:

- the corresponding form factor,
- the charge radius,
- all higher moments $\langle r^n \rangle = \int dr\, r^n \rho(r)$,
- all ratios $\langle r^{2n} \rangle / \langle r^2 \rangle^n$.

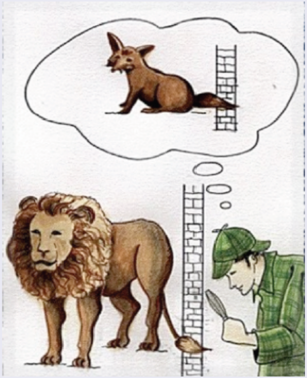
How do these ratios behave when n is large ?

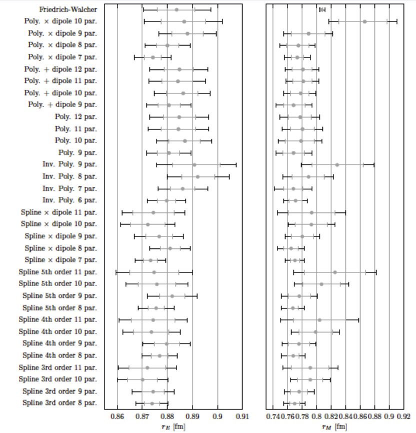
- Taylor expand G_E:
$$G_E(Q^2) = 1 - Q^2 \langle r^2 \rangle /6 + Q^4 \langle r^4 \rangle /120 - \ldots$$
- **Higher moments** are increasing with order, hence giving a **large contribution** to $G_E(Q^2)$.
- No reason for the $\langle r^2 \rangle$ term to dominate!

Nucleon charge radius.

Extraction of the charge radius from elastic scattering measurements.

Extrapolations. . .





Bernauer *et al.* , arXiv:1307.6227

Nucleon charge radius: an unexpected result.

Can the proton really be 0.00000000000003 mm smaller than expected?

Pohl *et al.*, Nature **A466**, 213 (2010)

- 5.0 σ discrepancy with CODATA value !

Nucleon charge radius: an unexpected result.

Can the proton really be 0.0000000000003 mm smaller than expected?

Nucleon charge radius: an unexpected result.

Can the proton really be 0.0000000000003 mm smaller than expected?

CODATA Recommended Values of the Fundamental Physical Constants: 2006*

Peter J. Mohr[†], Barry N. Taylor[‡], and David B. Newell[§],

National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8420, USA

(Dated: February 2, 2008)

This paper gives the 2006 self-consistent set of values of the basic constants and conversion factors of physics and chemistry recommended by the Committee on Data for Science and Technology (CODATA) for international use. Further, it describes in detail the adjustment of the values of the constants, including the selection of the final set of input data based on the results of least-squares analyses. The 2006 adjustment takes into account the data considered in the 2002 adjustment as well as the data that became available between 31 December 2002, the closing date of that adjustment, and 31 December 2006, the closing date of the new adjustment. The new data have led to a significant reduction in the uncertainties of many recommended values. The 2006 set replaces the previously recommended 2002 CODATA set and may also be found on the World Wide Web at physics.nist.gov/constants.

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*This report was prepared by the authors under the auspices of the CODATA Task Group on Fundamental Constants. The members

Nucleon charge radius: an unexpected result.

Can the proton really be 0.0000000000003 mm smaller than expected?

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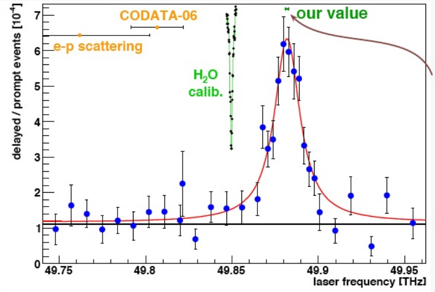
The resonance: discrepancy, sys., stat.

Discrepancy:
$5.0\,\sigma \;\leftrightarrow\; \sim 75\ \mathrm{GHz} \;\leftrightarrow\; \delta\nu/\nu = 1.5\times10^{-3}$



Fitted position:
700 MHz unc.
$\rightarrow\ r_p$

Fitted width as predicted

Systematics: 300 MHz
Statistics: 700 MHz

Laser frequency known with 300 MHz uncertainty

ETH — A. Antognini, CERN 10.08.2010 – p.21

Nucleon charge radius: an unexpected result.

Can the proton really be 0.00000000000003 mm smaller than expected?

Contributions to the μp Lamb shift

#	Contribution	Value	Unc.
3	Relativistic one loop VP	205.0282	
4	NR two-loop electron VP	1.5081	
5	Polarization insertion in two Coulomb lines	0.1509	
6	NR three-loop electron VP	0.00529	
7	Polarisation insertion in two and three Coulomb lines (corrected)	0.00223	
8	Three-loop VP (total, uncorrected)		
9	Wichmann-Kroll	-0.00103	
10	Light by light electron loop ((Virtual Delbrück)	0.00135	0.00135
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	-0.00500	0.0010
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	-0.00150	
13	Mixed electron and muon loops	0.00007	
14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	0.01077	0.00038
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	0.000047	
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	-0.000015	
17	Recoil contribution	0.05750	
18	Recoil finite size	0.01300	0.001
19	Recoil correction to VP	-0.00410	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	-0.66770	
21	Muon Lamb shift 4th order	-0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	-0.04497	
23	Recoil of order α^6	0.00030	
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	-0.00960	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability)	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	0.00019	
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	-0.00001	
	Sum	206.0573	0.0045

ETH A. Antognini, CERN 10.08.2010 – p.25

See more.

Nucleon charge radius: fully relativistic treatment.

Localization of a quantum relativistic system.

- From **uncertainty principle**: minimal spread in momentum / energy in a **confined** system.

Nucleon charge radius: fully relativistic treatment.

Localization of a quantum relativistic system.

- From **uncertainty principle**: minimal spread in momentum / energy in a **confined** system.
- If the energy levels of the confined system are high enough, **pair creation** is possible.

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Nucleon charge radius: fully relativistic treatment.

Localization of a quantum relativistic system.

- From **uncertainty principle**: minimal spread in momentum / energy in a **confined** system.
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- Pair creation may **prevent the localization** of a particle with a high resolution.

Nucleon charge radius: fully relativistic treatment.

Localization of a quantum relativistic system.

- From **uncertainty principle**: minimal spread in momentum / energy in a **confined** system.
- If the energy levels of the confined system are high enough, **pair creation** is possible.
- Pair creation may **prevent the localization** of a particle with a high resolution.
- Discussions about nucleon radius refers to a **specific prescription**.

Nucleon charge radius: fully relativistic treatment.

Quantum relativistic localization (Burkardt, Phys. Rev. **D62**, 071503 (2000)).

- **Wave packet** for spinless mass m particle localized at $\vec{R}$:

$$\left|\vec{R}\right\rangle = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\, e^{i\vec{p}\cdot\vec{R}}\psi(\vec{p})\,|\vec{p}\rangle \text{ with } E_p = \sqrt{\vec{p}^2+m^2}$$

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$$\int \mathrm{d}^3\vec{r}\, e^{i\vec{q}\cdot\vec{r}}\left\langle \vec{R}\left|\rho(\vec{r})\right|\vec{R}\right\rangle = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{\psi^*(\vec{p}+\vec{q})\psi(\vec{p})}{\sqrt{E_pE_{p+q}}}\left\langle \vec{p}\,'\left|\rho(\vec{0})\right|\vec{p}\right\rangle$$

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- Interpretation
- Nucleon charge radius

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- 5-dimensional Wigner distribution

- 3D Fourier transform of charge distribution:

$$\int \mathrm{d}^3\vec{r}\, e^{i\vec{q}\cdot\vec{r}} \left\langle \vec{R} \left| \rho(\vec{r}) \right| \vec{R} \right\rangle = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \frac{E_p + E_{p+q}}{2\sqrt{E_p E_{p+q}}} \psi^*(\vec{p}+\vec{q})\psi(\vec{p}) F(q^2)$$

- Three types of contributions

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- Three types of contributions

  Form factor **sensitivity** form factor's shape: cannot take F out of the integral.

$$q^0 = \sqrt{(\vec{p}+\vec{q})^2 + M^2} - \sqrt{\vec{p}^2 + M^2}$$

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Wave packet: Sensitivity to **spatial distribution** of the wave packet.

Relativistic effects: Nonrelativistic limit $\vec{p}^2 \ll m^2$:

$$E_p \simeq m + \frac{\vec{p}^2}{2m} \text{ and } \frac{E_p + E_{p+q}}{2\sqrt{E_p E_{p+q}}} \simeq 1$$

Nucleon charge radius: fully relativistic treatment.

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$$\int \mathrm{d}^3\vec{r}\, e^{i\vec{q}\cdot\vec{r}} \left\langle \vec{R}\,|\rho(\vec{r})|\,\vec{R} \right\rangle = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \frac{E_p + E_{p+q}}{2\sqrt{E_p E_{p+q}}} \psi^*(\vec{p}+\vec{q})\psi(\vec{p})F(q^2)$$

- Three types of contributions
  - Form factor: **sensitivity** form factor's shape: cannot take F out of the integral.
  - Wave packet: Sensitivity to **spatial distribution** of the wave packet.
  - Relativistic effects: Nonrelativistic limit $\vec{p}^2 \ll m^2$:
- 3D Fourier transform of charge distribution is F when:
  - Wave packet is very broad in momentum space.
  - Nonrelativistic limit.

Nucleon charge radius: fully relativistic treatment.

Quantum relativistic localization (Burkardt, Phys. Rev. **D62**, 071503 (2000)).

- Expand 3D Fourier transform of charge distribution:

$$\int \mathrm{d}^3\vec{r}\, e^{i\vec{q}\cdot\vec{r}} \left\langle \vec{R}\,|\rho(\vec{r})|\,\vec{R}\right\rangle = \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} \frac{E_p + E_{p+q}}{2\sqrt{E_p E_{p+q}}} \psi^*(\vec{p}+\vec{q})\psi(\vec{p}) F(q^2)$$

$$\simeq 1 + \frac{\langle r^2 \rangle}{6}\vec{q}^2 - \frac{\langle r^2 \rangle}{6} \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} |\psi(\vec{p})|^2 \frac{(\vec{q}\cdot\vec{p})^2}{E_p^2}$$

$$+ \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} |\vec{q}\cdot\nabla\psi(\vec{p})|^2 - \frac{1}{8} \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3} |\psi(\vec{p})|^2 \frac{(\vec{q}\cdot\vec{p})^2}{E_p^4}$$

- **Relativistic corrections** appear with terms $\propto (\vec{q}\cdot\vec{p})^2/E_p^2$ or $\vec{q}^2/E_p^2$.
- In a reference frame where E_p is large and $\vec{q}^2$ and $\vec{p}\cdot\vec{q}$ are finite, these **corrections remain small**.

Nucleon charge radius: fully relativistic treatment.

Quantum relativistic localization in an infinite momentum frame.

- Reference frame with a **fast moving** particle along z axis:

$$p_\mu \simeq \left(P + \frac{m^2}{2P}, 0_\perp, P \right) \text{ for large } P$$

- In the **Bjorken frame** the 4-momentum of the exchanged photon is:

$$q_\mu = \left(\frac{Q^2}{2x_B P}, q_\perp, 0 \right)$$

- With this choice are kept **finite** when $P \to \infty$:

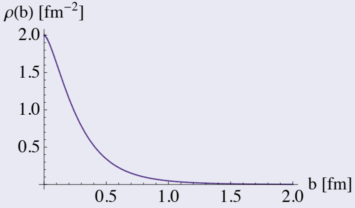
$$p \cdot q = \frac{Q^2}{2x_B} + \frac{m^2 Q^2}{4x_B P^2} \text{ and } q^2 = \left(\frac{Q^2}{2x_B P} \right)^2 - q_\perp^2$$

- In that frame the wave packet in completely **delocalized** in z direction and **sharply peaked** in transverse directions.
- Consistent relativistic def.: form factor $\equiv$ **2D Fourier transform** of charge distribution in **transverse plane**.

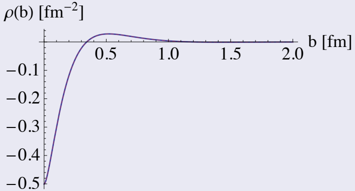
Nucleon charge radius: fully relativistic treatment.

Quantum relativistic localization in an infinite momentum frame.

Proton and neutron transverse charge densities



- proton

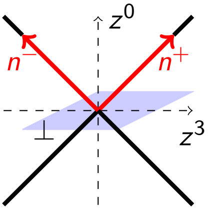


- neutron

Miller, Ann. Rev. Nucl. Part. Sci. **A60**, 1 (2010)

Light-cone coordinates.

Choice of a privileged axis along which particles have a large momentum.



- z axis defined by propagation of **fast moving** particles.
- Write $v^\mu = (v^+, \vec{v}_\perp, v^-)$ for a 4-vector v^μ with:

$$v^+ = \frac{v^0 + v^3}{\sqrt{2}} \text{ and } v^- = \frac{v^0 - v^3}{\sqrt{2}}$$

- Product of two 4-vectors v and w:

$$v \cdot w = v^+ w^- + v^- w^+ - \vec{v}_\perp \cdot \vec{w}_\perp$$

- Take two **light-like** 4-vectors $n_+ = (1, 0, 0, 1)$ and $n_- = (1, 0, 0, -1)$ sucht that:

$$n_+ \cdot n_- = 1 \text{ and } v^\pm = v \cdot n_\mp \text{ for any 4-vector } v^\mu$$

- For a particle moving at the speed of light in the $+z$ direction ($x^3 \simeq x^0$): $z^- \simeq 0$ and $z^+ \simeq \sqrt{2} x^0$.
- Interpret x^+ as **light-cone time**.

Light-cone Poincaré algebra.

Nonrelativistic properties of Quantum Field Theories on the light-cone.

- The Poincaré group is defined by:
  - 4 **translation** generators P^μ
  - 3 **spatial rotation** generators J^i
  - 3 **boost** generators K^i
- The 6 light-cone generators J^3, P^1, P^2, P^+, $(K^1 + J^2)/\sqrt{2}$, and $(K^2 - J^1)/\sqrt{2}$ leave **invariant** the surfaces of constant x^+.
- P^- generates translations in x^+ directions: **Hamiltonian**.
- The sub-algebra generated by these 7 generators is **isomorphic** to the algebra of **Galilean transformations** of 2D quantum mechanics:

$$
\begin{array}{ccl}
P^+ & \leftrightarrow & \text{Mass} \\
P^- & \leftrightarrow & \text{Hamiltonian} \\
J^3 & \leftrightarrow & \text{Rotations in transverse plane} \\
P^\perp & \leftrightarrow & \text{Translations in transverse plane}
\end{array}
$$

Quark Wigner distribution.

Full relativistic treatment.

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Wigner operator for quarks at fixed light-cone time $y^+ = 0$

$$\hat{\mathcal{W}}^q_\Gamma(\vec{b}_\perp, \vec{k}_\perp, x) =$$

$$\frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2 z_\perp}{(2\pi)^3} e^{i(xP^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)} \bar{q}\left(y - \frac{z}{2}\right)\Gamma \mathcal{L} q\left(y + \frac{z}{2}\right)\Big|_{z^+=0}$$

where:

Lorcé and Pasquini, Phys. Rev. **D84**, 014015 (2011)

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- p, p' incoming and outgoing hadron momenta, $P = (p + p')/2$,

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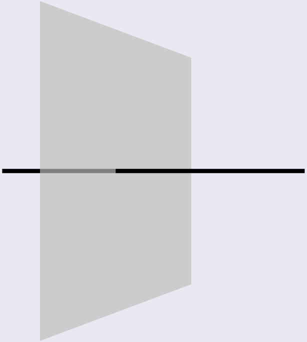

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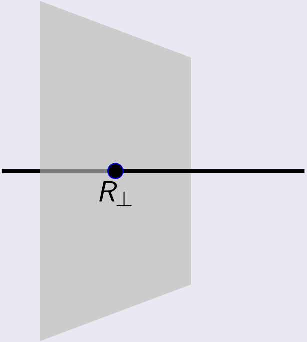

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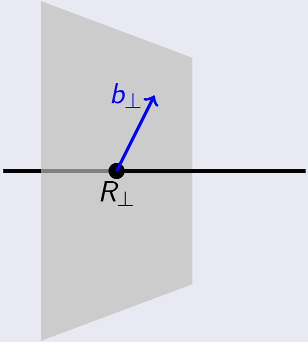

- Transverse center of momentum $R_\perp = \sum_i x_i r_{\perp i}$,
- Impact parameter $b_\perp$,

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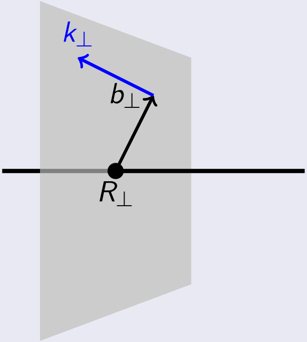

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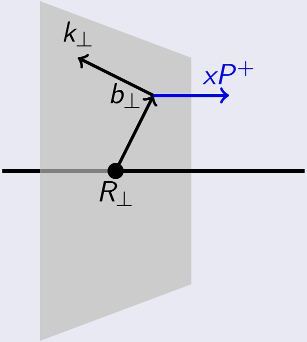

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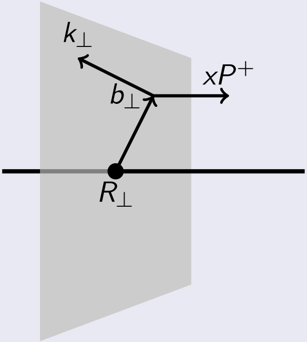
Quark Wigner distribution.

Transverse plane imaging.

Wigner operator for quarks at fixed light-cone time $y^+ = 0$

$$\hat{\mathcal{W}}^q_\Gamma(\vec{b}_\perp, \vec{k}_\perp, x) =$$

$$\frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2 z_\perp}{(2\pi)^3} e^{i(xP^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)} \bar{q}\left(y - \frac{z}{2}\right) \Gamma \mathcal{L} q \left(y + \frac{z}{2}\right)\Big|_{z^+=0}$$



- Transverse center of momentum $R_\perp = \sum_i x_i r_{\perp i}$,
- Impact parameter $b_\perp$,
- Transverse momentum $k_\perp$,
- Longitudinal momentum xP^+.

Quark Wigner distribution.

Wigner distributions as matrix elements of localized nucleon states.

CEA - Saclay

Anatomy of the nucleon

Phase space distributions
Aside on kinetic theory
Wigner distribution

Nucleon spatial structure
Elastic scattering
Interpretation
Nucleon charge radius

Quark Wigner distributions
Relativistic treatment
Light-cone physics
5-dimensional Wigner distribution

- Take a nucleon state $\left|p^+, \vec{p}_\perp, \vec{S}\right\rangle$ where $\vec{S}$ is the **polarization** of the nucleon.

Wigner distribution (quantum relativistic framework)

$$\mathcal{W}^q_\Gamma(\vec{b}_\perp, \vec{k}_\perp, x, \vec{S}) \equiv$$
$$\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} \left\langle p^+, \frac{\Delta_\perp}{2}, \vec{S} \middle| \hat{\mathcal{W}}^q_\Gamma(\vec{b}_\perp, \vec{k}_\perp, x) \middle| p^+, -\frac{\Delta_\perp}{2}, \vec{S} \right\rangle$$

- Wigner distributions are **2D Fourier transforms** of more general objects: Generalized Transverse Momentum Dependent parton distributions (GTMD).
- Leading twist: 16 GTMDs (complex-valued functions).

Meissner *et al.* , JHEP **0908**, 056 (2009), JHEP **0808**, 038 (2008)

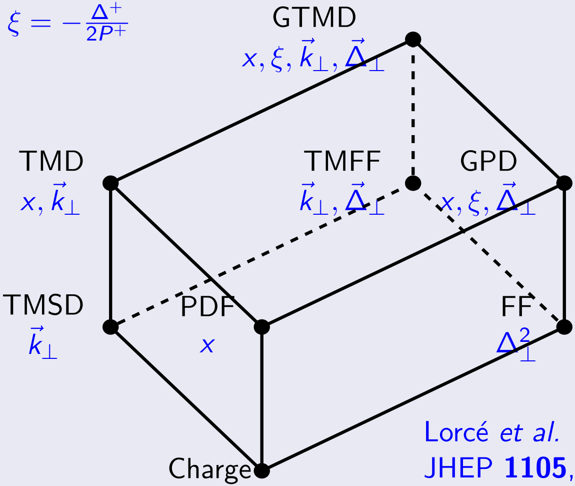
- Thus there are 16 Wigner distributions which are real-valued functions (leading twist).

The whole family of quark distributions.

An experimental channel allowing Wigner distribution measurements has not been identified yet.

Relations between quark distributions

$\xi = -\frac{\Delta^+}{2P^+}$



Lorcé *et al.*
JHEP **1105**, 041 (2011)

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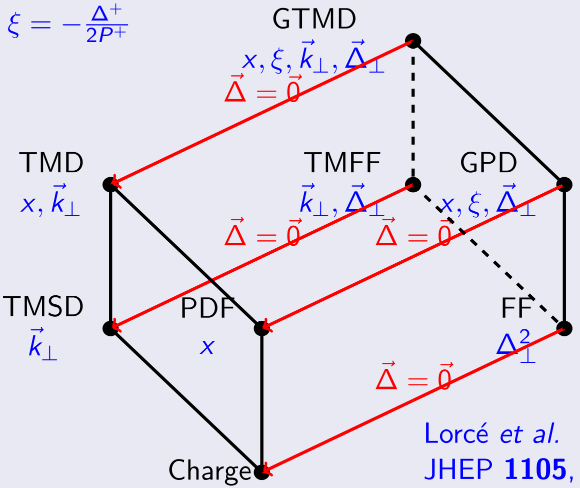

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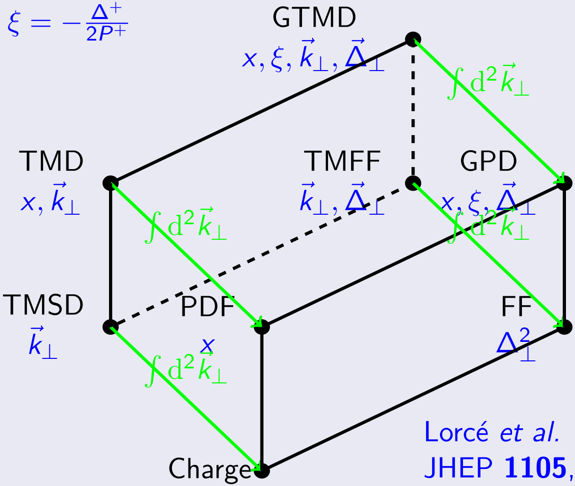

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- Elastic scattering
- Interpretation
- Nucleon charge radius

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- Relativistic treatment
- Light-cone physics
- 5-dimensional Wigner distribution

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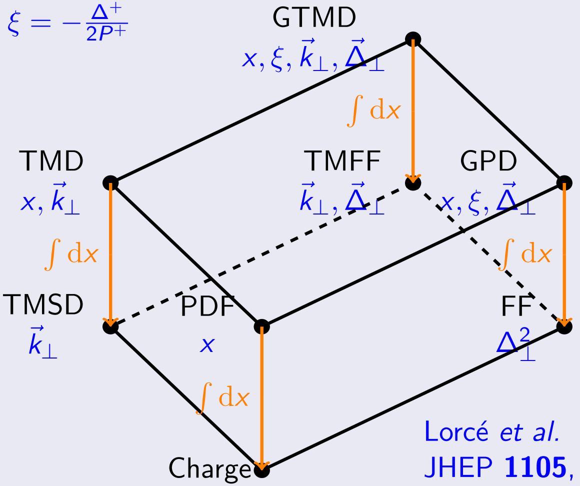

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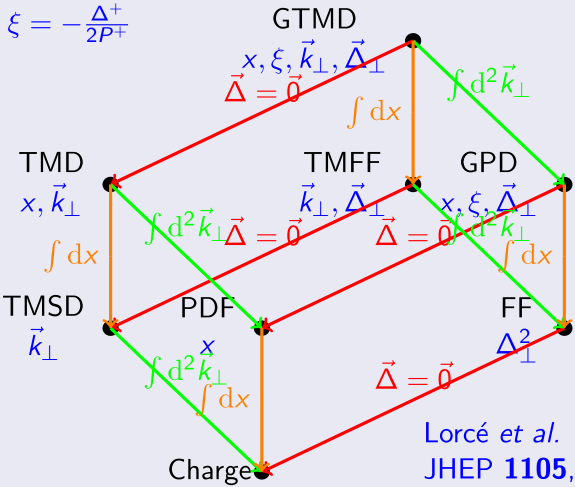

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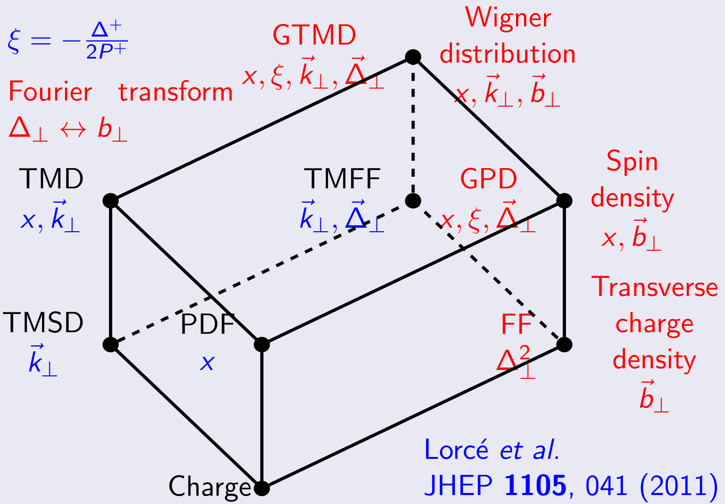

Lorcé *et al.*
JHEP **1105**, 041 (2011)

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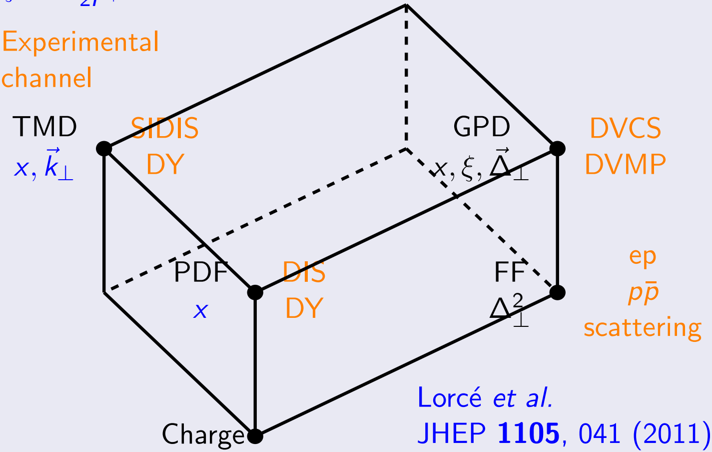
Experimental channel



Lorcé *et al.*
JHEP **1105**, 041 (2011)

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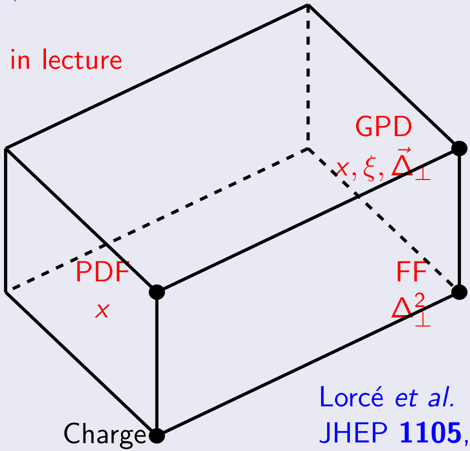
Covered in lecture



Lorcé *et al.*
JHEP **1105**, 041 (2011)

Hard probes

Reminder

Energy momentum tensor
Belinfante tensor
Form factors
Momentum sum rule
Angular momentum sum rule

Deep Inelastic Scattering
Kinematics
Structure functions
Compton tensor

Operator Product Expansion
Principle
Scaling
Definition of PDFs

Part. II
Hard probes and partonic content

Wednesday 2 Oct. 2013
10H30 - 11H30

Technical. All the words you have already heard but which were maybe left undefined.

What have we learned so far?

- The properties (**spin, charge, mass**, . . .) of the nucleon have been known for more than 80 years.
- There is no doubt today about the number of quarks and gluons, their charges and spins. Masses are getting known with an increasing accuracy.
- How can we **explain the characteristics** of the observed states in terms of those of QCD fundamental degrees of freedom?
- Heisenberg uncertainty principle and pair creation require **well-defined prescriptions** to localize quarks and gluons inside the nucleon.
- **Wigner distributions** are the suitable relativistic quantum mechanical generalizations of the 1-particle phase distribution of kinetic gaz theory.

What have we learned so far?

- Integrated Wigner distributions can be interpreted as quark space density in the **transverse plane** in an **infinite momentum frame**.
- No process has been identified so far to measure Wigner distributions, but some functions derived from Wigner distributions are **accessed experimentally**, *e.g.* :
  - Form Factors (FF),
  - Parton Distribution Functions (PDF),
  - Generalized Parton Distributions (GPD),
  - Transverse Momentum Dependent parton distributions (TMD).
- Wigner distributions provide information about quark localization and charge distribution (through form factors). Can we get more?

Spin and angular momentum densities.

Lorentz invariance in field theory (following Leader and Lorcé, arXiv:1309.4235).

- Consider an **infinitesimal** Lorentz transformation:

$$x^\mu \mapsto x'^\mu = \Lambda^\mu{}_\nu x^\nu = (\delta^\mu_\nu + \omega^\mu{}_\nu) x^\nu \text{ with } \omega_{\mu\nu} = -\omega_{\nu\mu}$$

- A family of fields $\big(\phi_n(x)\big)_n$ transforms as:

$$\phi_n(x) \mapsto \phi'_n(x) = \phi_n(x) - \frac{i}{2}\omega_{\mu\nu}(\Sigma^{\mu\nu})_n{}^m \phi_m(x)$$

where $(\Sigma^{\mu\nu})_n{}^m$ depends on the **spin** of the considered particle:

$$\begin{aligned} \text{Scalar} \quad & (\Sigma^{\mu\nu})_n{}^m = 0 \\ \text{Dirac} \quad & (\Sigma^{\mu\nu})_n{}^m = (\sigma^{\mu\nu})_n{}^m/2 \\ \text{Vector} \quad & (\Sigma^{\mu\nu})_n{}^m = i(\delta^\mu_n \eta^{\nu m} - \delta^\nu_m \eta^{\mu n}) \end{aligned}$$

where $\sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu]$ and $\eta_{\mu\nu}$ is the metric tensor.

- $(\Sigma^{\mu\nu})_n{}^m$ is **antisymmetric** w.r.t. $\mu \leftrightarrow \nu$.

Spin and angular momentum densities.

Lorentz invariance in field theory (following Leader and Lorcé, arXiv:1309.4235).

- Consider 6 **antisymmetric** generators $M^{\mu\nu}$ of Lorentz transformations:

$$i[M^{\mu\nu}, \phi_n] = (x^\mu \delta^\nu - x^\nu \delta^\mu)\phi_n - i(\Sigma^{\mu\nu})_n{}^m \phi_m$$

- The operators associated to the **spatial components** of $M^{\mu\nu}$ generate rotations about x, y and z axis:

$$\Lambda^{\mu\nu} = \left(\begin{array}{c|ccc} 1 & \omega_{01} & \omega_{02} & \omega_{03} \\ \hline -\omega_{01} & 1 & -\omega_{12} & \omega_{13} \\ -\omega_{02} & \omega_{21} & 1 & -\omega_{23} \\ -\omega_{03} & -\omega_{13} & \omega_{23} & 1 \end{array}\right) \rightarrow \begin{array}{c} M^{yz} \\ M^{zx} \\ M^{xy} \end{array}$$

- Three conserved **angular momentum operators** $J^i, (i = 1, 2, 3)$:

$$J^i = \frac{1}{2}\epsilon_{ijk} M^{jk}$$

- Operators M^{0i} generate **boosts** along x, y and z axis.

Spin and angular momentum densities.

Lorentz invariance in field theory (following Leader and Lorcé, arXiv:1309.4235).

- In case of a **Dirac particle**:

$$[\vec{J}, \psi] = -\left(\vec{r} \times \frac{\vec{\nabla}}{i}\right)\psi - \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}\psi$$

- From Noether's theorem, existence of a conserved **canonical angular momentum density**:

$$M^{\mu\nu\rho}(x) = \underbrace{x^\nu T^{\mu\rho}(x) - x^\rho T^{\mu\nu}(x)}_{\text{Orbital Angular Momentum (OAM)}} \underbrace{- i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} (\Sigma^{\nu\rho})_n{}^m \phi_m(x)}_{\text{Spin}}$$

- Fields **with spin** $\Sigma^{\mu\nu}$: $T^{\mu\nu}$ **not symmetric** w.r.t. $\mu \leftrightarrow \nu$.

Exercise II.1

Show that the energy-momentum tensor of a scalar field theory is symmetric.

Spin and angular momentum densities.

Canonical vs Belinfante tensors (following Leader and Lorcé, arXiv:1309.4235).

CEA - Saclay

Hard probes

Reminder

Energy momentum tensor

Belinfante tensor

Form factors

Momentum sum rule

Angular momentum sum rule

Deep Inelastic Scattering

Kinematics

Structure functions

Compton tensor

Operator Product Expansion

Principle

Scaling

Definition of PDFs

- Einstein field equations of **General relativity**:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi\mathcal{G} T_{\mu\nu}$$

Spin and angular momentum densities.

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- Einstein field equations of **General relativity**:

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\substack{\text{symmetric} \\ \text{by assumption}}} = 8\pi \mathcal{G} T_{\mu\nu}$$

- The metric is **supposed** symmetric w.r.t. $\mu \leftrightarrow \nu$ and covariantly constant $\nabla_\lambda g_{\mu\nu} = 0$.

Spin and angular momentum densities.

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- Einstein field equations of **General relativity**:

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{symmetric by assumption}} = \underbrace{8\pi \mathcal{G} T_{\mu\nu}}_{\text{Necessarily symmetric}}$$

- The metric is **supposed** symmetric w.r.t. $\mu \leftrightarrow \nu$ and covariantly constant $\nabla_\lambda g_{\mu\nu} = 0$.

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- The metric is **supposed** symmetric w.r.t. $\mu \leftrightarrow \nu$ and covariantly constant $\nabla_\lambda g_{\mu\nu} = 0$.
- Define the **Belinfante** energy-momentum density:

$$T^{\mu\nu}_{\mathrm{Bel.}} \equiv \frac{1}{2}(T^{\mu\nu} + T^{\nu\mu}) + \frac{1}{2}\partial_\lambda (M^{\mu\nu\lambda}_{\mathrm{spin}} + M^{\nu\mu\lambda}_{\mathrm{spin}})$$

Belinfante, Physica **6**, 887 (1939)
Rosenfeld, Mem. Acad. Roy. Belg. **18**, 6 (1940)

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- $T^{\mu\nu}_{\mathrm{Bel.}}$ **symmetric** w.r.t. $\mu \leftrightarrow \nu$, conserved $\partial_\mu T^{\mu\nu}_{\mathrm{Bel.}} = 0$ (From the conservation of **total** angular momentum, get $T^{\rho\nu} - T^{\nu\rho} = \partial_\mu M^{\mu\nu\rho}_{\mathrm{spin}}$).

Spin and angular momentum densities.

Canonical vs Belinfante tensors (following Leader and Lorcé, arXiv:1309.4235).

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- $T^{\mu\nu}_{\mathrm{Bel.}}$ **symmetric** w.r.t. $\mu \leftrightarrow \nu$, **conserved** $\partial_\mu T^{\mu\nu}_{\mathrm{Bel.}} = 0$ (From the conservation of **total** angular momentum, get $T^{\rho\nu} - T^{\nu\rho} = \partial_\mu M^{\mu\nu\rho}_{\mathrm{spin}}$).
- $T^{\mu\nu}_{\mathrm{Bel.}}$ is also **gauge invariant**.

Energy momentum form factors.

Form factor decomposition from symmetry considerations.

- In all the following, use the **Belinfante expression** of the QCD energy momentum tensor:

$$T^{\mu\nu} = \sum_q \bar{q}\gamma^{(\mu} i \overleftrightarrow{\mathrm{D}}^{\nu)} q - F_a^{\mu\lambda} F^{a\mu}{}_{\lambda} - \eta^{\mu\nu} \mathcal{L}_{\mathrm{QCD}}$$

where $a^{(\mu} b^{\nu)} = (a^\mu b^\nu + a^\nu b^\mu)/2$.

- Consider two nucleon states $|N, p\rangle$ and $|N, p'\rangle$. Define:

$$P = \frac{p + p'}{2} \text{ and } \Delta = p' - p$$

- Write the most general structure for the nucleon matrix element $\langle N, P + \Delta/2 \,|\, T^{\mu\nu} \,|\, N, P - \Delta/2\rangle$ fulfilling:
  - **Lorentz** transformation as a second rank tensor,
  - Invariance under **time reversal**,
  - Invariance under **parity transformation**,
  - **Hermiticity**.

Energy momentum form factors.

The energy momentum tensor is parameterized in terms of 3 form factors.

Hard probes

Reminder

Energy momentum tensor

Belinfante tensor

Form factors

Momentum sum rule

Angular momentum sum rule

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Definition of PDFs

- Introduce three **energy momentum form factors**:

$$\left\langle N, P+\frac{\Delta}{2}\right| T^{\mu\nu} \left| N, P-\frac{\Delta}{2}\right\rangle = \bar{u}\left(P+\frac{\Delta}{2}\right)\left[A(t)\gamma^{(\mu}P^{\nu)} + B(t)P^{(\mu}i\sigma^{\nu)\lambda}\frac{\Delta_\lambda}{2M} + \frac{C(t)}{M}(\Delta^\mu\Delta^\nu - \Delta^2\eta^{\mu\nu})\right] u\left(P-\frac{\Delta}{2}\right)$$

with $t = \Delta^2$.

Ji, Phys. Rev. Lett. **78**, 610 (1997)

Energy momentum form factors.

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Hard probes

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with $t = \Delta^2$.

Ji, Phys. Rev. Lett. **78**, 610 (1997)

- Apply **Gordon identity** $2M\gamma^\mu = \left(2P^\mu + i\sigma^{\mu\nu}\Delta_\nu\right)$

$$\left\langle N, P+\frac{\Delta}{2}\right| T^{\mu\nu} \left| N, P-\frac{\Delta}{2}\right\rangle = \bar{u}\left(P+\frac{\Delta}{2}\right)\left[A(t)\frac{P^\mu P^\nu}{M} + (A(t)+B(t))P^{(\mu}i\sigma^{\nu)\lambda}\frac{\Delta_\lambda}{2M} + \frac{C(t)}{M}(\Delta^\mu\Delta^\nu - \Delta^2\eta^{\mu\nu})\right] u\left(P-\frac{\Delta}{2}\right)$$

Energy momentum form factors.
Momentum sum rule (1/2).

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- Compute average of 4-momentum operator between nucleon states of momentum P:

$$\langle N, P \,|\mathbb{P}^\nu|\, N, P\rangle \;=\; \langle P| \int \mathrm{d}^3\vec{r}\, T^{0\nu}(\vec{r})\,|P\rangle$$

CEA - Saclay

Hard probes

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Momentum sum rule (1/2).

- Compute average of 4-momentum operator between nucleon states of momentum P:

$$\begin{aligned}
\langle N, P \,|\mathbb{P}^\nu|\, N, P\rangle &= \langle P| \int \mathrm{d}^3\vec{r}\, T^{0\nu}(\vec{r})\,|P\rangle \\
&= \lim_{\Delta\to 0} \left\langle P+\frac{\Delta}{2}\right| \int \mathrm{d}^3\vec{r}\, T^{0\nu}(\vec{r}) \left|P-\frac{\Delta}{2}\right\rangle
\end{aligned}$$

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&= \lim_{\Delta\to 0}\left\langle P+\frac{\Delta}{2}\right|\int \mathrm{d}^3\vec{r}\, e^{-i\vec{r}\cdot\vec{\Delta}}\, T^{0\nu}(\vec{0})\left|P-\frac{\Delta}{2}\right\rangle
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$$
\begin{aligned}
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&= \lim_{\Delta\to 0} \left\langle P + \frac{\Delta}{2} \right| \int \mathrm{d}^3\vec{r}\, T^{0\nu}(\vec{r}) \left| P - \frac{\Delta}{2} \right\rangle \\
&= \lim_{\Delta\to 0} \left\langle P + \frac{\Delta}{2} \right| \int \mathrm{d}^3\vec{r}\, e^{-i\vec{r}\cdot\vec{\Delta}}\, T^{0\nu}(\vec{0}) \left| P - \frac{\Delta}{2} \right\rangle \\
&= \lim_{\Delta\to 0} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \left\langle P + \frac{\Delta}{2} \right| T^{0\nu}(\vec{0}) \left| P - \frac{\Delta}{2} \right\rangle
\end{aligned}
$$

Energy momentum form factors.

Momentum sum rule (1/2).

- Compute average of 4-momentum operator between nucleon states of momentum P:

$$
\begin{aligned}
\langle N, P \,|\mathbb{P}^\nu|\, N, P\rangle &= \langle P| \int \mathrm{d}^3\vec{r}\, T^{0\nu}(\vec{r})\,|P\rangle \\
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&= \lim_{\Delta\to 0} \left\langle P + \frac{\Delta}{2} \right| \int \mathrm{d}^3\vec{r}\, e^{-i\vec{r}\cdot\vec{\Delta}}\, T^{0\nu}(\vec{0}) \left| P - \frac{\Delta}{2} \right\rangle \\
&= \lim_{\Delta\to 0} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \left\langle P + \frac{\Delta}{2} \right| T^{0\nu}(\vec{0}) \left| P - \frac{\Delta}{2} \right\rangle \\
&= (2\pi)^3 A(0) P^\nu (2P^0) \delta^{(3)}(\vec{0})
\end{aligned}
$$

Energy momentum form factors.

Momentum sum rule (1/2).

- Compute average of 4-momentum operator between nucleon states of momentum P:

$$
\begin{aligned}
\langle N, P | \mathbb{P}^\nu | N, P \rangle &= \langle P | \int \mathrm{d}^3\vec{r}\, T^{0\nu}(\vec{r}) | P \rangle \\
&= \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \right| \int \mathrm{d}^3\vec{r}\, T^{0\nu}(\vec{r}) \left| P - \frac{\Delta}{2} \right\rangle \\
&= \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \right| \int \mathrm{d}^3\vec{r}\, e^{-i\vec{r}\cdot\vec{\Delta}}\, T^{0\nu}(\vec{0}) \left| P - \frac{\Delta}{2} \right\rangle \\
&= \lim_{\Delta \to 0} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \left\langle P + \frac{\Delta}{2} \right| T^{0\nu}(\vec{0}) \left| P - \frac{\Delta}{2} \right\rangle \\
&= A(0) P^\nu (2P^0)(2\pi)^3 \delta^{(3)}(\vec{0}) \\
&= A(0) P^\nu \langle P | P \rangle
\end{aligned}
$$

Energy momentum form factors.

Momentum sum rule (2/2).

- Average of 4-momentum operator between nucleon states of momentum P:

$$\frac{\langle N, P \,|\mathbb{P}^\nu|\, N, P \rangle}{\langle P \,|\, P \rangle} = A(0) P^\nu$$

- Energy momentum **conservation**: $A(0) = 1$.
- **Sum rule** for quark and gluon contributions A_q and A_g:

$$A_q(0) + A_g(0) = 1$$

Energy momentum form factors.

Angular momentum sum rule (1/4).

- Average of J^3 between nucleon states in **rest frame**:

$$\left\langle P \left| J^3 \right| P \right\rangle = \left\langle P \left| \int \mathrm{d}^3\vec{r} \left[r^1 T^{02}(\vec{r}) - r^2 T^{01}(\vec{r}) \right] \right| P \right\rangle$$

Energy momentum form factors.

Angular momentum sum rule (1/4).

- Average of J^3 between nucleon states in **rest frame**:

$$\langle P \,|J^3|\, P \rangle = \left\langle P \left| \int \mathrm{d}^3\vec{r} \left[r^1 T^{02}(\vec{r}) - r^2 T^{01}(\vec{r}) \right] \right| P \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \left| \int \mathrm{d}^3\vec{r}\, r^i T^{0j}(\vec{r}) \right| P - \frac{\Delta}{2} \right\rangle$$

Energy momentum form factors.

Angular momentum sum rule (1/4).

- Average of J^3 between nucleon states in **rest frame**:

$$\left\langle P \left| J^3 \right| P \right\rangle = \left\langle P \left| \int \mathrm{d}^3\vec{r}\, \left[r^1\, T^{02}(\vec{r}) - r^2\, T^{01}(\vec{r}) \right] \right| P \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \right| \int \mathrm{d}^3\vec{r}\, r^i\, T^{0j}(\vec{r}) \left| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \right| \int \mathrm{d}^3\vec{r}\, r^i\, e^{-i\vec{r}\cdot\vec{\Delta}}\, T^{0j}(\vec{0}) \left| P - \frac{\Delta}{2} \right\rangle$$

Energy momentum form factors.

Angular momentum sum rule (1/4).

- Average of J^3 between nucleon states in **rest frame**:

$$\left\langle P \left| J^3 \right| P \right\rangle = \left\langle P \left| \int \mathrm{d}^3\vec{r} \left[r^1 T^{02}(\vec{r}) - r^2 T^{01}(\vec{r}) \right] \right| P \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \left| \int \mathrm{d}^3\vec{r}\, r^i T^{0j}(\vec{r}) \right| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \left| \int \mathrm{d}^3\vec{r}\, r^i e^{-i\vec{r}\cdot\vec{\Delta}} T^{0j}(\vec{0}) \right| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \left| \int \mathrm{d}^3\vec{r} \frac{1}{-i} \frac{\partial}{\partial \Delta^i} e^{-i\vec{r}\cdot\vec{\Delta}} T^{0j}(\vec{0}) \right| P - \frac{\Delta}{2} \right\rangle$$

Energy momentum form factors.

Angular momentum sum rule (1/4).

- Average of J^3 between nucleon states in **rest frame**:

$$\langle P | J^3 | P \rangle = \left\langle P \left| \int \mathrm{d}^3\vec{r} \left[r^1 T^{02}(\vec{r}) - r^2 T^{01}(\vec{r}) \right] \right| P \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \right| \int \mathrm{d}^3\vec{r}\, r^i T^{0j}(\vec{r}) \left| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \right| \int \mathrm{d}^3\vec{r}\, r^i e^{-i\vec{r}\cdot\vec{\Delta}} T^{0j}(\vec{0}) \left| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \right| \int \mathrm{d}^3\vec{r} \frac{1}{-i} \frac{\partial}{\partial \Delta^i} e^{-i\vec{r}\cdot\vec{\Delta}} T^{0j}(\vec{0}) \left| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left[i \frac{\partial}{\partial \Delta^i} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \right] \left\langle P + \frac{\Delta}{2} \right| T^{0j}(\vec{0}) \left| P - \frac{\Delta}{2} \right\rangle$$

Hard probes

Reminder

Energy momentum tensor
- Belinfante tensor
- Form factors
- Momentum sum rule
- Angular momentum sum rule

Deep Inelastic Scattering
- Kinematics
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- Compton tensor

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CEA - Saclay

Hard probes

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Energy momentum form factors.

Angular momentum sum rule (1/4).

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$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \left| \int \mathrm{d}^3\vec{r}\, r^i T^{0j}(\vec{r}) \right| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \left| \int \mathrm{d}^3\vec{r}\, r^i e^{-i\vec{r}\cdot\vec{\Delta}} T^{0j}(\vec{0}) \right| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left\langle P + \frac{\Delta}{2} \left| \int \mathrm{d}^3\vec{r} \frac{1}{-i} \frac{\partial}{\partial \Delta^i} e^{-i\vec{r}\cdot\vec{\Delta}} T^{0j}(\vec{0}) \right| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} \left[i \frac{\partial}{\partial \Delta^i} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \right] \left\langle P + \frac{\Delta}{2} \left| T^{0j}(\vec{0}) \right| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta \to 0} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \left(-i \frac{\partial}{\partial \Delta^i} \right) \Bigg[\left(A(t) + B(t) \right)$$

$$\times \bar{u}\left(P + \frac{\Delta}{2} \right) P^{(0} i\sigma^{j)\lambda} \frac{\Delta_\lambda}{2M} u\left(P - \frac{\Delta}{2} \right) + \mathcal{O}(\Delta^0) + \mathcal{O}(\Delta^2) \Bigg]$$

Energy momentum form factors.

Angular momentum sum rule (1/4).

- Average of J^3 between nucleon states in **rest frame**:

$$\left\langle P \left| J^3 \right| P \right\rangle = \left\langle P \left| \int \mathrm{d}^3\vec{r} \left[r^1 T^{02}(\vec{r}) - r^2 T^{01}(\vec{r}) \right] \right| P \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta\to 0} \left\langle P + \frac{\Delta}{2} \left| \int \mathrm{d}^3\vec{r}\, r^i T^{0j}(\vec{r}) \right| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta\to 0} \left\langle P + \frac{\Delta}{2} \left| \int \mathrm{d}^3\vec{r}\, r^i e^{-i\vec{r}\cdot\vec{\Delta}} T^{0j}(\vec{0}) \right| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta\to 0} \left\langle P + \frac{\Delta}{2} \left| \int \mathrm{d}^3\vec{r} \frac{1}{-i} \frac{\partial}{\partial \Delta^i} e^{-i\vec{r}\cdot\vec{\Delta}} T^{0j}(\vec{0}) \right| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta\to 0} \left[i \frac{\partial}{\partial \Delta^i} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \right] \left\langle P + \frac{\Delta}{2} \left| T^{0j}(\vec{0}) \right| P - \frac{\Delta}{2} \right\rangle$$

$$= \epsilon_{ij3} \lim_{\Delta\to 0} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \left(-i \frac{\partial}{\partial \Delta^i} \right) \left[(A(t) + B(t)) \right.$$

$$\left. \times \bar{u}\left(P + \frac{\Delta}{2}\right) P^{(0} i\sigma^{j)\lambda} \frac{\Delta_\lambda}{2M} u\left(P - \frac{\Delta}{2}\right) + \mathcal{O}(\Delta^0) + \mathcal{O}(\Delta^2) \right]$$

Energy momentum form factors.

Angular momentum sum rule (2/4).

- Average of J^3 between nucleon states in **rest frame**:

$$\langle P \left| J^3 \right| P \rangle = \epsilon_{ij3} \lim_{\Delta \to 0} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \left(-i \frac{\partial}{\partial \Delta^i} \right) \Bigg[(A(t) + B(t))$$

$$\times \bar{u}\left(P + \frac{\Delta}{2}\right) P^{(0} i \sigma^{j)\lambda} \frac{\Delta_\lambda}{2M} u\left(P - \frac{\Delta}{2}\right) + \mathcal{O}(\Delta^0) + \mathcal{O}(\Delta^2) \Bigg]$$

$$= \epsilon_{ij3} (2\pi)^3 \delta^{(3)}(\vec{0}) \left[A(0) + B(0) \right] \bar{u}(P) \frac{\left[P^0 \sigma^{ji} + P^j \sigma^{0i} \right]}{2M} u(P)$$

Energy momentum form factors.

Angular momentum sum rule (2/4).

- Average of J^3 between nucleon states in **rest frame**:

$$\langle P | J^3 | P \rangle = \epsilon_{ij3} \lim_{\Delta \to 0} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \left(-i \frac{\partial}{\partial \Delta^i} \right) \Big[(A(t) + B(t)) \times \bar{u}\left(P + \frac{\Delta}{2}\right) P^{(0} i \sigma^{j)\lambda} \frac{\Delta_\lambda}{2M} u\left(P - \frac{\Delta}{2}\right) + \mathcal{O}(\Delta^0) + \mathcal{O}(\Delta^2) \Big]$$

$$= \epsilon_{ij3} (2\pi)^3 \delta^{(3)}(\vec{0}) \left[A(0) + B(0) \right] \bar{u}(P) \frac{\left[P^0 \sigma^{ji} + P^j \sigma^{0i} \right]}{2M} u(P)$$

$$= (2\pi)^3 \delta^{(3)}(\vec{0}) \left[A(0) + B(0) \right] \bar{u}(P) \frac{-2M\sigma^{12}}{2M} u(P)$$

Energy momentum form factors.

Angular momentum sum rule (2/4).

Hard probes

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Energy momentum tensor
- Belinfante tensor
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- Momentum sum rule
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Deep Inelastic Scattering
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- Structure functions
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- Average of J^3 between nucleon states in **rest frame**:

$$\left\langle P \left| J^3 \right| P \right\rangle = \epsilon_{ij3} \lim_{\Delta \to 0} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \left(-i \frac{\partial}{\partial \Delta^i} \right) \Bigg[\big(A(t) + B(t) \big) \times \bar{u} \left(P + \frac{\Delta}{2} \right) P^{(0} i \sigma^{j)\lambda} \frac{\Delta_\lambda}{2M} u \left(P - \frac{\Delta}{2} \right) + \mathcal{O}(\Delta^0) + \mathcal{O}(\Delta^2) \Bigg]$$

$$= \epsilon_{ij3} (2\pi)^3 \delta^{(3)}(\vec{0}) \big[A(0) + B(0) \big] \bar{u}(P) \frac{\big[P^0 \sigma^{ji} + P^j \sigma^{0i} \big]}{2M} u(P)$$

$$= (2\pi)^3 \delta^{(3)}(\vec{0}) \big[A(0) + B(0) \big] \bar{u}(P) \frac{-2M\sigma^{12}}{2M} u(P)$$

$$= (2\pi)^3 \delta^{(3)}(\vec{0}) \big[A(0) + B(0) \big] \frac{1}{2} \bar{u}(P) u(P)$$

Energy momentum form factors.

Angular momentum sum rule (2/4).

- Average of J^3 between nucleon states in **rest frame**:

$$\left\langle P \left| J^3 \right| P \right\rangle = \epsilon_{ij3} \lim_{\Delta \to 0} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \left(-i \frac{\partial}{\partial \Delta^i} \right) \Big[(A(t) + B(t))$$

$$\times \bar{u} \left(P + \frac{\Delta}{2} \right) P^{(0} i \sigma^{j)\lambda} \frac{\Delta_\lambda}{2M} u \left(P - \frac{\Delta}{2} \right) + \mathcal{O}(\Delta^0) + \mathcal{O}(\Delta^2) \Big]$$

$$= \epsilon_{ij3} (2\pi)^3 \delta^{(3)}(\vec{0}) \left[A(0) + B(0) \right] \bar{u}(P) \frac{\left[P^0 \sigma^{ji} + P^j \sigma^{0i} \right]}{2M} u(P)$$

$$= (2\pi)^3 \delta^{(3)}(\vec{0}) \left[A(0) + B(0) \right] \bar{u}(P) \frac{-2M\sigma^{12}}{2M} u(P)$$

$$= (2\pi)^3 \delta^{(3)}(\vec{0}) \left[A(0) + B(0) \right] \frac{1}{2} \bar{u}(P) u(P)$$

$$= \frac{1}{2} \left[A(0) + B(0) \right] (2\pi)^3 \delta^{(3)}(\vec{0}) 2M$$

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Energy momentum form factors.

Angular momentum sum rule (2/4).

- Average of J^3 between nucleon states in **rest frame**:

$$\langle P \,|J^3|\, P\rangle = \epsilon_{ij3} \lim_{\Delta \to 0} (2\pi)^3 \delta^{(3)}(\vec{\Delta}) \left(-i \frac{\partial}{\partial \Delta^i} \right) \Bigg[(A(t) + B(t))$$

$$\times \bar{u}\left(P + \frac{\Delta}{2}\right) P^{(0} i\sigma^{j)\lambda} \frac{\Delta_\lambda}{2M} u\left(P - \frac{\Delta}{2}\right) + \mathcal{O}(\Delta^0) + \mathcal{O}(\Delta^2) \Bigg]$$

$$= \epsilon_{ij3} (2\pi)^3 \delta^{(3)}(\vec{0}) \left[A(0) + B(0) \right] \bar{u}(P) \frac{\left[P^0 \sigma^{ji} + P^j \sigma^{0i} \right]}{2M} u(P)$$

$$= (2\pi)^3 \delta^{(3)}(\vec{0}) \left[A(0) + B(0) \right] \bar{u}(P) \frac{-2M\sigma^{12}}{2M} u(P)$$

$$= (2\pi)^3 \delta^{(3)}(\vec{0}) \left[A(0) + B(0) \right] \frac{1}{2} \bar{u}(P) u(P)$$

$$= \frac{1}{2} \left[A(0) + B(0) \right] (2\pi)^3 \delta^{(3)}(\vec{0}) 2M$$

$$= \frac{1}{2} \left[A(0) + B(0) \right] \langle P \,|\, P \rangle$$

Energy momentum form factors.

Angular momentum sum rule (3/4).

- Average of J^3 between nucleon states in **rest frame**:

$$\frac{\langle P | J^3 | P \rangle}{\langle P | P \rangle} = \frac{1}{2}\left[A(0) + B(0)\right]$$

- **Angular momentum** conservation: $A(0) + B(0) = 1$.
- From **energy** conservation get $B(0) = 0$.
- **Sum rules** for quarks and gluons:

$$J_q = \frac{1}{2}\left[A_q(0) + B_q(0)\right] \text{ (idem } J_g\text{) and } \frac{1}{2} = J_q + J_g$$

- Decomposition into **spin and orbital parts**:

$$\underbrace{\frac{1}{2}}_{\text{nucleon spin}} = \sum_q \underbrace{\underbrace{\left(\frac{1}{2}\Delta q\right.}_{\text{Quark spin contribution}} + \underbrace{\left. L_q\right)}_{\text{Quark OAM contribution}}}_{\frac{1}{2}\left[A_q(0)+B_q(0)\right]} + \underbrace{J_g}_{\text{Gluon contribution}}$$

See Ji sum rule.

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Energy momentum form factors.

Angular momentum sum rule (4/4).

- Question: How to access **experimentally** the energy momentum form factors?

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Energy momentum form factors.

Angular momentum sum rule (4/4).

- Question: How to access **experimentally** the energy momentum form factors?
- Spin 2 probe: *graviton*?! **Hopeless!**

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Energy momentum form factors.

Angular momentum sum rule (4/4).

- Question: How to access **experimentally** the energy momentum form factors?
- Spin 2 probe: *graviton*?! **Hopeless!**
- Consider a **light-like** vector n:

$$\left\langle P+\frac{\Delta}{2}\left|T_q^{\mu\nu}(0)\right|P-\frac{\Delta}{2}\right\rangle n_\mu n_\nu = \left\langle P+\frac{\Delta}{2}\right|\bar{q}\gamma^{(\mu}i\overleftrightarrow{\mathrm{D}}^{\nu)}q-\eta^{\mu\nu}\mathcal{L}_{\mathrm{QCD}}\left|P-\frac{\Delta}{2}\right\rangle n_\mu n_\nu$$

Energy momentum form factors.

Angular momentum sum rule (4/4).

- Question: How to access **experimentally** the energy momentum form factors?
- Spin 2 probe: *graviton*?! **Hopeless!**
- Consider a **light-like** vector n:

$$\left\langle P+\frac{\Delta}{2}\left|T_q^{\mu\nu}(0)\right|P-\frac{\Delta}{2}\right\rangle n_\mu n_\nu = \left\langle P+\frac{\Delta}{2}\left|\bar{q}\gamma^{(\mu}i\overleftrightarrow{\mathrm{D}}^{\nu)}q-\eta^{\mu\nu}\mathcal{L}_{\mathrm{QCD}}\right|P-\frac{\Delta}{2}\right\rangle n_\mu n_\nu$$

- Terms symmetric w.r.t. $\mu \leftrightarrow \nu$ vanish after contraction with $n_\mu n_\nu$ (notation $\Delta^+ \equiv -2\xi P^+$):

$$\frac{1}{P^{+2}}\left\langle P+\frac{\Delta}{2}\left|\bar{q}(0)\gamma^{(\mu}i\overleftrightarrow{\mathrm{D}}^{\nu)}q(0)\right|P-\frac{\Delta}{2}\right\rangle n_\mu n_\nu = \bar{u}\left(P+\frac{\Delta}{2}\right) \times\left[\frac{A_q(t)+4\xi^2 C_q(t)}{M}+\left(A_q(t)+B_q(t)\right)i\frac{\sigma^{+\lambda}\Delta_\lambda}{2MP^+}\right]u\left(P-\frac{\Delta}{2}\right)$$

Energy momentum form factors.

Angular momentum sum rule (4/4).

- Question: How to access **experimentally** the energy momentum form factors?
- Spin 2 probe: *graviton*?! **Hopeless!**
- Consider a **light-like** vector n:

$$\left\langle P + \frac{\Delta}{2} \middle| T_q^{\mu\nu}(0) \middle| P - \frac{\Delta}{2} \right\rangle n_\mu n_\nu \quad = \quad \left\langle P + \frac{\Delta}{2} \middle| \bar{q}\gamma^{(\mu} i \overleftrightarrow{\mathrm{D}}^{\nu)} q - \eta^{\mu\nu} \mathcal{L}_{\mathrm{QCD}} \middle| P - \frac{\Delta}{2} \right\rangle n_\mu n_\nu$$

- Terms symmetric w.r.t. $\mu \leftrightarrow \nu$ vanish after contraction with $n_\mu n_\nu$ (notation $\Delta^+ \equiv -2\xi P^+$):

$$\frac{1}{P^{+2}} \left\langle P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^{(\mu} i \overleftrightarrow{\mathrm{D}}^{\nu)} q(0) \middle| P - \frac{\Delta}{2} \right\rangle n_\mu n_\nu \quad = \quad \bar{u}\left(P + \frac{\Delta}{2}\right)$$

$$\times \left[\frac{A_q(t) + 4\xi^2 C_q(t)}{M} + \left(A_q(t) + B_q(t)\right) i \frac{\sigma^{+\lambda}\Delta_\lambda}{2MP^+} \right] u\left(P - \frac{\Delta}{2}\right)$$

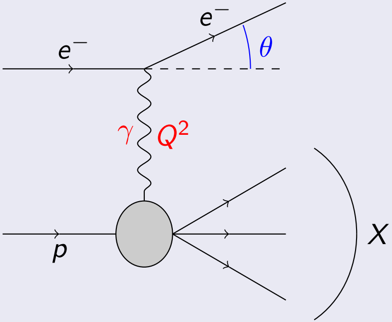
- Experimental access to this matrix element?

See Ji sum rule.

Inelastic scattering.

Kinematics and standard notations.

Kinematics of inelastic scattering on the nucleon



$$
\begin{aligned}
q &\equiv k - k', \\
Q^2 &\equiv -q^2, \\
\nu &\equiv \frac{p \cdot q}{M}, \\
y &\equiv \frac{p \cdot q}{p \cdot k}, \\
x_B &\equiv \frac{Q^2}{2p \cdot q}, \\
\omega &\equiv \frac{1}{x_B}, \\
W^2 &\equiv (p+q)^2, \\
s &\equiv (p+k)^2.
\end{aligned}
$$

In the target rest frame $\theta \in [0, \pi]$ and:

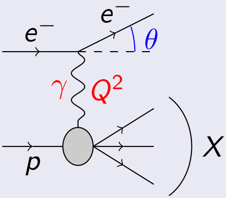
$$p \equiv (M, \vec{0}), \quad k \equiv (E, \vec{k}), \quad k' \equiv (E', \vec{k'}).$$

Inelastic scattering.

Kinematics, standard notations, orders of magnitude, variable ranges.

Kinematics of inelastic scattering on the nucleon

$$q \equiv k - k', \quad Q^2 \equiv -q^2, \quad \nu \equiv \frac{p \cdot q}{M}, \quad y \equiv \frac{p \cdot q}{p \cdot k},$$



$$x_B \equiv \frac{Q^2}{2p \cdot q}, \quad \omega \equiv \frac{1}{x_B}, \quad W^2 \equiv (p+q)^2, \quad s \equiv (p+k)^2.$$

Exercise II.2

Show that $0 \leq x_B \leq 1$ and that $x_B = 1$ corresponds to the case of elastic scattering. Explain why ν and y are respectively called *energy loss* and *fractional energy loss*. Prove that $\nu \geq 0$ and $0 \leq y \leq 1$.

Deep Inelastic scattering.

Amplitude at Born order.

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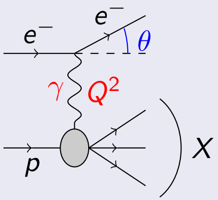
Deep Inelastic Scattering in the Bjorken limit

Deep inelastic scattering is the scattering process with the following kinematic restrictions:

- $Q^2 \gg M^2$ (Deep),
- $W^2 \gg M^2$ (Inelastic).

Consider DIS in the **Bjorken limit**: finite x_B and $Q^2 \to \infty$.

Amplitude $\mathcal{M}(eN \to eX)$ at **Born order**



$$\mathcal{M}(eN \to eX) = \bar{u}(k')\gamma^\mu u(k)\frac{e^2}{q^2} \left\langle X \left| J_\mu^{\mathrm{e.m.}}(0) \right| N, p \right\rangle$$

Deep Inelastic scattering.

The amplitude at Born order depends on the Fourier transform of a commutator of electromagnetic currents between nucleon states.

- **Unpolarized** deep inelastic scattering cross section:

$$E' \frac{\mathrm{d}\sigma_{eN}}{\mathrm{d}^3\vec{k}'} = \frac{1}{32\pi^3(s - M^2)} \frac{e^2}{q^4} 4\pi L^{\mu\nu} W_{\mu\nu}$$

where $L^{\mu\nu} = 2(k^\mu k'^\nu + k^\nu k'^\mu - \eta^{\mu\nu} k \cdot k')$ is the **leptonic** tensor and $W_{\mu\nu}$ is the **hadronic** tensor:

$$\equiv \frac{1}{4\pi} \int \mathrm{d}X (2\pi)^4 \delta^{(4)}(p + q - p_X) \left\langle p \left| J_\mu^{\mathrm{e.m.}}(0) \right| p_X \right\rangle \left\langle p_X \left| J_\nu^{\mathrm{e.m.}}(0) \right| p \right\rangle$$

Deep Inelastic scattering.

The amplitude at Born order depends on the Fourier transform of a commutator of electromagnetic currents between nucleon states.

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$$= \frac{1}{4\pi} \int \mathrm{d}^4 y \, e^{iq\cdot y} \left\langle N, p \left| J_\mu^{\mathrm{e.m.}}(y) J_\nu^{\mathrm{e.m.}}(0) \right| N, p \right\rangle$$

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$$= \frac{1}{4\pi} \int \mathrm{d}^4 y \, e^{iq\cdot y} \left\langle N, p \left| [J^{\mathrm{e.m.}}_\mu(y), J^{\mathrm{e.m.}}_\nu(0)] \right| N, p \right\rangle \text{ since } M_X^2 \geq M^2.$$

Deep Inelastic scattering.

Aside: the Bjorken limit is the light-cone limit.

- **Spacetime region** probed by the Bjorken limit?

$$W_{\mu\nu} = \frac{1}{4\pi} \int \mathrm{d}^4 y \, e^{iq \cdot y} \left\langle N, p \middle| [J_\mu^{\text{e.m.}}(y), J_\nu^{\text{e.m.}}(0)] \middle| N, p \right\rangle$$

- From **causality**, $y^2 \geq 0$.
- In the lab. frame $\nu = q^0$ and $|\vec{q}| = \nu\sqrt{1 - q^2/\nu^2}$
- At fixed $x_B = -q^2/(2M\nu)$ and large q^2, ν:

$$q \cdot y = \nu \left[y^0 - \frac{\vec{q} \cdot \vec{y}}{|\vec{q}|} - M x_B \frac{\vec{q} \cdot \vec{y}}{|\vec{q}|} + \mathcal{O}\left(\frac{1}{\nu} \right) \right]$$

- $q \cdot y$ is **kept finite** with:

$$y^0 - \frac{\vec{q} \cdot \vec{y}}{|\vec{q}|} = \mathcal{O}\left(\frac{1}{\nu} \right) \text{ and } \frac{\vec{q} \cdot \vec{y}}{|\vec{q}|} = \mathcal{O}\left(\frac{1}{x_B} \right)$$

- From $(\vec{q} \cdot \vec{y}/|\vec{q}|)^2 \leq \vec{y}^2$, obtain:

$$0 \leq y^2 \leq \frac{\text{Const.}}{-q^2} \quad \Rightarrow \textbf{Light cone physics!}$$

Deep Inelastic scattering.

The information about hadron structure accessible through DIS is contained in 2 structure functions.

- In DIS all the information about the hadron structure is contained in the **hadronic** tensor $W_{\mu\nu}$:

$$W_{\mu\nu} = \frac{1}{4\pi} \int \mathrm{d}^4 y \, e^{iq \cdot y} \left\langle N, p \left| [J_\mu^{\mathrm{e.m.}}(y), J_\nu^{\mathrm{e.m.}}(0)] \right| N, p \right\rangle$$

- Parameterize the hadronic tensor taking into account:
  - Parity invariance,
  - Time reversal invariance,
  - Hermiticity,
  - Current conservation.

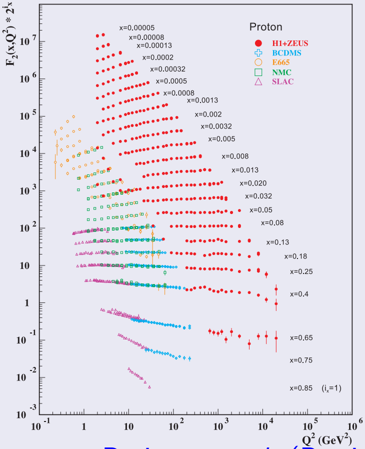
$$\begin{aligned} W_{\mu\nu} &= -F_1(x_B, Q^2) \left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \\ &\quad + \frac{F_2(x_B, Q^2)}{M\nu} \left(p_\mu - q_\mu \frac{p \cdot q}{q^2} \right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2} \right) \end{aligned}$$

- Experimentally F_1 and F_2 have a **weak dependence** on Q^2 in the valence region: **Bjorken scaling**.

Deep Inelastic scattering.

Measurements of F_2 and scaling violations.

World measurements of F_2



Beringer *et al.* (Particle Data Group), Phys. Rev. **D86**, 010001 (2012)

Deep Inelastic scattering.

Compton tensor.

- In DIS all the information about the hadron structure is contained in the **hadronic** tensor $W_{\mu\nu}$:

$$W_{\mu\nu} = \frac{1}{4\pi} \int \mathrm{d}^4 y \, e^{iq \cdot y} \left\langle N, p \middle| [J_\mu^{\mathrm{e.m.}}(y), J_\nu^{\mathrm{e.m.}}(0)] \middle| N, p \right\rangle$$

- Define the **Compton tensor** $\mathcal{T}_{\mu\nu}$ by:

$$\mathcal{T}_{\mu\nu} = \frac{i}{4\pi} \int \mathrm{d}^4 y \, e^{iq \cdot y} \left\langle N, p \middle| T\Big(J_\mu^{\mathrm{e.m.}}(y) J_\nu^{\mathrm{e.m.}}(0) \Big) \middle| N, p \right\rangle$$

- **Unitarity** of S-matrix ($S^\dagger S = 1$) with $S = 1 + iT$:

$$-i(T - T^\dagger) = T^\dagger T \qquad \text{(Optical theorem)}$$

- Sandwich between **2-particle states** $|p_1, p_2\rangle$:

$$\left\langle p_1, p_2 \middle| -i(T - T^\dagger) \middle| p_1, p_2 \right\rangle = \sum_X \left\langle p_1, p_2 \middle| T^\dagger \middle| X \right\rangle \left\langle X \middle| T \middle| p_1, p_2 \right\rangle$$

- Hadronic and Compton tensors are related:

$$W_{\mu\nu} = 2 \Im \mathcal{T}_{\mu\nu}$$

Deep Inelastic scattering.

Analytic properties of the Compton tensor and its structure functions.

- **Same symmetry properties** for $\mathcal{T}_{\mu\nu}$ and $W_{\mu\nu}$.
- Parameterize $\mathcal{T}_{\mu\nu}$ as:

$$\begin{aligned}\mathcal{T}_{\mu\nu} &= -T_1(x_B, Q^2)\left(\eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \\ &\quad + \frac{T_2(x_B, Q^2)}{M\nu}\left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right)\end{aligned}$$

- $\mathcal{T}_{\mu\nu}$ enjoys analytic properties: *The scattering amplitudes are the real boundary values of* **analytic functions of the Mandelstam variables** s, t, u *regarded as complex variables, with only such* **singularities as are demanded by the unitarity equations**.

Collins, *An Introduction to Regge Theory and High-Energy Physics*, CUP 1977

- **Discontinuity** for $1/x_B (= \omega) > 1$:

$$F_i(\omega) = 2\Im T_i(\omega + i0^+) \text{ for } i = 1, 2$$

Operator Product Expansion (OPE).

Principle.

- Products of fields at same position are generally **singular**.

See more.

- The product of two local fields $A(y)$ and $B(0)$ may be expressed in terms of **local operators** O_i such that:

$$\lim_{y \to 0} A(y)B(0) = \sum_i c_i(y) O_i(0)$$

See more.

Wilson, Phys. Rev. **179**, 1499 (1969)

- The OPE has the following properties:
  - It is an **equality between operators**: for all $|\alpha\rangle$ and $|\beta\rangle$

$$\lim_{y \to 0} \langle \alpha | A(y)B(0) | \beta \rangle = \sum_i c_i(y) \langle \alpha | O_i(0) | \beta \rangle$$

  - Each operator O_i has the **quantum numbers** of AB.
  - The **singular behavior** is contained in the **Wilson coefficients** c_i (dim = **mass dimension**):

$$c_i(y) \propto |y|^{\mathrm{dim}(O_i) - \mathrm{dim}(A) - \mathrm{dim}(B)} \text{ (up to logs)}$$

Operator Product Expansion.

Momentum representation and light-cone expansion.

- Switch to **momentum representation**:

$$\int \mathrm{d}^4y\, e^{iq\cdot y} A(y)B(0) = \sum_{i,n} \int \mathrm{d}^4y\, e^{iq\cdot y} c_{n,i}(q^2) O_{n,i}(0) \text{ when } q \to \infty$$

- The **Bjorken regime** corresponds to small y^2 (and not small y^0, y^1, y^2 and y^3).

$$A(y)B(0) = \sum_i c_{n,i}(y^2) y_{\mu_1} \ldots y_{\mu_n} O^{\mu_1 \ldots \mu_n}_{n,i}(0) \text{ when } y^2 \to 0$$

Operator Product Expansion on the light-cone.

Fourier transforming light-cone OPE is expanding in powers of $1/x_B$.

- Sandwich light-cone OPE between nucleon states $|p\rangle$:

$$\langle p| \int \mathrm{d}^4 y \, e^{iq\cdot y} A(y) B(0) \, |p\rangle$$

$$= \sum_{i,n} \left\langle p \left| O_{n,i}^{\mu_1 \ldots \mu_n}(0) \right| p \right\rangle \int \mathrm{d}^4 y \, e^{iq\cdot y} y_{\mu_1} \ldots y_{\mu_n} c_{n,i}(y^2)$$

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$$= \sum_{i,n} \langle p\,| O_{n,i}(0) |\, p\rangle \left(\frac{1}{-i} p_\mu \frac{\partial}{\partial q_\mu} \right)^n \int \mathrm{d}^4y\, e^{iq\cdot y} c_{n,i}(y^2)$$

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$$= \sum_n \frac{1}{x_B^n} \sum_i \langle p \,| O_{n,i}(0) |\, p \rangle \, i^n Q^{2n} \frac{\mathrm{d}^n}{\mathrm{d}(Q^2)^n} c_{n,i}(Q^2)$$

Operator Product Expansion on the light-cone.

Definition of twist. Scaling and leading twist.

- **Scaling behavior** of Wilson coefficients:

$$c_{n,i}(y^2) \propto \left(\frac{1}{y^2}\right)^{\frac{6+n-\mathrm{dim}(O_{n,i})}{2}}$$

because $\mathrm{dim}(J_\mu^{\mathrm{e.m.}} J_\nu^{\mathrm{e.m.}}) = 6$.

- **Fourier transform**:

$$c_{n,i}(Q^2) \propto \left(Q^2\right)^{\frac{6+n-\mathrm{dim}(O_{n,i})-4}{2}}$$

- Relevant variable: **twist** $\tau = \mathrm{dim}(O_{n,i}) - n$

Gross and Treiman, Phys. Rev. **D4**, 1059 (1971)

- At **leading twist** $\tau = 2$ light-cone expansion of Compton tensor does not depend on Q^2 any more (up to logs).
- Recover **Bjorken scaling** from field theory!

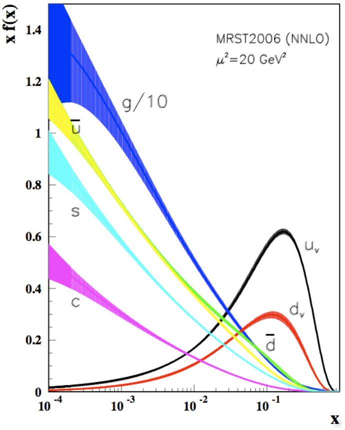
Operator Product Expansion and scaling violations.

Logarithmic corrections to Bjorken scaling can be computed with the OPE.

- **Factorization** between **short distance, process dependent** properties in Wilson coefficients and **large distance, universal** properties in operators.
- This result can be obtained by carefully controlling the **mass singularities** in Green functions.
- The limits of **vanishing masses** and **large momentum transfer** Q give rise to the same effects.
- The Operators O_i should be renormalized as products of local fields, and depends on a **renormalization scale** μ.
- Since structure functions are observable, the Wilson coefficients c_i also depend on this scale μ so as to compensate the μ-dependence of the operators O_i.
- These corrections can be computed in **perturbative QCD**, contribute with $\log Q^2/\mu^2$ terms, and are described by DGLAP equations.

Operator Product Expansion and scaling violations.

Extractions of PDFs from experimental data.



1. Choose a functional form for PDFs at a **low scale** μ_0 with **free parameters**.
2. Use DGLAP equation to **evolve** the PDFs from the low scale μ_0 to a scale $\mu \simeq Q$ where data are available.
3. At this scale Q compute structure functions and compare to the data.
4. Repeat for all data sets.
5. **Adjust free parameters**, typically by χ^2-fitting.

Operator Product Expansion on the light-cone.

What are the leading twist operators?

- Reminder: In OPE take **all** operators with the **same quantum numbers** as original fields.
- In QCD: six **towers** of **twist-2** operators forming totally symmetric representations of the Lorentz group:

$$
\begin{aligned}
O_q^{\mu\mu_1\ldots\mu_n} &= \bar{q}\gamma^{(\mu} i \overleftrightarrow{D}^{\mu_1} \ldots i \overleftrightarrow{D}^{\mu_n)} q, \\
\tilde{O}_q^{\mu\mu_1\ldots\mu_n} &= \bar{q}\gamma^{(\mu}\gamma_5 i \overleftrightarrow{D}^{\mu_1} \ldots i \overleftrightarrow{D}^{\mu_n)} q, \\
O_{qT}^{\mu\nu\mu_1\ldots\mu_n} &= \bar{q}\sigma^{\mu(\nu} i \overleftrightarrow{D}^{\mu_1} \ldots i \overleftrightarrow{D}^{\mu_n)} q, \\
O_g^{\mu\mu_1\ldots\mu_n\nu} &= F^{(\mu\lambda} i \overleftrightarrow{D}^{\mu_1} \ldots i \overleftrightarrow{D}^{\mu_n} F_\lambda{}^{\nu)}, \\
\tilde{O}_g^{\mu\mu_1\ldots\mu_n\nu} &= -iF^{(\mu\lambda} i \overleftrightarrow{D}^{\mu_1} \ldots i \overleftrightarrow{D}^{\mu_n} \tilde{F}_\lambda{}^{\nu)}, \\
O_{gT}^{\mu\nu\mu_1\ldots\mu_n\lambda\rho} &= F^{(\mu\lambda} i \overleftrightarrow{D}^{\mu_1} \ldots i \overleftrightarrow{D}^{\mu_n} F^{\nu)\rho}.
\end{aligned}
$$

- Compton tensor also describe **two-photon processes** $\Rightarrow$ same operators appear in the description of DVCS.

Operator Product Expansion on the light-cone.

Interpretation of twist = mass dimension - spin.

- Reminder: the OPE involve matrix elements such as $\left\langle p \left| O^{\mu_1 \ldots \mu_n}_{n,i}(0) \right| p \right\rangle$.
- The operator $O_{n,i}(0)$ can be assumed to be **completely symmetric** since its matrix element between nucleon states with **same momentum** p is proportional to $p^{\mu_1} \ldots p^{\mu_n}$.
- The operator $O_{n,i}(0)$ can be assumed to be **traceless**: the trace term will come with a $\eta^{\mu_i \mu_j}$ factor, which contributes to the matrix element as $p_{\mu_i} p^{\mu_i} = M^2$ and gives rise to power-suppressed terms in the OPE (**target mass corrections**).
- **Completely symmetric traceless** operators with n indices corresponds to spin-n fields.
- Hence twist is **mass dimension minus spin**.

The relevance of leading twist operators.

The matrix elements of twist-2 operators are related to the Mellin moments of DIS structure functions F_1 and F_2.

- Light-cone OPE of tensor Compton schematically yields:

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Reminder

Energy momentum tensor
Belinfante tensor
Form factors
Momentum sum rule
Angular momentum sum rule

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Structure functions
Compton tensor

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Principle
Scaling
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The relevance of leading twist operators.

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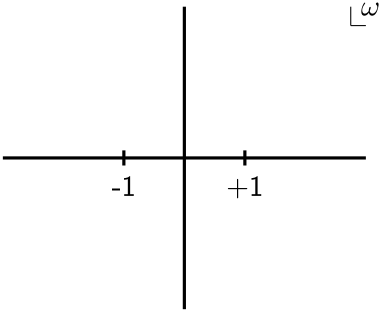

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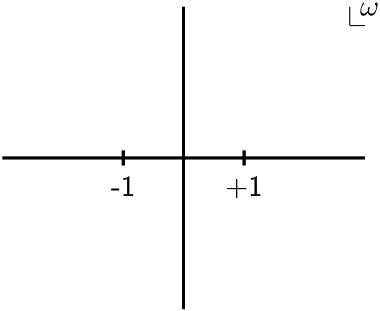

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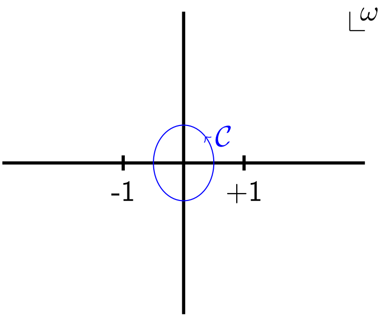

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- Get coefficient of $1/x_B^n$:

$$\frac{1}{2i\pi} \int_{\mathcal{C}} \frac{\mathrm{d}\omega}{\omega^{n+1}} T\left(J^{\mathrm{e.m.}}J^{\mathrm{e.m.}}\right)$$

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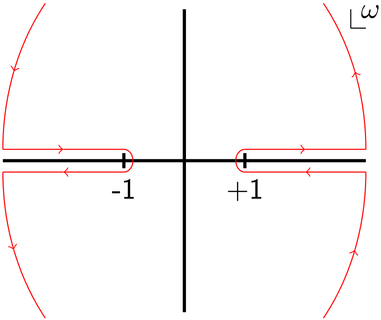

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- **Deform** contour.

Hard probes

Reminder

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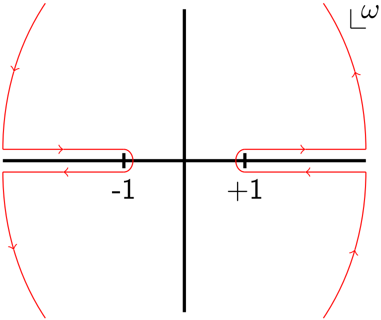

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- **Deform** contour.

- Use $F_i(\omega) = 2\Im T_i(\omega + i0^+)$ for $i = 1, 2$:

$$\text{Coef of } \frac{1}{x_B^n} = \frac{2}{\pi}\int_0^{+1} \frac{\mathrm{d}x}{x^{n+1}} F(x, Q^2)$$

The relevance of leading twist operators.

Parton Distribution Functions defined as matrix operators.

Exercise II.3

Assume $f(x)$ is known from its Mellin moments $M_n = \int \mathrm{d}x\, x^n f(x)$. Use $\int_{-\infty}^{+\infty} \mathrm{d}x\, x^n e^{+i\nu x} = 2\pi(-i)^n \delta^{(n)}(\nu)$ to show that:

$$f(x) = \sum_{n=0}^{\infty} M_n \delta^{(n)}(x) \frac{(-1)^n}{n!}$$

- Definition of PDF $q(x)$ as a **bilocal matrix element**:

$$q(x)\bar{u}(p)\gamma^+ u(p) = p^+ \int \frac{\mathrm{d}z^-}{4\pi} e^{ixp^+z^-} \langle N, p| \, \bar{q}\left(-\frac{z}{2}\right) \not{n} q \left(+\frac{z}{2}\right) |N, p\rangle$$

- **Matrix element definition**: can be used for models or first principles evaluations.
- Mellin moments of PDFs are matrix elements of **local fields** and can be computed on the lattice.

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- How are GPDs defined ?
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Part. III
3D imaging and beyond

Thursday 3 Oct. 2013
8H30 - 10H

Phenomenological status of 3D imaging in the most advanced case.

What have we learned so far?

- **Deep Inelastic Scattering** (DIS) can be described in terms of two **structure functions** $F_1(x_B, Q^2)$ and $F_2(x_B, Q^2)$.
- In the **Bjorken regime** (large Q^2 and fixed x_B) these functions show a logarithmic dependence on Q^2 controlled by DGLAP evolution equations.
- The Bjorken limit corresponds to the limit of points **arbitrarily close to the light-cone**.
- The **Operator Product Expansion** (OPE) on the light-cone organize nonperturbative contributions in terms of a **twist expansion** where the twist is the mass dimension of a field minus its spin.
- Parton Distribution Functions are defined by their **Mellin moments**, which are related to **leading twist** operators.

What have we learned so far?

- The **Belinfante** energy momentum tensor $T^{\mu\nu}$ is **symmetric**, **conserved** and **gauge-invariant**.
- The matrix element of the Belinfante operator sandwiched between nucleon states involve three **energy momentum form factors** $A(t)$, $B(t)$ and $C(t)$ for quarks and gluons.
- The form factors $A_q(0)$ and $A_g(0)$ describe the **sharing of longitudinal momentum** between quarks and gluons inside the nucleon.
- The factors $B_q(0)$ and $B_g(0)$ describe the **sharing of angular momentum** between quarks and gluons inside the nucleon.
- The energy momentum form factors $A(t)$, $B(t)$ and $C(t)$ can be accessed through the matrix element of a **quark twist-2** operator sandwiched between nucleon states.

Definition of GPDs.

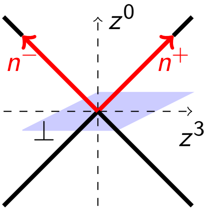
Matrix elements of twist-2 bilocal operators.

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle p' \left| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)\right| p\right\rangle_{z^+=0, z_\perp=0}$$

$$= \frac{1}{2P^+}\left[H^q \bar{u}(p')\gamma^+ u(p) + E^q \bar{u}(p') \frac{i\sigma^{+\alpha}\Delta_\alpha}{2M} u(p)\right]$$

$$\tilde{F}^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle p' \left| \bar{q}\left(-\frac{z}{2}\right)\gamma^+\gamma_5 q\left(\frac{z}{2}\right)\right| p\right\rangle_{z^+=0, z_\perp=0}$$

$$= \frac{1}{2P^+}\left[\tilde{H}^q \bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q \bar{u}(p') \frac{\gamma^5\Delta^+}{2M} u(p)\right]$$



References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)),
Ji, Phys. Rev. Lett. **78**, 610 (1997),
Radyushkin, Phys. Lett. **B380**, 417 (1996).

3D imaging

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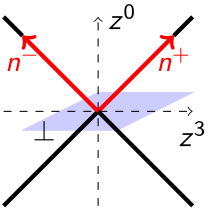
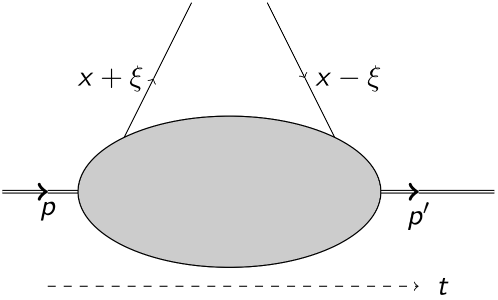


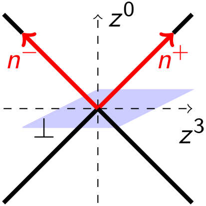
12 GPDs at twist 2

- Partons with a **light-like** separation.
- **Quarks**, **gluon** and **transversity** GPDs.
- $\mathrm{GPD}^{q,g} = \mathrm{GPD}^{q,g}(x, \xi, t)$.

Definition of GPDs.

Matrix elements of twist-2 bilocal operators.





Interpretation

- $x \in [\xi, 1]$: q emitted + q absorbed.
- $x \in [-\xi, +\xi]$: $\bar{q}$ emitted + q absorbed.
- $x \in [-1, -\xi]$: $\bar{q}$ emitted + $\bar{q}$ absorbed.

GPDs and Orbital Angular Momentum (1/2).

Towards a measurement of the **quark OAM contribution** to the nucleon spin.

- Reminder 1: Angular momentum sum rule

$$\frac{\langle P \,|J^3_q|\, P\rangle}{\langle P\,|P\rangle} = \frac{1}{2}\left[A_q(0) + B_q(0)\right] = \frac{1}{2}\Delta q + L_q$$

Back to angular momentum sum rule.

- Reminder 2: Energy momentum tensor

$$\frac{1}{P^{+2}}\left\langle P+\frac{\Delta}{2}\middle|\bar{q}(0)\gamma^{(+}i\overleftrightarrow{D}^{+)}q(0)\middle|P-\frac{\Delta}{2}\right\rangle = \bar{u}\left(P+\frac{\Delta}{2}\right)$$

$$\times\left[\frac{A_q(t)+4\xi^2 C_q(t)}{M} + \left(A_q(t)+B_q(t)\right)i\frac{\sigma^{+\lambda}}{P^+}\frac{\Delta_\lambda}{2M}\right]u\left(P-\frac{\Delta}{2}\right)$$

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Back to tensor matrix element.

- Compute GPDs Mellin moment of order 1:

$$\frac{1}{P^+}\bar{u}(p')\left[\int \mathrm{d}x\, xH^q(x,\xi,t)\gamma^+ + \int \mathrm{d}x\, xE^q(x,\xi,t)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}\right]u(p)$$

$$= \int \frac{dz^-}{2\pi}\int \mathrm{d}x\, xe^{ixP^+z^-}\left\langle p' \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)\middle| p\right\rangle_{z^+=0, z_\perp=0}$$

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$$\bar{u}(p')\left[\int \mathrm{d}x\, x H^q(x,\xi,t)\frac{1}{M} + \int \mathrm{d}x\, x(H^q + E^q)(x,\xi,t)\frac{i\sigma^{+\lambda}\Delta_\lambda}{2MP^+}\right] u(p)$$

$$= \frac{+i}{P^{+2}}\frac{\partial}{\partial z^-}\left\langle p' \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| p \right\rangle_{|z=0}$$

GPDs and Orbital Angular Momentum (1/2).

Towards a measurement of the **quark OAM contribution** to the nucleon spin.

- Reminder 1: Angular momentum sum rule

$$\frac{\langle P | J_q^3 | P \rangle}{\langle P | P \rangle} = \frac{1}{2}\left[A_q(0) + B_q(0)\right] = \frac{1}{2}\Delta q + L_q$$

Back to angular momentum sum rule.

- Reminder 2: Energy momentum tensor

$$\frac{1}{P^{+2}}\left\langle P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^{(+}i\overleftrightarrow{D}^{+)}q(0) \middle| P - \frac{\Delta}{2} \right\rangle = \bar{u}\left(P + \frac{\Delta}{2}\right)$$

$$\times\left[\frac{A_q(t) + 4\xi^2 C_q(t)}{M} + \left(A_q(t) + B_q(t)\right)i\frac{\sigma^{+\lambda}}{P^+}\frac{\Delta_\lambda}{2M}\right]u\left(P - \frac{\Delta}{2}\right)$$

Back to tensor matrix element.

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$$= \frac{1}{P^{+2}}\langle p' | \bar{q}(0)\gamma^+ i\overleftrightarrow{D}^+ q(0) | p \rangle$$

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$$\frac{1}{P^{+2}}\left\langle P+\frac{\Delta}{2}\middle|\bar{q}(0)\gamma^{(+}i\overleftrightarrow{D}^{+)}q(0)\middle|P-\frac{\Delta}{2}\right\rangle = \bar{u}\left(P+\frac{\Delta}{2}\right)$$

$$\times\left[\frac{A_q(t)+4\xi^2 C_q(t)}{M} + \left(A_q(t)+B_q(t)\right)i\frac{\sigma^{+\lambda}}{P^+}\frac{\Delta_\lambda}{2M}\right]u\left(P-\frac{\Delta}{2}\right)$$

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GPDs and Orbital Angular Momentum (2/2).

Towards a measurement of the **quark OAM contribution** to the nucleon spin.

- Link between GPDs and energy momentum form factors:

$$\int \mathrm{d}x\, xH^q(x,\xi,t) = A_q(t) + 4\xi^2 C_q(t)$$

$$\int \mathrm{d}x\, xE^q(x,\xi,t) = B_q(t) - 4\xi^2 C_q(t)$$

- Spin sum rule:

$$\int \mathrm{d}x\, x\big(H^q(x,\xi,0) + E^q(x,\xi,0)\big) = A_q(0) + B_q(0) = 2J_q$$

Ji sum rule

$$2J^q = \int_0^1 dx\, x[q(x) + \bar{q}(x)] + \int_{-1}^{+1} dx\, xE^q(x,0,0)$$

$$= \Delta q + 2L^q$$

Ji, Phys. Rev. Lett. **78**, 210 (1997)

1114 citations on 3/10/2013...

Properties (1/2).

Generalization of nucleon Form Factors and Parton Distribution Functions.

3D imaging

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$$= \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-}\left\langle p' \left| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)\right| p\right\rangle_{z^+=0, z_\perp=0}$$

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$$= \frac{1}{2}\int \frac{dz^-}{2\pi} 2\pi\delta(P^+z^-)\langle p' \left| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)\right| p\rangle_{z^+=0, z_\perp=0}$$

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$$= \lim_{\Delta\to 0}\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\left\langle p'\left|\bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)\right|p\right\rangle_{z^+=0, z_\perp=0}$$

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$$\int_{-1}^{+1} dx\, H^q(x\xi, t) = F_1^q(t)$$

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$$H^q(x, 0, 0) = q(x)$$

$$\frac{1}{2P^+}\bar{u}(p')\left[\quad H^q(x,0,0)\gamma^+\right]u(p)$$

$$= \frac{1}{2P^+}\bar{u}(P)q(x)\gamma^+ u(P)$$

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$$H^q(x, 0, 0) = q(x)$$

Exercise III.1

Prove the **polynomiality** property of GPDs:

$$\int_{-1}^{+1} dx\, x^n H^q(x, \xi, t) = \text{polynomial in } \xi$$

As in the case of nucleon form factors, this properties is tied to **Lorentz invariance**.

Properties (2/2).

3d imaging of nucleon's partonic content.

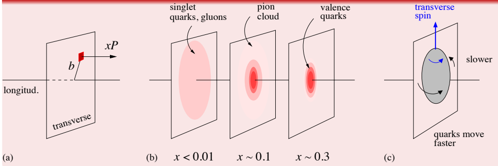
- **Probabilistic interpretation** of Fourier transform of GPD$(x, \xi = 0, t)$ in **transverse plane**.

$$\rho(x, b_\perp, \lambda, \lambda_N) = \frac{1}{2}\left[H(x, b_\perp^2) + \frac{b_\perp^j \epsilon_{ji} S_\perp^i}{M} \frac{\partial E}{\partial b_\perp^2}(x, b_\perp^2) + \lambda\lambda_N \tilde{H}(x, b_\perp^2)\right]$$

- Notations : quark helicity λ, nucleon longitudinal polarization λ_N and nucleon transverse spin $S_\perp$.

Burkardt, Phys. Rev. **D62**, 071503 (2000)

Obtain this 3d picture from exclusive measurements ?



Weiss, AIP Conf. Proc. **1149**, 150 (2009)

A partonic picture of hadronic processes (1/2).

Exclusive processes, **factorization** and **universality**.

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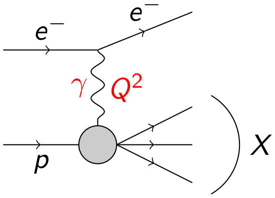

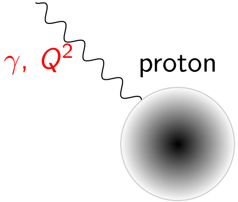

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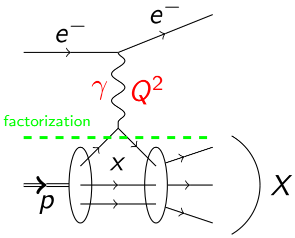

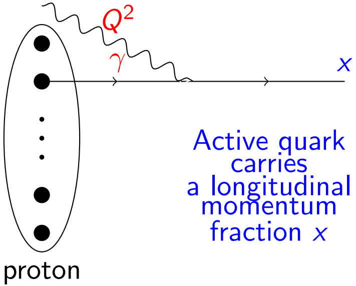

Active quark carries a longitudinal momentum fraction x

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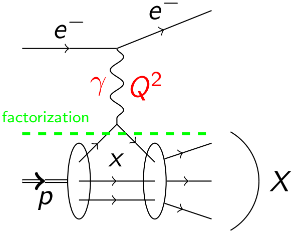
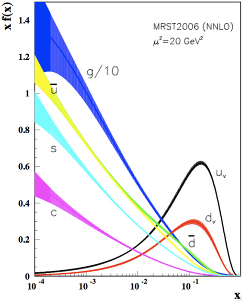
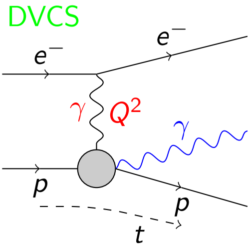
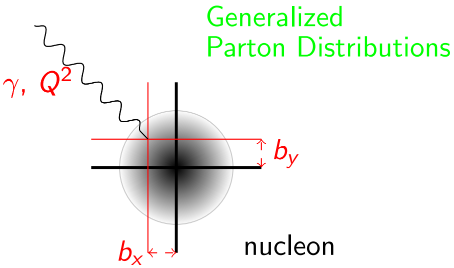




A partonic picture of hadronic processes (1/2).

Exclusive processes, **factorization** and **universality**.





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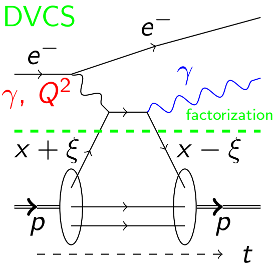
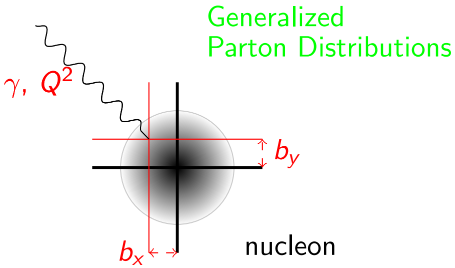
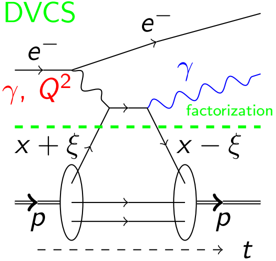




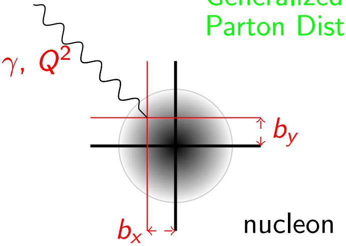
A partonic picture of hadronic processes (1/2).

Exclusive processes, **factorization** and **universality**.

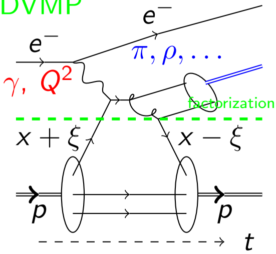
DVCS



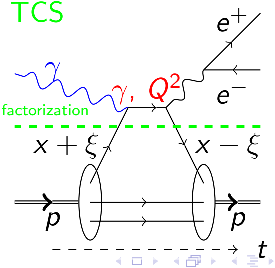
Generalized
Parton Distributions



DVMP



TCS



A partonic picture of hadronic processes (2/2).

Exclusive processes, **factorization** and **universality**.

Bjorken regime : large Q^2 and fixed $xB \simeq 2\xi/(1+\xi)$

- Partonic interpretation relies on **factorization theorems**.
- All-order proofs for DVCS, TCS and some DVMP.
- GPDs depend on a (arbitrary) factorization scale μ_F.
- **Consistency** requires the study of **different channels**.

- GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^{1} dx\, C\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F)$$

for a given GPD F.

- Integration kernels C have been worked out at NLO.

Belitsky and Müller, Phys. Lett. **B417**, 129 (1998)

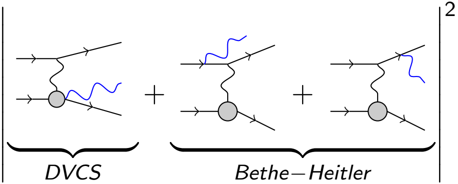
- CFF $\mathcal{F}$ is a **complex function**.

Measurement principle is intrinsically quantum.

Quantum interference and amplification.

- The DVCS and Bethe-Heitler (BH) processes have the **same incoming and outgoing states** :

$$\sigma(ep \to ep\gamma) =$$



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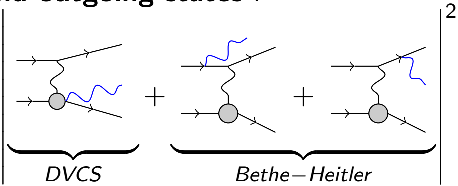
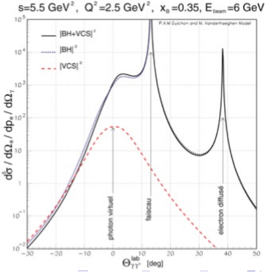


- Measurement of the **interference**.
- Control of BH thanks to **form factors**.



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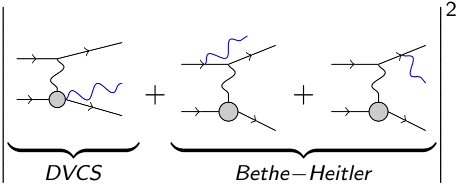
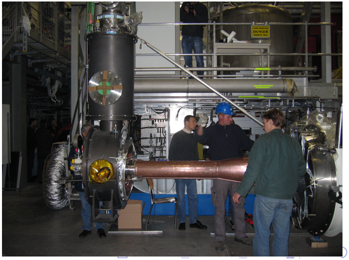
$\sigma(ep \to ep\gamma) =$ 

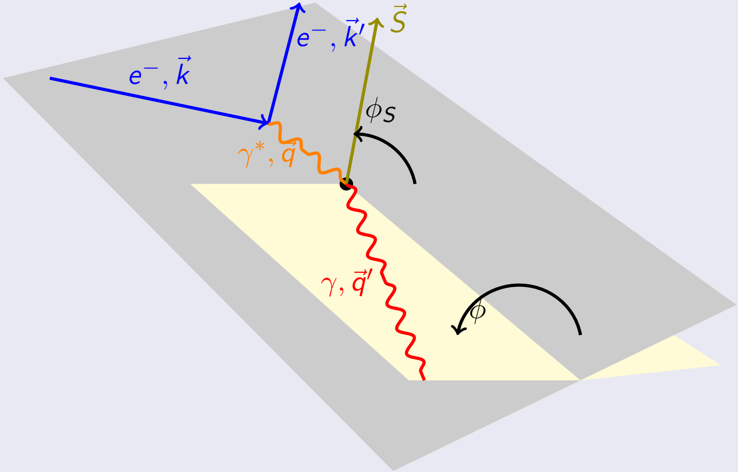
- Measurement of the **interference**.
- Control of BH thanks to **form factors**.
- **Polarized** beam or target.



Definition of observables (1/4).

Harmonic structure of $ep \to ep\gamma$ amplitude.

Deeply Virtual Compton Scattering



Definition of observables (2/4).

Harmonic structure of $ep \to ep\gamma$ amplitude.

- Study the **harmonic structure** of $ep \to ep\gamma$ amplitude.

  Diehl *et al.*, Phys. Lett. **B411**, 193 (1997)

- Angle ϕ between leptonic and hadronic planes

$$|\mathcal{M}_{\mathrm{BH}}|^2 \propto \frac{1}{|t|}\frac{1}{P(\cos\phi)}\sum_{n=0}^{3}\left[c_n^{\mathrm{BH}}\cos(n\phi)+s_n^{\mathrm{BH}}\sin(n\phi)\right]$$

$$|\mathcal{M}_{\mathrm{DVCS}}|^2 \propto \sum_{n=0}^{3}\left[c_n^{\mathrm{DVCS}}\cos(n\phi)+s_n^{\mathrm{DVCS}}\sin(n\phi)\right]$$

$$\mathcal{M}_{\mathrm{I}} \propto \frac{1}{|t|}\frac{1}{P(\cos\phi)}\sum_{n=0}^{3}\left[c_n^{\mathrm{I}}\cos(n\phi)+s_n^{\mathrm{I}}\sin(n\phi)\right]$$

- Use expressions for s_n for c_n with **exact treatment** of all contributions apart from OPE in the hadronic tensor.

  Guichon and Vanderhaeghen (2008)

Definition of observables (3/4).

Single and double asymmetries.

- Combined beam-spin and charge asymmetries :

$$\begin{aligned} d\sigma^{h_e, Q_e}(\phi) &= d\sigma_{\mathrm{UU}}(\phi)\left[1 + h_e A_{\mathrm{LU,DVCS}}(\phi)\right. \\ &\left. + Q_e h_e A_{\mathrm{LU,I}}(\phi) + Q_e A_{\mathrm{C}}(\phi)\right] \end{aligned}$$

- Single beam-spin asymmetry :

$$A^{Q_e}_{\mathrm{LU}}(\phi) = \frac{d\sigma^{Q_e}_{\rightarrow} - d\sigma^{Q_e}_{\leftarrow}}{d\sigma^{Q_e}_{\rightarrow} + d\sigma^{Q_e}_{\leftarrow}}$$

- Relation between observables :

$$A^{Q_e}_{\mathrm{LU}}(\phi) = \frac{Q_e A_{\mathrm{LU,I}}(\phi) + A_{\mathrm{LU,DVCS}}(\phi)}{1 + Q_e A_{\mathrm{C}}(\phi)}$$

- Compute Fourier coefficients of asymmetries.

Definition of observables (4/4).

What are the probed combinations of CFFs ?

Typical kinematics

Experiment	Kinematics		
	x_B	Q^2 [GeV^2]	t [GeV^2]
HERA	0.001	8.00	-0.30
COMPASS	0.05	2.00	-0.20
HERMES	0.09	2.50	-0.12
CLAS	0.19	1.25	-0.19
HALL A	0.36	2.30	-0.23

Definition of observables (4/4).

What are the probed combinations of CFFs ?

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Typical kinematics

Experiment	Kinematics			
	x_B	Q^2 [GeV^2]	t [GeV^2]	$-t/Q^2$
HERA	0.001	8.00	-0.30	0.04
COMPASS	0.05	2.00	-0.20	0.10
HERMES	0.09	2.50	-0.12	0.05
CLAS	0.19	1.25	-0.19	0.15
HALL A	0.36	2.30	-0.23	0.10

Definition of observables (4/4).

What are the probed combinations of CFFs ?

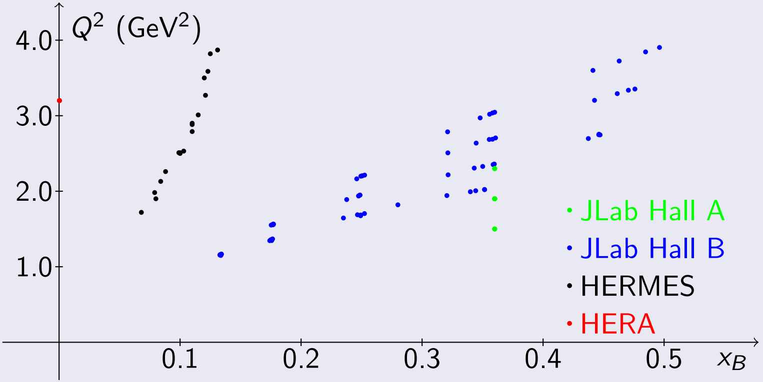
Selection of observables

Experiment	Observable	Normalized CFF dependence
HERMES	$A_{\mathrm{C}}^{\cos 0\phi}$	$\mathrm{Re}\mathcal{H} + 0.06\mathrm{Re}\mathcal{E} + 0.24\mathrm{Re}\widetilde{\mathcal{H}}$
	$A_{\mathrm{C}}^{\cos \phi}$	$\mathrm{Re}\mathcal{H} + 0.05\mathrm{Re}\mathcal{E} + 0.15\mathrm{Re}\widetilde{\mathcal{H}}$
	$A_{\mathrm{LU,I}}^{\sin \phi}$	$\mathrm{Im}\mathcal{H} + 0.05\mathrm{Im}\mathcal{E} + 0.12\mathrm{Im}\widetilde{\mathcal{H}}$
	$A_{\mathrm{UL}}^{+,\sin \phi}$	$\mathrm{Im}\widetilde{\mathcal{H}} + 0.10\mathrm{Im}\mathcal{H} + 0.01\mathrm{Im}\mathcal{E}$
CLAS	$A_{\mathrm{LU}}^{-,\sin \phi}$	$\mathrm{Im}\mathcal{H} + 0.06\mathrm{Im}\mathcal{E} + 0.21\mathrm{Im}\widetilde{\mathcal{H}}$
	$A_{\mathrm{UL}}^{-,\sin \phi}$	$\mathrm{Im}\widetilde{\mathcal{H}} + 0.12\mathrm{Im}\mathcal{H} + 0.04\mathrm{Im}\mathcal{E}$
HALL A	$\sigma^{\cos 0\phi}$	$1 + 0.05\mathrm{Re}\mathcal{H} + 0.007\mathcal{H}\mathcal{H}^*$
	$\sigma^{\cos \phi}$	$1 + 0.12\mathrm{Re}\mathcal{H} + 0.05\mathrm{Re}\widetilde{\mathcal{H}}$

Kinematic region of existing DVCS measurements.

Looking for the Bjorken regime.

What is large Q^2 ?



- World data cover **complementary kinematic regions**.

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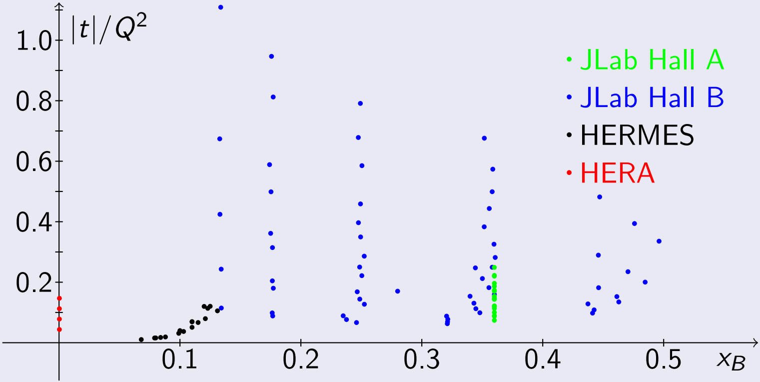

- World data cover **complementary kinematic regions**.
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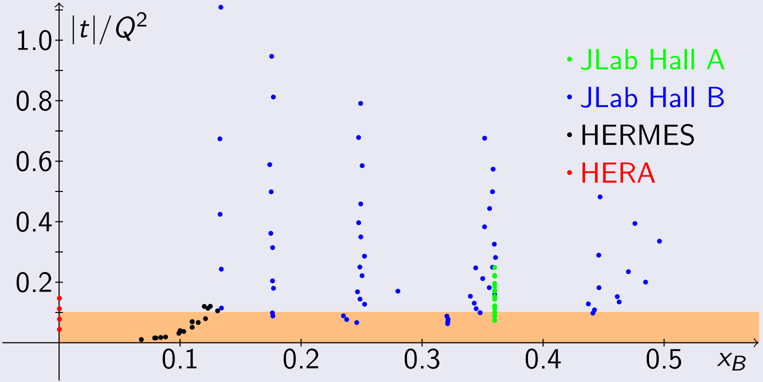

- World data cover **complementary kinematic regions**.
- Q^2 is **not so large** for most of the data.
- Higher twists, finite-t and target mass corrections ?

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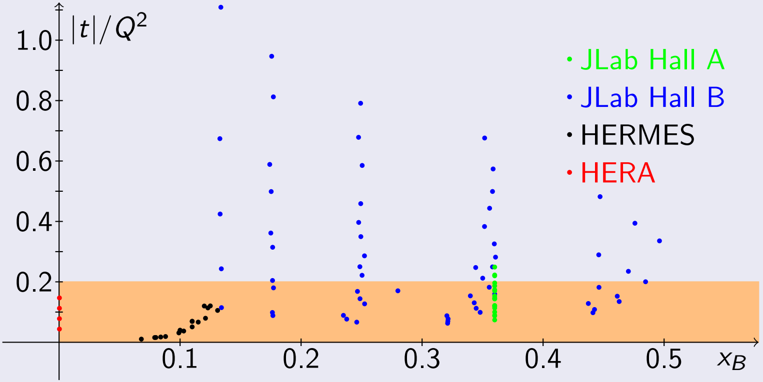

- World data cover **complementary kinematic regions**.
- Q^2 is **not so large** for most of the data.
- Higher twists, finite-t and target mass corrections ?

Approximations.

First systematic study of DVCS polarized and unpolarized observables.

Unless explicitly stated

- Work at twist 2 accuracy.
- Use LO expression of kernel $C(x, \xi)$.
- No finite$-t$ or target mass corrections (higher twist).

Double Distribution models.

Enforcing the polynomiality condition.

- Simplify by considering **spinless hadron**.

$$H^q(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left\langle P+\frac{\Delta}{2}\right| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)\left|P-\frac{\Delta}{2}\right\rangle$$

where $\xi = -\Delta^+/(2P^+)$ is the skewness.

- Define **Double Distributions** (DDs) F^q and G^q:

$$\left\langle P+\frac{\Delta}{2}\right| \bar{q}\left(-\frac{z}{2}\right)\gamma_\mu q\left(\frac{z}{2}\right)\left|P-\frac{\Delta}{2}\right\rangle_{z^2=0}$$

$$= 2P_\mu \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha\, e^{-i\beta(Pz)+i\alpha\frac{(\Delta z)}{2}} F^q(\beta,\alpha,t)$$

$$-\Delta_\mu \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha\, e^{-i\beta(Pz)+i\alpha\frac{(\Delta z)}{2}} G^q(\beta,\alpha,t) + \text{ higher twist terms}$$

where Ω is the rhombus defined by $|\alpha|+|\beta| \leq 1$.

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where $\xi = -\Delta^+/(2P^+)$ is the skewness.

- Define **Double Distributions** (DDs) F^q and G^q:
- Relation between GPDs and DDs:

$$H^q(x,\xi,t) = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\, \delta(x-\beta-\alpha\xi)\big(F^q(\beta,\alpha)+\xi G^q(\beta,\alpha,t)\big)$$

- Polynomiality is **automatically fulfilled**.
- DDs look like PDFs for the variable β, and Distribution Amplitudes (DAs) for the variable α.

Exercise III.2

Derive the relation between GPDs and DDs. Apply the factorized Ansatz (*see next slide*). What happens for $n \to \infty$?

Double Distribution models.

GK model (Goloskokov and Kroll).

- **Factorized Ansatz**. For $i = g$, sea or val :

$$
\begin{aligned}
H_i(x, \xi, t) &= \int_{|\alpha|+|\beta|\leq 1} d\beta d\alpha \, \delta(\beta + \xi\alpha - x) f_i(\beta, \alpha, t) \\
f_i(\beta, \alpha, t) &= e^{b_i t} \frac{1}{|\beta|^{\alpha' t}} h_i(\beta) \pi_{n_i}(\beta, \alpha) \\
\pi_{n_i}(\beta, \alpha) &= \frac{\Gamma(2n_i + 2)}{2^{2n_i+1}\Gamma^2(n_i + 1)} \frac{(1 - |\beta|)^2 - \alpha^2]^{n_i}}{(1 - |\beta|)^{2n_i+1}}
\end{aligned}
$$

- Expressions for h_i and n_i :

$$
\begin{aligned}
h_g(\beta) &= |\beta| g(|\beta|) & n_g &= 2 \\
h^q_{\mathrm{sea}}(\beta) &= q_{\mathrm{sea}}(|\beta|)\mathrm{sign}(\beta) & n_{\mathrm{sea}} &= 2 \\
h^q_{\mathrm{val}}(\beta) &= q_{\mathrm{val}}(\beta)\Theta(\beta) & n_{\mathrm{val}} &= 1
\end{aligned}
$$

- Designed to study DVMP. Expect better comparison to data at small x_B.

Goloskokov and Kroll, Eur. Phys. J. **C42**, 281 (2005)

See more.

Mellin-Barnes representation.

KM model (Kumericki and Müller) (1/2).

- Start again from t-channel **partial-wave expansion**:

$$H_+(x,\xi) = 2\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{l=0\\ \text{even}}}^{n+1} B_{nl}\theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right)C_n^{3/2}\left(\frac{x}{\xi}\right)P_l\left(\frac{1}{\xi}\right)$$

- From $C_n^{3/2}$ define rescaled polynomials $c_n(x,\xi)$ to **recover Mellin moments** when $\xi \to 0$.
- From orthogonality relation on $C_n^{3/2}$ define **orthogonal polynomials** $p_n(x,\xi)$ such that:

$$\int_{-1}^{+1} dx\, c_n(x,\xi)p_m(x,\xi) = (-1)^n\delta_{nm}$$

- Write partial-wave expansion:

$$H_+(x,\xi) = \sum_{n=0}^{\infty}(-1)^n p_n(x,\xi)H_n(\xi)$$

Mellin-Barnes representation.

KM model (Kumericki and Müller) (2/2).

- Start from partial-wave expansion:

$$H_+(x,\xi) = \sum_{n=0}^{\infty} (-1)^n p_n(x,\xi) H_n(\xi)$$

- Resum by means of **Sommerfeld - Watson transform**:

$$H_+(x,\xi) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \, \frac{1}{\sin \pi j} p_j(x,\xi) H_j(\xi)$$

Müller and Schäfer, Nucl. Phys. **B379**, 1 (2006)

- Express CFF $\mathcal{H}$ in terms of moments H_j:

$$\mathcal{H}(\xi) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \, \frac{1}{\xi^{j+1}} \left[i + \tan\left(\frac{\pi j}{2}\right) \right] [C_j^0 + \ldots] H_j(\xi)$$

- Regge modeling of $H_j(\xi)$ moments.

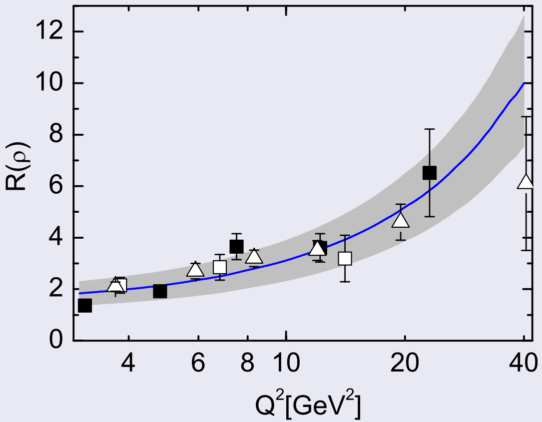
Kumericki and Müller, Nucl. Phys. **B841**, 1 (2009)

See dual model.

Goloskokov-Kroll (GK) model on DVMP.

The GK model **was tuned** to analyse DVMP.

σ_L/σ_T for ρ^0 at $W = 90$ GeV

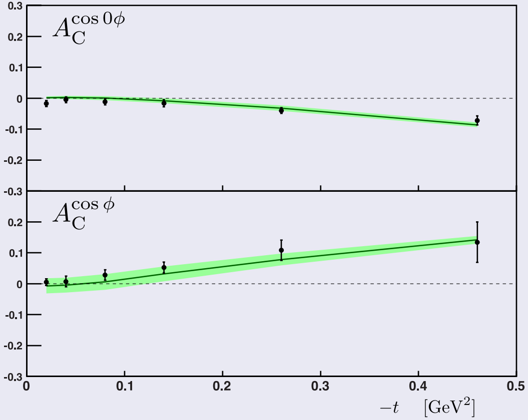


Goloskokov and Kroll, Eur. Phys. J. **C53**, 281 (2005)

Goloskokov-Kroll (GK) model on DVCS.

No parameter of the GK model **was tuned** to analyse DVCS.

Beam Charge Asymmetry, HERMES



Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

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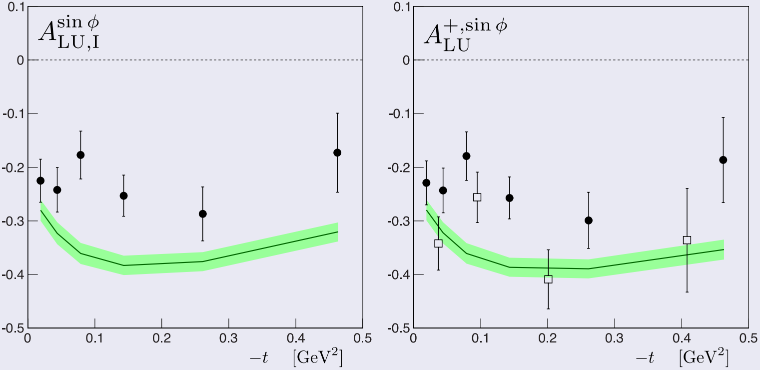
Beam Spin Asymmetry, HERMES



Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

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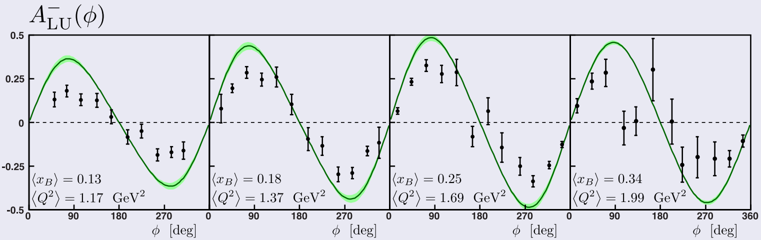
No parameter of the GK model **was tuned** to analyse DVCS.

Beam Spin Asymmetry, CLAS

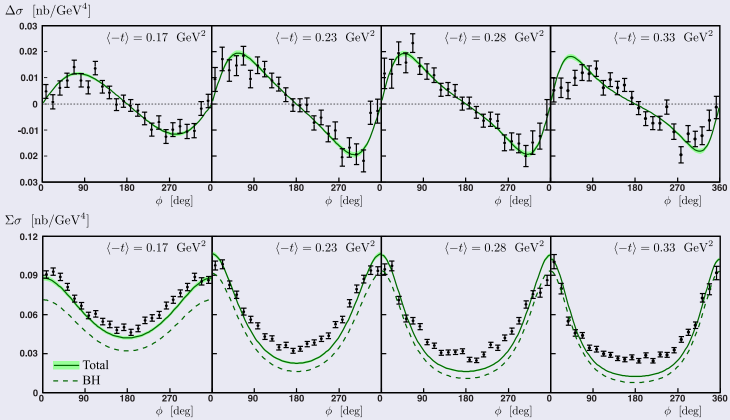


Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

Goloskokov-Kroll (GK) model on DVCS.

No parameter of the GK model **was tuned** to analyse DVCS.

Helicity-dependent and independent cross sections, JLab Hall A



Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

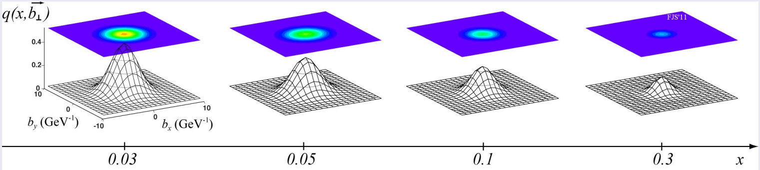
Goloskokov-Kroll (GK) model on DVCS.

No parameter of the GK model **was tuned** to analyse DVCS.

Spin structure with GK model (quoted at 4 GeV^2)

- $J^u \simeq 0.250$, $J^d \simeq 0.020$, $J^s \simeq 0.015$, $J^g \simeq 0.214$
- $\sum_{q,g} J^{q,g} \simeq 1/2$

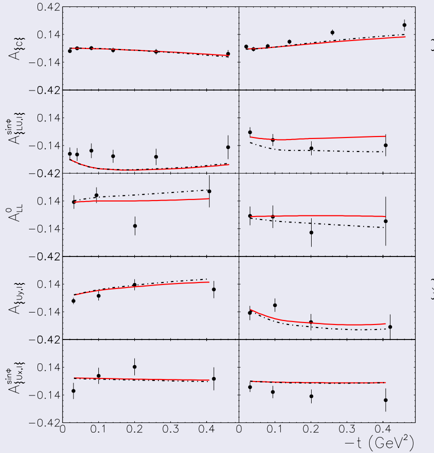
3D nucleon structure with GK model



Constraining models from DVCS data.

Double Distribution model (Goloskokov and Kroll (GK)).

GK model compared to HERMES data



Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

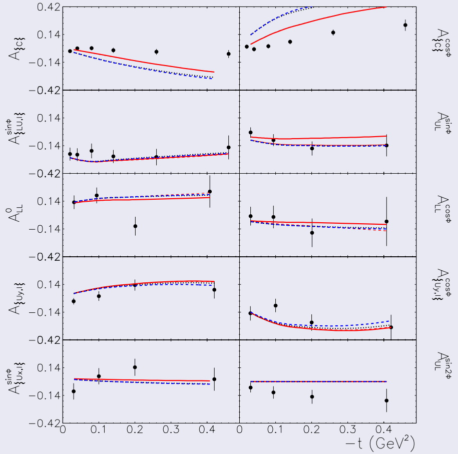
- Only H
- All GPDs

Goloskokov and Kroll, Eur. Phys. J. **C42**, 281 (2005)

Constraining models from DVCS data.

Double Distribution model (Vanderhaeghen, Guichon and Guidal (VGG)).

VGG model compared to HERMES data



Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

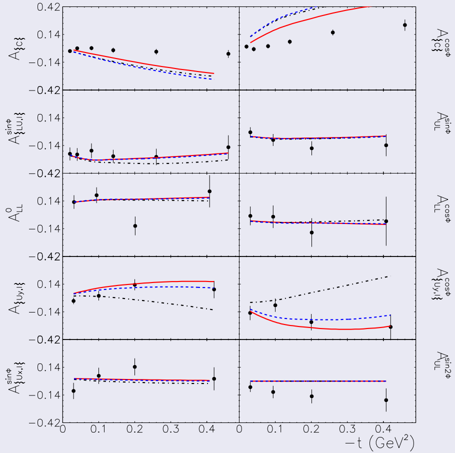
- Only H
- $H + E$ $(J_u, J_d) = (0.3, 0.)$
- $H + E$ $(J_u, J_d) = (0., 0.3)$

Guidal *et al.* , Phys. Rev. **D72**, 054013 (2005)

Constraining models from DVCS data.

Dual model (Polyakov and Vanderhaeghen).

Dual model compared to HERMES data



Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

- Only H
- $H+E$ (with a given model of $E(x,0,t)$)
- $H+E$ (with another model of $E(x,0,t)$)

Polyakov and Shuvaev, hep-ph/0207153

Polyakov, Phys. Lett. **B659**, 542 (2008)

Constraining models from DVCS data.

Mellin - Barnes representation (Kumericki and Müller).

3D imaging

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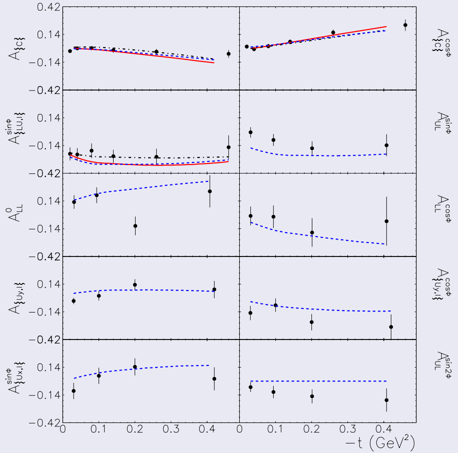
KM model compared to HERMES data



Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

- Without JLab Hall A data
- With JLab Hall A data
- With HERMES polarized target data

Müller and Schäfer, Nucl. Phys. **B739**, 1 (2006)

Kumericki and Müller, Nucl. Phys. **B841**, 1 (2009)

From QCD first principles to experimental data.

Very good theoretical control, but not easy to implement!

- **All-order proof of factorization** of DVCS amplitude.
  Collins and Freund, Phys. Rev. **D59**, 074009 (1999)
- Hard scattering kernel computed at **next-to-leading order** at leading twist.
  Belistky and Müller, Phys. Lett. **B417**, 129 (1998)
- Evolution equations computed at **next-to-leading order**.
  Belitsky *et al.* , Nucl. Phys. **B574**, 347 (2000) and ref. therein
- Finite-t and target mass corrections computed at **leading order**: kinematic power corrections to **twist 4 accuracy**.
  Braun *et al.* , Phys. Rev. Lett. **109**, 242001 (2012)

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  Braun *et al.* , Phys. Rev. Lett. **109**, 242001 (2012)

GPD "measurements" ?

- **Already achieved**: experimentally constrained models.
- Next step: **Measured transverse plane images**.

Ways to improve comparison.

Satisfactory agreement but needs improvement in the valence region...

- Implementation of GPD evolution.

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Moutarde *et al.* , Phys. Rev. **D87**, 054029 (2013)

See more.

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Moutarde *et al.* , Phys. Rev. **D87**, 054029 (2013)

See more.

- Resummation.

Altinoluk *et al.*, arXiv:1309.2508 [hep-ph]

See more.

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See more.

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Altinoluk *et al.*, arXiv:1309.2508 [hep-ph]

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- Modification of the profile function.

Mezrag *et al.* , Phys. Rev. **D88**, 014001 (2013)

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  ▸ See more.

- Resummation.

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  ▸ See more.

- Modification of the profile function.

  Mezrag *et al.* , Phys. Rev. **D88**, 014001 (2013)

  ▸ See more.

- Finite$-t$ and target mass corrections. Problem recently solved for DVCS.

  Braun *et al.* , Phys. Rev. Lett. **109**, 242001 (2012)

  ▸ See more.

CEA - Saclay

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How many parameters to parameterize GPDs?

Naive counting from a Double Distribution model (1/3).

- Radyushkin's **Factorized Ansatz** + t-dependence from nucleon **form factor** F_1:

$$\begin{aligned} H^q(x,\xi,t) &= \int_{|\alpha|+|\beta|\leq 1} d\beta d\alpha\, \delta(\beta+\xi\alpha-x) f^q(\beta,\alpha,t) \\ f^q(\beta,\alpha,t) &= F_1^q(t)h(\beta)\pi_n(\beta,\alpha) \\ \pi_n(\beta,\alpha) &= \frac{\Gamma(2n+2)}{2^{2n+1}\Gamma^2(n+1)} \frac{(1-|\beta|)^2-\alpha^2]^n}{(1-|\beta|)^{2n+1}} \end{aligned}$$

- Expressions for h and n :

$$\begin{aligned} h^q_{\mathrm{sea}}(\beta) &= q_{\mathrm{sea}}(|\beta|)\mathrm{sign}(\beta) \qquad & n_{\mathrm{sea}} &= 1 \\ h^q_{\mathrm{val}}(\beta) &= q_{\mathrm{val}}(\beta)\Theta(\beta) \qquad & n_{\mathrm{val}} &= 1 \end{aligned}$$

- Add D-term at $z = x/\xi$:

$$D(z) \simeq (1-z^2)\Big(-4.C_1^{3/2}(z) - 1.2C_3^{3/2}(z) - 0.4C_5^{3/2}(z)\Big)$$

Vanderhaeghen *et al.*, Phys. Rev. **D60**, 094017 (1999)

How many parameters to parameterize GPDs?

Naive counting from a Double Distribution model (2/3).

- Parton Distribution Function:

$$q(x) = Ax^{\eta_1}(1-x)^{\eta_2}(1+\epsilon\sqrt{x}+\gamma x)$$

Martin *et al.*, Eur. Phys. J. **C63**, 189 (2009)
5 parameters per quark flavor

- Kelly parameterization of form factor $(\tau = t/(4M^2))$:

$$F_1^q(t) = \frac{1+a\tau}{1+b\tau+c\tau^2+d\tau^3}$$

Kelly *et al.*, Phys. Rev. **C70**, 068202 (2004)
4 parameters per quark flavor

- Profile function parameter n :

$$\pi_n(\beta,\alpha) = \frac{\Gamma(2n+2)}{2^{2n+1}\Gamma^2(n+1)}\frac{(1-|\beta|)^2-\alpha^2]^n}{(1-|\beta|)^{2n+1}}$$

Mezrag *et al.*, Phys. Rev. **D88**, 014001 (2013)
1 parameters per quark flavor

How many parameters to parameterize GPDs?

Naive counting from a Double Distribution model (3/3).

- Naive counting leads to **9 parameters per quark flavor**!
- Not fully realistic:
  - No **correlations between** x **and** t...
  - ...But generalized form factors computed on the lattice exhibit different t-dependence.

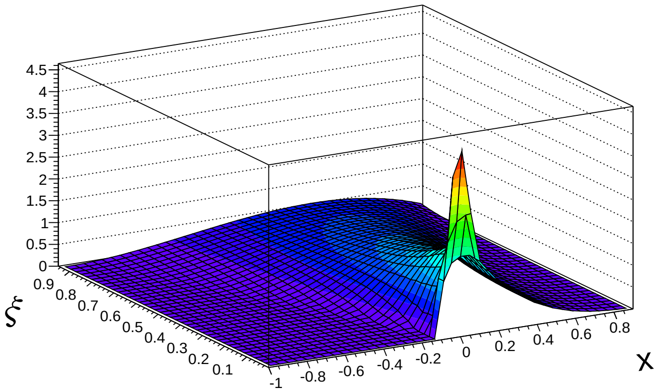
Hägler, Phys. Rept. **490**, 49 (2010)

- Expect $\simeq$ 30 - 40 parameters for u, d, s and g from naive counting, **not considering higher-twist GPDs**.
- Strategy:
  - Find **educated parameterization** (few free parameters) to proceed with **usual** χ^2**-minimization**.
  - Use **uneducated parameterization** (lot of free parameters) but proceed with **alternative fitting procedures** (neural networks? etc.)

From principles to actual data.

Direct experimental access to a restricted domain.

GPD H at $t = -0.23$ GeV^2 and $Q^2 = 2.3$ GeV^2.

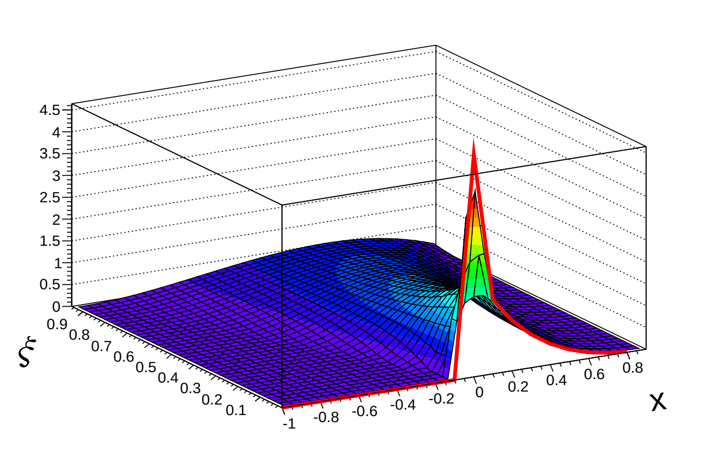


GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

Direct experimental access to a restricted domain.

Need to know $H(x, \xi = 0, t)$ to do transverse plane imaging.

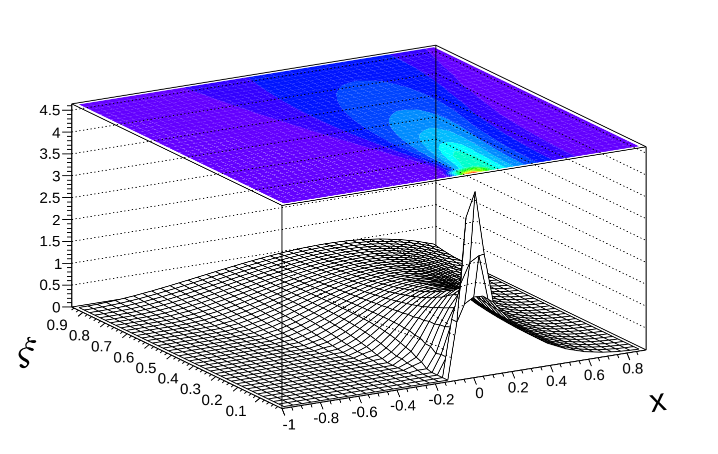


GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

Direct experimental access to a restricted domain.

What is the physical region?

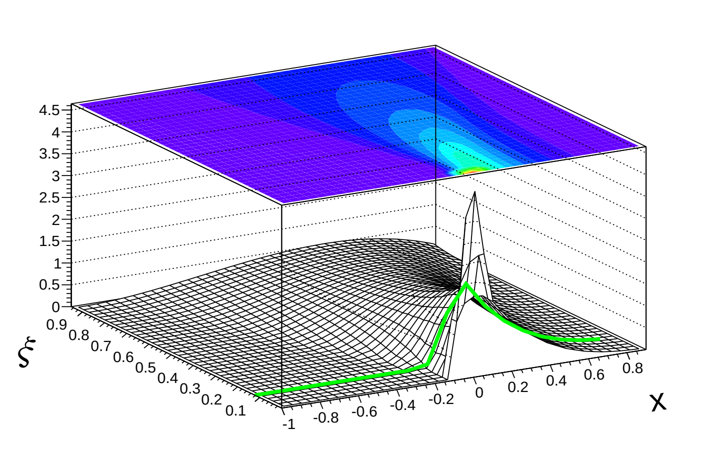


GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

Direct experimental access to a restricted domain.

$\xi_{\min}$ from finite beam energy.

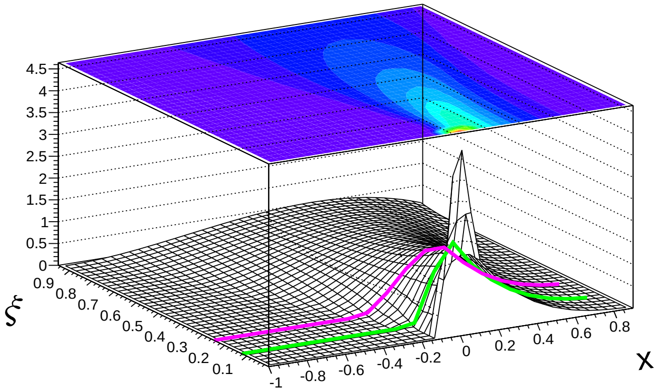


GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

Direct experimental access to a restricted domain.

$\xi_{\max}$ from kinematic constraint on 4-momentum transfer.

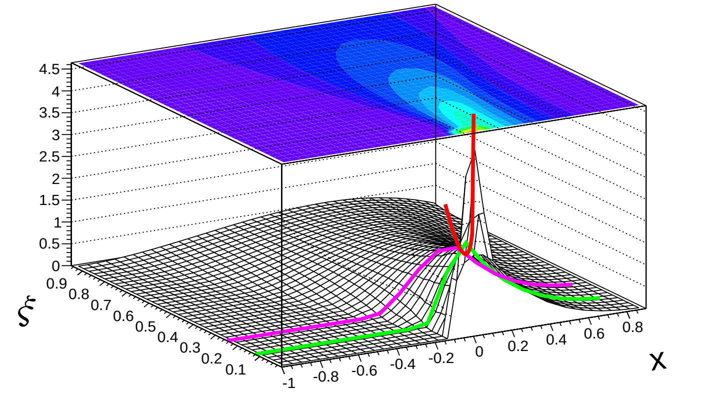


GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

Direct experimental access to a restricted domain.

The cross-over line $x = \xi$.

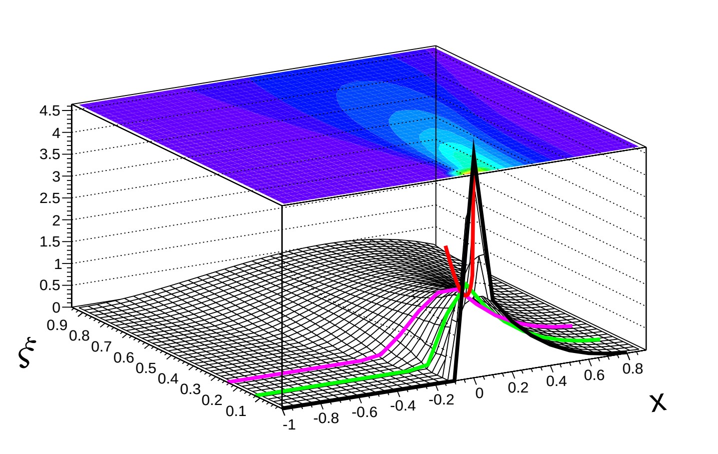


GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

Direct experimental access to a restricted domain.

The black curve is what is needed for transverse plane imaging!

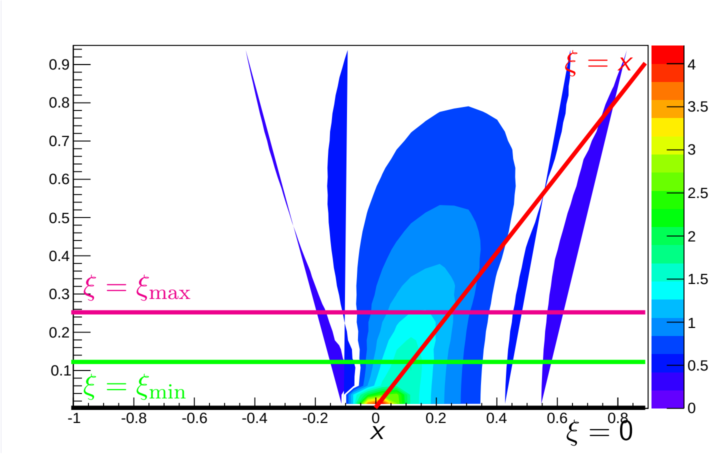


GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

Direct experimental access to a restricted domain.

Density plot of H at $t = -0.23$ GeV^2 and $Q^2 = 2.3$ GeV^2



GPD model: see Kroll *et al.* , Eur. Phys. J. **C73**, 2278 (2013)

3D imaging

Reminder

Theoretical framework
How are GPDs defined ?
How are GPDs measured ?

From theory to measure
Models
Universality tests
Probing models on DVCS
Going further

From measure to theory
Degrees of freedom
Kinematic restrictions
Fitting strategies
Model-dependence vs accuracy

Near future
Experimental developments
GPD toolkit

Overview of current extraction methods.

Problems: Model dependence ? Degrees of freedom ? Extrapolations ?

Local fits

Take each kinematic bin independently of the others. Extraction of $Re\mathcal{H}$, $Im\mathcal{H}$, ... as independent parameters.

Global fit

Take all kinematic bins at the same time. Use a parametrization of GPDs or CFFs.

Hybrid : Local / global fit

Start from local fits and add smoothness assumption.

Neural networks

Already used for PDF fits. Exploratory stage for GPDs.

See more.

3D imaging

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Summary of first extractions.

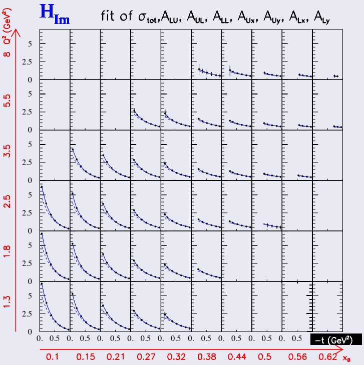
Feasibility of twist-2 analysis of existing data.

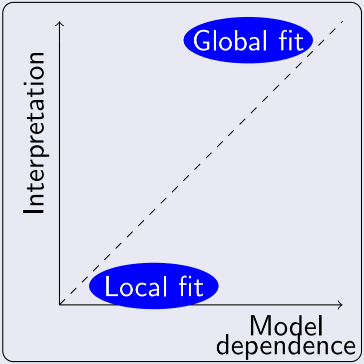
- **Dominance** of twist 2 and **validity** of a GPD analysis of DVCS data.
- $Im\mathcal{H}$ **best determined**. Large uncertainties on $Re\mathcal{H}$.
- However sizable **higher twist contamination** for DVCS measurements.
- Already some indications about the **invalidity** of the H-dominance hypothesis with **unpolarized data**.
- Clear signs that one or several things are missing !

JLab's 12 GeV upgrade.

Dealing with $\mathcal{O}(1\ \%)$ statistical accuracy.

CLAS 12 pseudo-data (M. Guidal and H. Avakian)



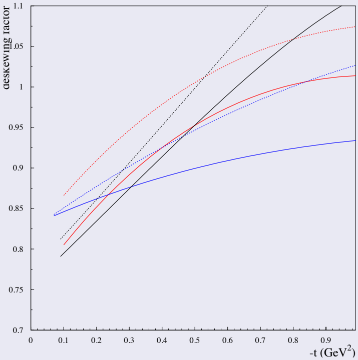


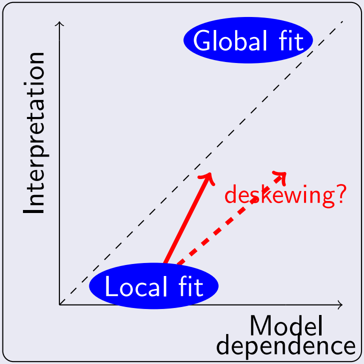
Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

JLab's 12 GeV upgrade.

Dealing with $\mathcal{O}(1\ \%)$ statistical accuracy.

Model-estimate of $H(\xi, 0, t)/H(\xi, \xi, t)$





Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

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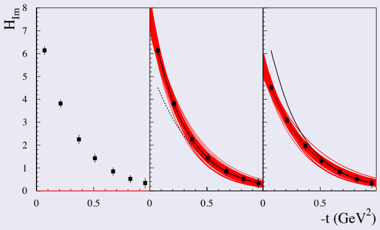
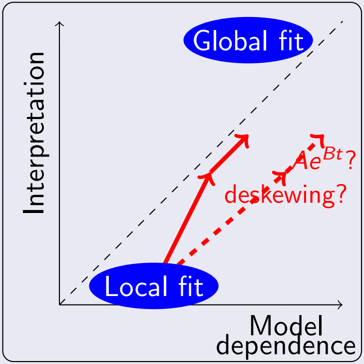
JLab's 12 GeV upgrade.

Dealing with $\mathcal{O}(1\ \%)$ statistical accuracy.

Extracted $Im\mathcal{H}$ as function of t and Ae^{Bt} fit



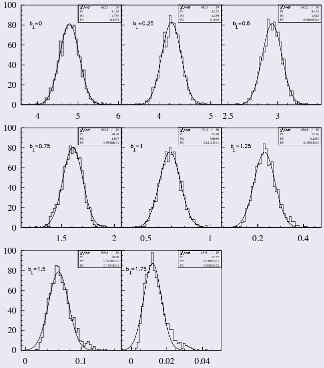


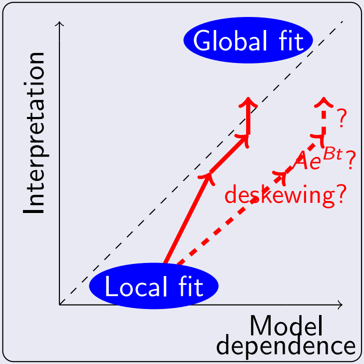
Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

JLab's 12 GeV upgrade.

Dealing with $\mathcal{O}(1\ \%)$ statistical accuracy.

2D Fourier transform of fit function (error propagation)



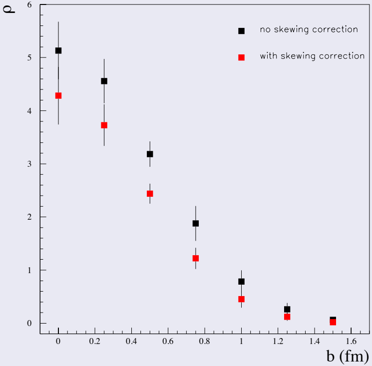


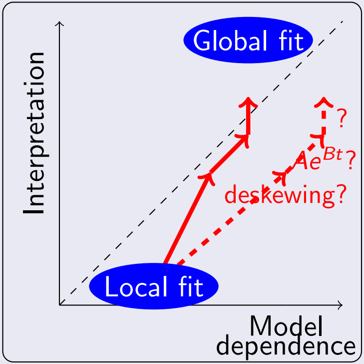
Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

JLab's 12 GeV upgrade.

Dealing with $\mathcal{O}(1\ \%)$ statistical accuracy.

$b_\perp$-dependence of spatial density



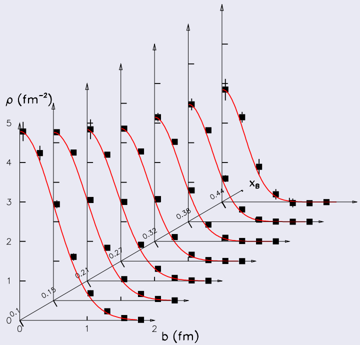


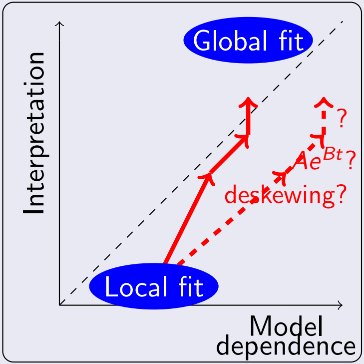
Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

JLab's 12 GeV upgrade.

Dealing with $\mathcal{O}(1\ \%)$ statistical accuracy.

Spatial density as function of x_B



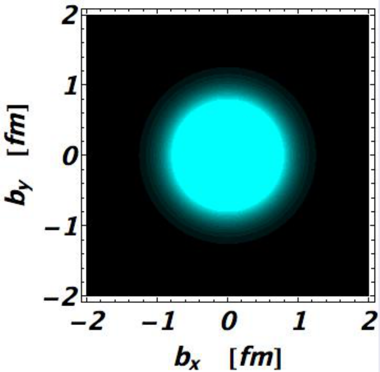


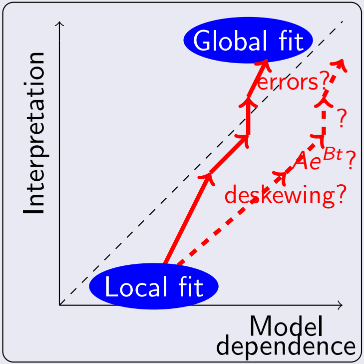
Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

JLab's 12 GeV upgrade.

Dealing with $\mathcal{O}(1\ \%)$ statistical accuracy.

Contour plot of spatial charge density





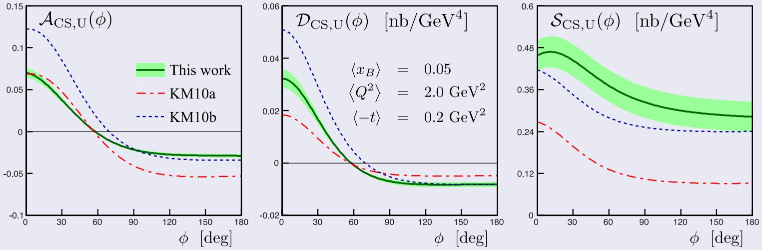
Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

COMPASS-II.

Kinematic domain in between collider and fixed-target experiments.

- Observables with **beam spin** and **beam charge** differences.

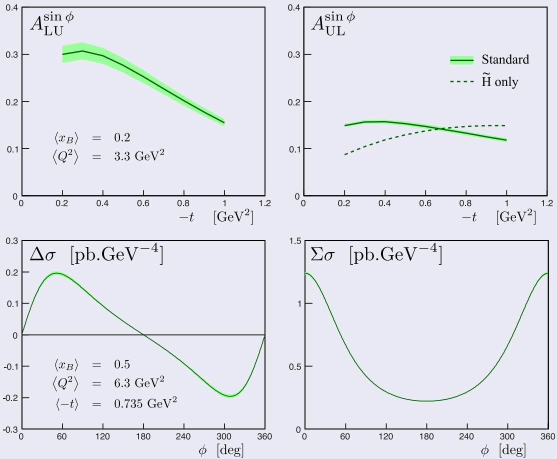
GK model prediction for COMPASS-II



JLab's 12 GeV upgrade.

Dealing with 1 % statistical accuracy.

GK model prediction for JLab 12 GeV



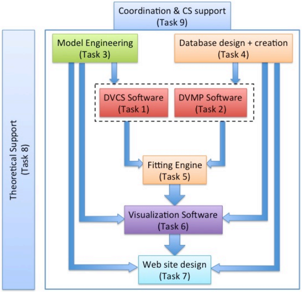
GPD phenomenology toolkit.

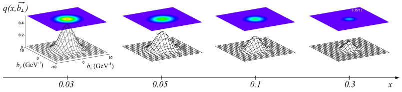
The path between models and data.

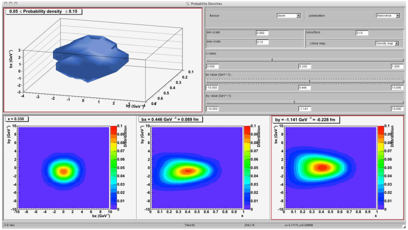
1. Comprehensive **database of experimental results**.
2. Comprehensive **database of theoretical predictions**.
3. **Fitting engine**.
4. **Propagation** of statistic and systematic **uncertainties**.
5. **Visualizing software** to compare experimental results and model expectations.
6. Connection to **experimental set-up descriptions** to design new experiments.
7. **Interactive website** providing free access to model and experimental values.

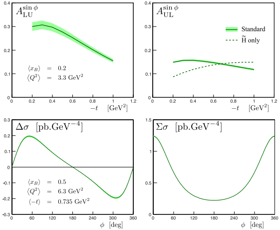
GPD phenomenology toolkit.

Platform structure, existing pieces and planned development.









Conclusions.

Investigation of the nucleon structure has dramatically changed our understanding of the strong interaction.

- Today our picture of the nucleon as a system built up from quarks and gluons is vastly different from the naive quark model image.
- **Numerous progress** have been made in **recent years** in clarifying concepts and obtaining quantitative information on nucleon structure.
- The next generation of experimental facilities will provide a great wealth of information **with unprecedented accuracy**.
- 3D nucleon structure is one of the **science highlights** of a possible future Electron Ion Collider.
- Nucleon structure is a very lively field where theoretical progress goes along with experimental advances.

Prospect: Electron Ion Collider.

Motivation.

- A collider is needed to provide kinematic reach well into the **gluon-dominated regime**.
- Electron beams are needed to bring to bear the **unmatched precision** of the electromagnetic interaction as a probe.
- Polarized nucleon beams are needed to determine the **correlations** of sea quark and gluon distributions **with the nucleon spin**.
- Heavy ion beams are needed to provide precocious access to the regime of **saturated gluon densities** and offer a precise dial in the study of propagation-length for color charges in nuclear matter.

Accardi *et al.* , *EIC White Paper*, arXiv:1212.1701

Prospect: Electron Ion Collider.

Technical goals.

The EIC machine designs are aimed at achieving:

- **Highly polarized** ($\sim 70\%$) electron and nucleon beams.
- **Ion beams** from deuteron to the heaviest nuclei (uranium or lead).
- Variable center of mass energies from $\sim 20 - \sim 100$ GeV, upgradable to ~ 150 GeV.
- **High collision luminosity** $\sim 10^{33-34}$ cm^{-2}s^{-1}.
- Possibilities of having more than one interaction region.

Accardi *et al.* , *EIC White Paper*, arXiv:1212.1701

Prospect: Electron Ion Collider.

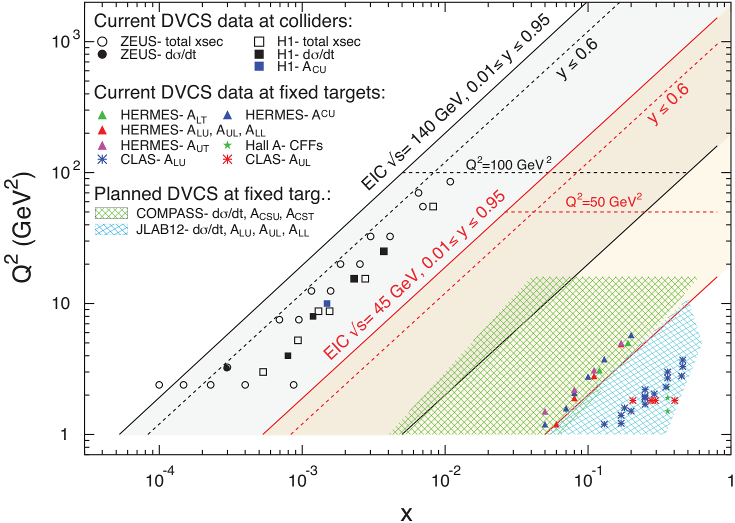
Key measurements for transverse plane parton imaging at stages I and II.

Deliverables	Observables	What we learn	Requirements
GPDs of sea quarks and gluons	DVCS and J/ψ, ρ^0, ϕ production cross-section and polarization asymmetries	transverse spatial distrib. of sea quarks and gluons; total angular momentum and spin-orbit correlations	$\int dt\, L \sim 10$ to 100fb^{-1}; leading proton detection; polarized e^- and p beams; wide range of x and Q^2; range of beam energies; e^+ beam valuable for DVCS
GPDs of valence and sea quarks	electro-production of π^+, K and ρ^+, K^*	dependence on quark flavor and polarization	

Accardi *et al.* , *EIC White Paper*, arXiv:1212.1701

Prospect: Electron Ion Collider.

Existing and planned measurements of DVCS in the x, Q^2 plane.



Accardi *et al.* , *EIC White Paper*, arXiv:1212.1701

Thank you for your attention!

Questions? Comments? etc. $\Rightarrow$ herve.moutarde@cea.fr

Appendix

On the discovery of the nucleon spin.

Experiment may be conducted without establishing thermal equilibrium between molecules with even J and those with odd J: separate gases which do not interconvert.

Modern Quantum Mechanical Solution (David Dennison, 1927)

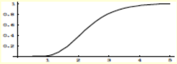
Consider a hydrogen molecule (rigid rotator —two degrees of freedom)

$$E_n = n(n+1)\frac{h^2}{8\pi^2 J}, \quad n = 0,1,2\,\dots$$

The requirements of wave function symmetries and nuclear spin $\Rightarrow$ **two varieties** of molecular hydrogen:

parahydrogen

orthohydrogen



specific heats of para- and orthohydrogen are quite different at low temperatures

ORTHOHYDROGEN $\Leftrightarrow$ PARAHYDROGEN TRANSITION IS SLOW

Gearhart, APS, Saint Louis, Apr. 2008

See also Tomonaga, *The story of spin*, Chicago, 1997.

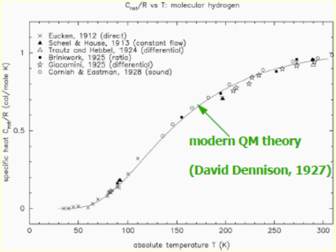
On the discovery of the nucleon spin.

Experiment may be conducted without establishing thermal equilibrium between molecules with even J and those with odd J: separate gases which do not interconvert.

THEORY AND EXPERIMENT BY THE LATE 1920s

To calculate the specific heat of molecular hydrogen:

Treat hydrogen as a mixture of para- and orthohydrogen, in the ratio 1:3 (room temperature ratio for spin 1/2 fermions).



Dennison's theory predicts a value for the moment of inertia of molecular hydrogen much larger than the accepted value in 1925.

Gearhart, APS, Saint Louis, Apr. 2008

See also Tomonaga, *The story of spin*, Chicago, 1997.

Back to lecture.

A brief history of the nucleon
Discovery
Nucleon spatial structure
Nucleon charge radius
Operator Product Expansion
Principle
From theory to measure
Models
Going further
From measure to theory
Fitting strategies

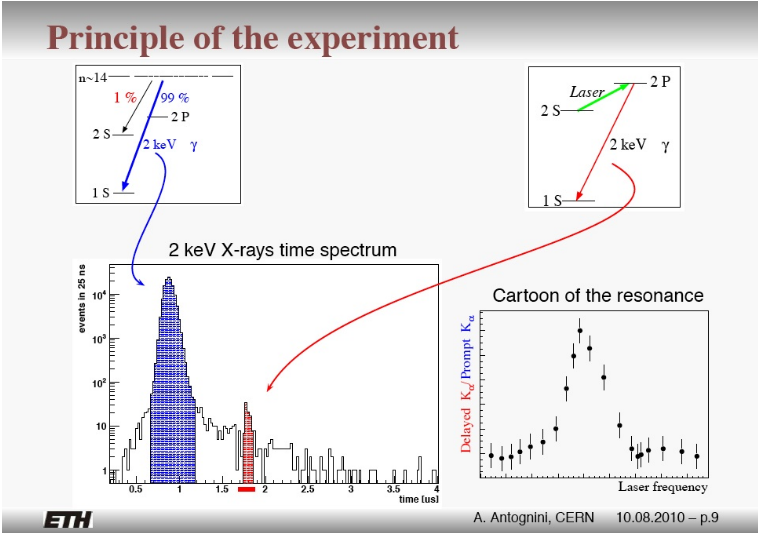
Muonic hydrogen: "Lamb shift" measurement.

Experimental principle and design.

- New beam line to produce μ^- with a low kinetic energy: $\simeq$ 5 keV.
- Muons are stopped in gazeous H_2 at low density (1 hPa). Formation of highly excited **muonic hydrogen** ($n = 14$).
- Desexcitation of μp in state 1S (99 %). The population of the 2S state (1 %) is long-lived.
- Transitions 2S $\rightarrow$ 2P induced by laser. Fast desexcitation 2P $\rightarrow$ 1S by emission of a X photon (1.9 keV).

Muonic hydrogen: "Lamb shift" measurement.

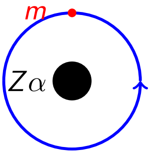
Experimental principle and design.



Why muonic hydrogen?

Reminders : Bohr radius, Rydberg constant, etc.

- Classical mechanics :



- Compensation of **coulombic** and **central** forces:

$$m\frac{v^2}{r} = \frac{Z\alpha\hbar c}{r^2}$$

- Mechanical energy $= f(r)$:

$$E = -\frac{Z\alpha\hbar c}{2r}$$

- Quantization of $L = mvr$:

$$\text{Bohr radius } a_0 = \frac{\hbar}{mc\alpha} \quad \text{and} \quad E = -\frac{(Z\alpha)^2 mc^2}{2n^2}$$

- Rydberg constant:

$$R_\infty = \alpha^2 \frac{m_e c}{2h}$$

$$hcR_\infty = 13.605\,691\,93\;(34)$$

Why muonic hydrogen?

Lamb shift (following Welton, Phys. Rev. **74**, 1157 (1948)).

- Field fluctuations $\Rightarrow$ **spread of the charge** of the orbiting particle:

$$< r^2 > \simeq \frac{1}{m^2}\frac{2\alpha}{\pi}\log(Z\alpha)^{-2}$$

- Charge radius $< r^2 > \neq 0 \Rightarrow$ **perturbation of coulombic potential** $V = -Z\alpha/r$:

$$\delta V = \frac{2\pi}{3} Z\alpha < r^2 > \delta(\vec{r})$$

- Consequence: correction of energy level S :

$$\Delta E = \langle nS|\delta V|nS\rangle \simeq |\Psi_n(0)|^2 \frac{2\pi(Z\alpha)}{3} < r^2 >$$

- Remark: non-relativistic wavefunction $\psi_{2S}(0) = 1/\sqrt{8\pi a_0^3}$
- **Stronger effect** for muonic hydrogen..

Back to lecture.

Operator Product Expansion (OPE).

Ill-defined field products (following Lehman, Riv. Nuovo Cim. **11**, 342 (1954)).

- Consider a scalar field $\phi(x)$ (mass m) and a complete set of states $|p, \alpha\rangle$ (momentum p, quantum number α).
- Use $\phi(x) = e^{i\mathbb{P}\cdot x}\phi(0)e^{-i\mathbb{P}\cdot x}$ to write:

$$\begin{aligned}\langle 0 | \phi(x)^2 | 0 \rangle &= \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\sum_\alpha \langle 0 | \phi(x) | p, \alpha\rangle \langle p, \alpha | \phi(x) | 0\rangle \\ &= \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{1}{2E_{\vec{p}}}\sum_\alpha |\langle 0 | \phi(0) | p, \alpha\rangle|^2\end{aligned}$$

- Take a **1-particle state** $|p, \alpha\rangle$. Then $|\langle 0 | \phi(0) | p, \alpha\rangle|^2$ depends on $p^2 = m^2$ and:

$$\langle 0 | \phi(x)^2 | 0 \rangle \geq \sum_\alpha |\langle 0 | \phi(0) | \text{1-particle}, \alpha\rangle|^2 \int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{1}{2E_{\vec{p}}} = +\infty$$

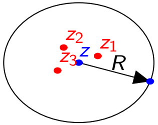
- $\phi(x)^2$ is **not mathematically well-defined**.

Back to lecture.

Nonperturbative proof of the OPE (1/2).

(Following Weinberg, *The quantum theory of fields*, CUP, 1996).

$$|\mathbf{x_i} - \mathbf{x_j}| \ll \mathbf{R} \ll |\mathbf{z} - \mathbf{y_k}| \quad \forall \mathbf{i}, \mathbf{j}, \mathbf{k}$$



- $A_1(z_1)$, $A_2(z_2)$, ...: $z_i \to z$.
- $B_1(y_1)$, $B_2(y_2)$, ...: fixed y_i.
- Operators built from fields ϕ_n.

- Surround point z with ball $\mathcal{B}(R)$ and decompose action:

$$\langle 0 \left| T\left(A_1(z_1) A_2(z_2) \dots B_1(y_1) B_2(y_2) \dots \right) \right| 0 \rangle$$

$$= \int \prod_{u \notin \mathcal{B}(R), n} \mathrm{d}\phi_n(u) B_1(y_1) B_2(y_2) \dots \exp\left(i \int_{u \notin \mathcal{B}(R)} \mathrm{d}^4 u \, \mathcal{L} \right)$$

$$\times \int \prod_{u \in \mathcal{B}(R), n} \mathrm{d}\phi_n(u) A_1(z_1) A_2(z_2) \dots \exp\left(i \int_{u \in \mathcal{B}(R)} \mathrm{d}^4 u \, \mathcal{L} \right)$$

Nonperturbative proof of the OPE (2/2).

(Following Weinberg, *The quantum theory of fields*, CUP, 1996).

- Fields inside the ball merge smoothly with fields outside the ball at the surface of the ball.
- Path integrals over fields inside and outside the ball do not affect each other.
- Path integral over fields inside the ball can be expressed in terms of values and derivatives of fields on the surface of the ball.
- For a small ball, express these values and derivatives in terms of Taylor expansion of fields located at the ball center z with coefficients depending on $z_i - z$.
- Take the limit $R \to 0$.

Back to lecture.

Double Distribution models.

VGG model (Vanderhaeghen, Guichon and Guidal).

- **Factorized Ansatz.**

$$\begin{aligned} H(x,\xi,t) &= \int_{|\alpha|+|\beta|\leq 1} d\beta d\alpha\, \delta(\beta+\xi\alpha-x) f(\beta,\alpha,t) \\ f(\beta,\alpha,t) &= \frac{1}{|\beta|^{\alpha'(1-\beta)t}} h(\beta)\pi_n(\beta,\alpha) \\ \pi_n(\beta,\alpha) &= \frac{\Gamma(2n+2)}{2^{2n+1}\Gamma^2(n+1)} \frac{(1-|\beta|)^2-\alpha^2]^n}{(1-|\beta|)^{2n+1}} \end{aligned}$$

- Expressions for h and n :

$$\begin{aligned} h_g(\beta) &= |\beta| g(|\beta|) & n_g &= 1 \\ h^q_{\mathrm{sea}}(\beta) &= q_{\mathrm{sea}}(|\beta|)\mathrm{sign}(\beta) & n_{\mathrm{sea}} &= 1 \\ h^q_{\mathrm{val}}(\beta) &= q_{\mathrm{val}}(\beta)\Theta(\beta) & n_{\mathrm{val}} &= 1 \end{aligned}$$

- Add D-term at $z = x/\xi$:

$$D(z) \simeq (1-z^2)\Big(-4.C_1^{3/2}(z) - 1.2C_3^{3/2}(z) - 0.4C_5^{3/2}(z)\Big)$$

Back to lecture.

Guidal *et al.*, Phys. Rev. **D72**, 054013 (2005)

Dual model.

PV model (Polyakov and Vanderhaeghen) (1/2).

- t-channel **partial-wave expansion**:

$$H_+(x,\xi) = 2\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{l=0\\ \text{even}}}^{n+1} B_{nl}\theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right)C_n^{3/2}\left(\frac{x}{\xi}\right)P_l\left(\frac{1}{\xi}\right)$$

- Introduce **forward-like** functions $Q_k(x,t)$ defined by:

$$B_{n\ n+1-k}(t) = \int_0^1 dx\, x^n Q_k(x,t)$$

- Resum series by means of **Shuvaev transform**:

$$H_+(x,\xi,t) = \sum_{k=0}^{\infty}\int_0^1 dy\, K^{(k)}(x,\xi,y)Q_k(y,t)$$

with **known kernels** $K^{(k)}$.

Polyakov and Shuvaev, hep-ph/0207153

Dual model.

PV model (Polyakov and Vanderhaeghen) (2/2).

- Define **quintessence** from forward-like functions:

$$N(x, t) = \sum_{n=0}^{\infty} x^{2n} Q_{2n}(x, t)$$

- $Re\mathcal{H}$ and $Im\mathcal{H}$ depend **on** $\mathbf{N(x, t)}$ **only**:

Dual model.

PV model (Polyakov and Vanderhaeghen) (2/2).

- Define **quintessence** from forward-like functions:

$$N(x,t) = \sum_{n=0}^{\infty} x^{2n} Q_{2n}(x,t)$$

- $Re\mathcal{H}$ and $Im\mathcal{H}$ depend **on $\mathbf{N(x,t)}$ only**:

$$Re\mathcal{H} = 2\int_0^{\frac{1-\sqrt{1-\xi^2}}{\xi}} \frac{dx}{x} N(x,t) \left[\frac{1}{\sqrt{1-2x/\xi+x^2}} + \frac{1}{\sqrt{1+2x/\xi+x^2}} - \frac{2}{\sqrt{1+x^2}} \right]$$

$$Im\mathcal{H} = 2\int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^{1} \frac{dx}{x} N(x,t) \frac{1}{\sqrt{2x/\xi-x^2-1}}$$

Polyakov and Shuvaev, hep-ph/0207153

Polyakov, Phys. Lett. **B659**, 542 (2008)

Dual model.

PV model (Polyakov and Vanderhaeghen) (2/2).

- Define **quintessence** from forward-like functions:

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Polyakov and Shuvaev, hep-ph/0207153

Polyakov, Phys. Lett. **B659**, 542 (2008)

- The relation between $Im\mathcal{H}$ and $N(x,t)$ can be inverted:

$$N(x,t) = \frac{1}{\pi} \frac{x(1-x^2)}{(1+x)^{\frac{3}{2}}} \int_{\frac{2x}{1+x}}^{1} \frac{d\xi}{\xi^{\frac{3}{2}}} \frac{1}{\sqrt{\xi - \frac{2x}{1+x^2}}} \left(\frac{1}{2} - \xi \frac{d}{\xi} \right) Im\mathcal{H}(\xi,t)$$

Dual model.

PV model (Polyakov and Vanderhaeghen) (2/2).

- Define **quintessence** from forward-like functions:

$$N(x, t) = \sum_{n=0}^{\infty} x^{2n} Q_{2n}(x, t)$$

- $Re\mathcal{H}$ and $Im\mathcal{H}$ depend **on $\mathbf{N}(\mathbf{x}, \mathbf{t})$ only**:

  Polyakov and Shuvaev, hep-ph/0207153

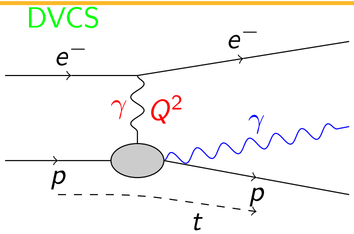
  Polyakov, Phys. Lett. **B659**, 542 (2008)

- The relation between $Im\mathcal{H}$ and $N(x, t)$ can be inverted:
- The quintessence function contains **all the information** that can be extracted from DVCS **at leading order**.
- Here model forward-like function Q_0 from PDFs with Regge-type Ansatz: $q(x, t) = q(x)e^{-\alpha' t}$.

Back to lecture.

Timelike and spacelike Compton Scattering.

Scattering amplitudes and their partonic interpretation.

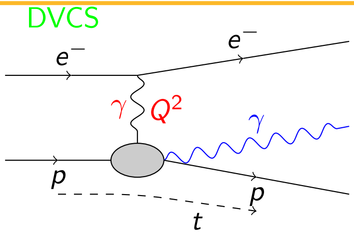


Compton Form Factors (CFF)

- Parametrize amplitudes.

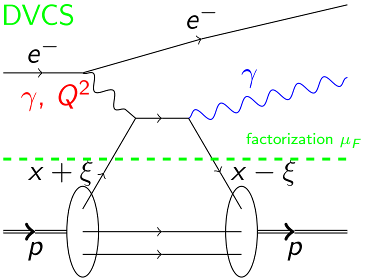
Timelike and spacelike Compton Scattering.

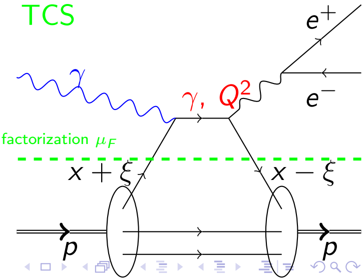
Scattering amplitudes and their partonic interpretation.



Compton Form Factors (CFF)

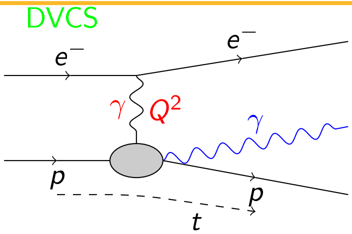
- Parametrize amplitudes.
- Evaluation at LO.





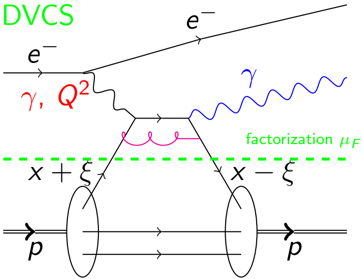
Timelike and spacelike Compton Scattering.

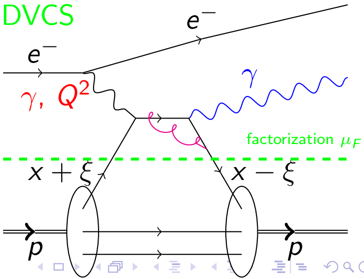
Scattering amplitudes and their partonic interpretation.



Compton Form Factors (CFF)

- Parametrize amplitudes.
- Evaluation at LO.
- Evaluation at NLO.





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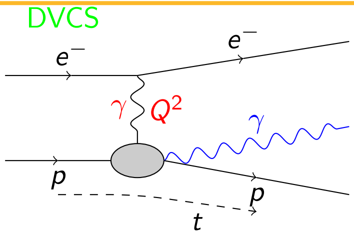
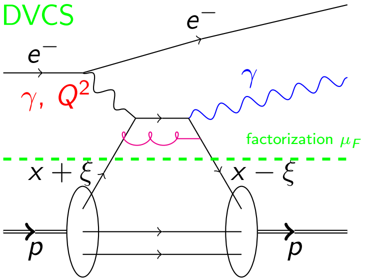
Timelike and spacelike Compton Scattering.

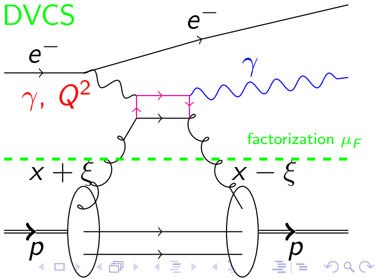
Scattering amplitudes and their partonic interpretation.



Compton Form Factors (CFF)

- Parametrize amplitudes.
- Evaluation at LO.
- Evaluation at NLO.
- Other diagrams at NLO, including gluon GPDs.





Explicit expressions of Compton Form Factors.

Quark and gluon contributions to the CFF $\mathcal{H}$ at LO and NLO (at fixed t).

- Convolution of singlet GPD $H_q^+(x) \equiv H_q(x) - H_q(-x)$:

$$\mathcal{H}_q(\xi, Q^2) = \int_{-1}^{+1} dx\, H_q^+(x, \xi, \mu_F)\;\; T_q\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) + \int_{-1}^{+1} dx\, H_g(x, \xi, \mu_F)\;\; T_g\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right)$$

Belistky and Müller, Phys. Lett. **B417**, 129 (1998)
Pire *et al.* , Phys. Rev. **D83**, 034009 (2011)

Explicit expressions of Compton Form Factors.

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- Convolution of singlet GPD $H_q^+(x) \equiv H_q(x) - H_q(-x)$:

$$\mathcal{H}_q(\xi, Q^2) \stackrel{\text{LO}}{=} \int_{-1}^{+1} dx\, H_q^+(x, \xi, \mu_F)\ C_0^q(x, \xi) + \int_{-1}^{+1} dx\, H_g(x, \xi, \mu_F)\ 0$$

Belistky and Müller, Phys. Lett. **B417**, 129 (1998)
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- Integration yields **imaginary** parts to $\mathcal{H}$:

$$Im\mathcal{H}_q(\xi, Q^2) \stackrel{\text{LO}}{=} \pi H_q^+(\xi, \xi, \mu_F)$$

Explicit expressions of Compton Form Factors.

Quark and gluon contributions to the CFF $\mathcal{H}$ at LO and NLO (at fixed t).

- Convolution of singlet GPD $H_q^+(x) \equiv H_q(x) - H_q(-x)$:

$$\mathcal{H}_q(\xi, Q^2) \stackrel{\text{NLO}}{=} \int_{-1}^{+1} dx\, H_q^+(x, \xi, \mu_F) \left[C_0^q + C_1^q + \frac{1}{2} \ln \frac{|Q^2|}{\mu_F^2} C_{\text{Coll}}^q \right] + \int_{-1}^{+1} dx\, H_g(x, \xi, \mu_F) \left(0 + C_1^g + \frac{1}{2} \ln \frac{|Q^2|}{\mu_F^2} C_{\text{Coll}}^g \right)$$

Belistky and Müller, Phys. Lett. **B417**, 129 (1998)
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- Integration yields **imaginary** parts to $\mathcal{H}$:

$$Im\mathcal{H}_q(\xi, Q^2) \stackrel{\text{NLO}}{=} \mathcal{I}(\xi) H_q^+(\xi, \xi, \mu_F) + \int_{-1}^{+1} dx\, \mathcal{T}^q(x) \Big(H_q^+(x, \xi, \mu_F) - H_q^+(\xi, \xi, \mu_F) \Big) + \text{gluon contributions.}$$

Explicit expressions of Compton Form Factors.

Compton Scattering: LO vs NLO.

Imaginary part of Compton Form Factor $\mathcal{H}_q$ at NLO:

$$
\begin{aligned}
Im\mathcal{H}_q(\xi, Q^2) \overset{\text{NLO}}{=} & \ \mathcal{I}(\xi) H_q^+(\xi, \xi, \mu_F) \\
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\end{aligned}
$$

Due to $\mathcal{O}(\alpha_S(\mu_F))$ corrections:

Explicit expressions of Compton Form Factors.

Compton Scattering: LO vs NLO.

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Imaginary part of Compton Form Factor $\mathcal{H}_q$ at NLO:

$$\begin{aligned} Im\mathcal{H}_q(\xi, Q^2) \overset{\text{NLO}}{=} & \ \mathcal{I}(\xi) H_q^+(\xi, \xi, \mu_F) \\ & + \int_{-1}^{+1} dx\, \mathcal{T}^q(x) \Big(H_q^+(x, \xi, \mu_F) - H_q^+(\xi, \xi, \mu_F) \Big) \\ & + \text{gluon contributions.} \end{aligned}$$

Due to $\mathcal{O}(\alpha_S(\mu_F))$ corrections:

- $Im\mathcal{H}_q$ is **no more equal** to $\pi H_q^+(x = \xi, \xi)$ (LO):

Explicit expressions of Compton Form Factors.

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- $Im\mathcal{H}_q$ is **no more equal** to $\pi H_q^+(x = \xi, \xi)$ (LO):
  - Multiplicative factor $\mathcal{I}$ depends on ξ.

Explicit expressions of Compton Form Factors.

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\end{aligned}
$$

Due to $\mathcal{O}(\alpha_S(\mu_F))$ corrections:

- $Im\mathcal{H}_q$ is **no more equal** to $\pi H_q^+(x = \xi, \xi)$ (LO):
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  - Integral with **off-diagonal terms**.

Explicit expressions of Compton Form Factors.

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  - Integral with **off-diagonal terms**.
  - $Im\mathcal{H}_q$ **contains** gluon contributions.

Explicit expressions of Compton Form Factors.

Compton Scattering: LO vs NLO.

Imaginary part of Compton Form Factor $\mathcal{H}_q$ at NLO:

$$
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  - Integral with **off-diagonal terms**.
  - $Im\mathcal{H}_q$ **contains** gluon contributions.
- **No more direct link** to H_q even in valence region where $H_q(-\xi, \xi)$ is expected to be small.

Explicit expressions of Compton Form Factors.

Compton Scattering: LO vs NLO.

Imaginary part of Compton Form Factor $\mathcal{H}_q$ at NLO:

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Im\mathcal{H}_q(\xi, Q^2) \overset{\text{NLO}}{=} & \ \mathcal{I}(\xi) H_q^+(\xi, \xi, \mu_F) \\
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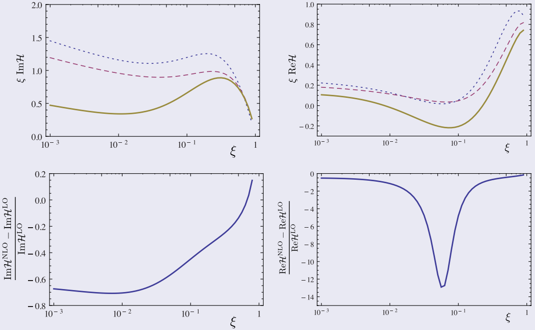
- **No more direct link** to H_q even in valence region where $H_q(-\xi, \xi)$ is expected to be small.

Question: What is the size of these $\mathcal{O}(\alpha_S(\mu_F))$ corrections?

Next-to-Leading Order computations.

Large gluon contributions, maximum in the HERMES / COMPASS region.

$\mathcal{H}$ at LO and NLO ($t = -0.1$ GeV^2, $Q^2 = \mu_F^2 = 4.$ GeV^2)



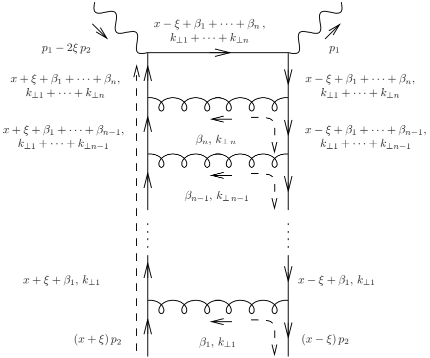
Moutarde *et al.* , Phys. Rev. **D87**, 054029 (2013)

dotted: LO dashed: NLO quark corrections solid: full NLO

Back to lecture.

Resummation of DVCS.

Resum the leading logarithmic singularity at all orders around $x = \xi$.



Back to lecture.

Altinoluk *et al.* , arXiv:1309.2508, JHEP **1210**, 049 (2012)

Notation: $D = \sqrt{\frac{\alpha_s C_F}{2\pi}}$.

$$(C_0 + C_1)^{res} = \frac{e_q^2}{x - \xi + i\epsilon}\left\{ \cosh\left[D \log\left(\frac{\xi - x}{2\xi} - i\epsilon \right)\right] - \frac{D^2}{2}\left[9 + 3\frac{\xi - x}{x + \xi}\log\left(\frac{\xi - x}{2\xi} - i\epsilon \right)\right]\right\} - (x \to -x)\,.$$

Profile function modification.

Different ways of applying Radyushkin's Double Distribution Ansatz (RDDA).

- **Ambiguity** in the definition of DD: The following transformation

$$\begin{aligned} F^q(\beta,\alpha) &\rightarrow F^q(\beta,\alpha) + \frac{\partial\sigma^q}{\partial\alpha}(\beta,\alpha), \\ G^q(\beta,\alpha) &\rightarrow G^q(\beta,\alpha) - \frac{\partial\sigma^q}{\partial\beta}(\beta,\alpha), \end{aligned}$$

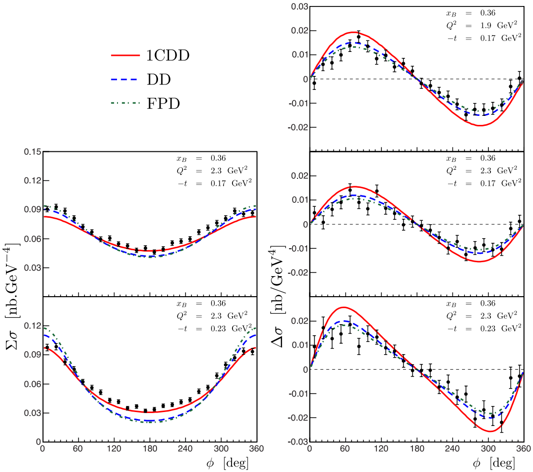
  gives rise to the **same** GPD models.

- This equivalence is **broken** when applying the RDDA, which can be done in infinitely many ways.

- Comparison to data to make the best Ansatz.

Profile function modification.

Different ways of applying Radyushkin's Double Distribution Ansatz (RDDA).



Mezrag *et al.* , Phys. Rev. **D88**, 014001 (2013)

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Finite-t and target mass corrections.

Recently derived. Preliminary phenomenological estimates.

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where H, E and $X = H + E$ are generalized parton distributions

$$F \otimes C \equiv \int dx\, F(x,\xi,t)\, C(x,\xi)\,.$$

and the coefficient functions $C_k^{\pm}(x,\xi)$ are given by the following expressions:

$$\boxed{\begin{aligned}
C_0^{\pm}(x,\xi) &= \frac{1}{\xi + x - i\epsilon} \pm \frac{1}{\xi - x - i\epsilon}\,,\\
C_1^{\pm}(x,\xi) &= \frac{1}{x-\xi}\ln\left(\frac{\xi+x}{2\xi} - i\epsilon\right) \pm (x \leftrightarrow -x)\,,\\
C_2^{\pm}(x,\xi) &= \left\{\frac{1}{\xi+x}\left[\mathrm{Li}_2\left(\frac{\xi-x}{2\xi} + i\epsilon\right) - \mathrm{Li}_2(1)\right] \pm (x \leftrightarrow -x)\right\} + \frac{1}{2}C_1^{\pm}(x,\xi)\,.
\end{aligned}}$$

- Complete results available for all helicity amplitudes
- Factorization checked to $1/Q^2$ accuracy
- Gauge and translation invariance checked to $1/Q^2$ accuracy

Back to lecture.

Finite-t and target mass corrections.

Recently derived. Preliminary phenomenological estimates.

- **General remarks**
  - The singularities at $x = \pm\xi$ in $1/Q^2$ corrections are *weaker* (logarithmic) as compared to the leading term. Thus factorization is not endangered.
  - For scalar targets, target mass corrections only enter via $|P_\perp|^2 \sim |t - t_{\min}|$; naive Nachtmann-type corrections are always overcompensated by finite-t effects.
  - For the nucleon, the main new effect is that Compton form factor $\mathcal{H}$ receives a sizable mass correction proportional to E-distribution, and vice versa.

Back to lecture.

A brief history of the nucleon
Discovery

Nucleon spatial structure
Nucleon charge radius

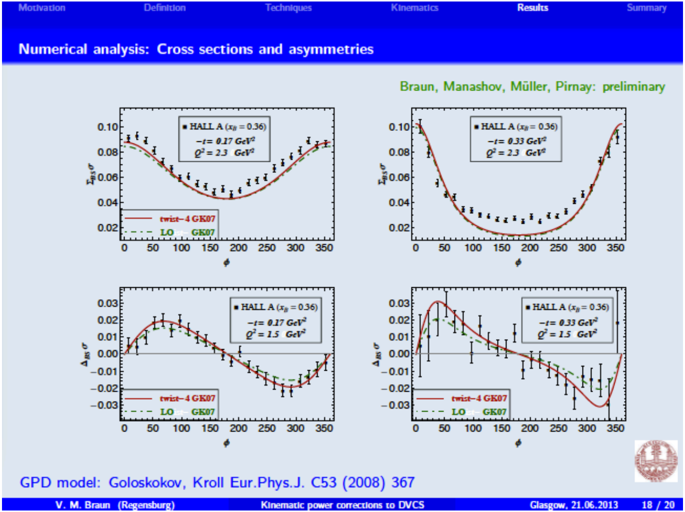
Operator Product Expansion
Principle

From theory to measure
Models
Going further

From measure to theory
Fitting strategies

Finite-t and target mass corrections.

Recently derived. Preliminary phenomenological estimates.



Back to lecture.

Overview of current extraction methods.

Problems: Model dependence ? Degrees of freedom ? Extrapolations ?

Local fits

Take each kinematic bin independantly of the others.
Extraction of $Re\mathcal{H}$, $Im\mathcal{H}$, ... as independent parameters.

Global fit

Take all kinematic bins at the same time. Use a parametrization of GPDs or CFFs.

Hybrid : Local / global fit

Start from local fits and add smoothness assumption.

Neural networks

Already used for PDF fits. Exploratory stage for GPDs.

Overview of current extraction methods.

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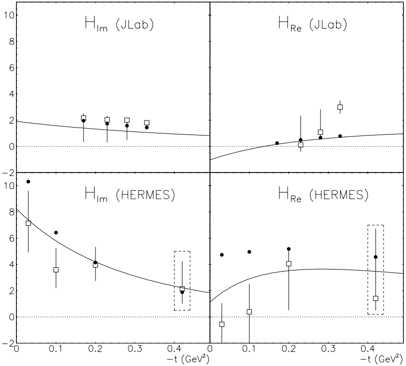

- □ : "7-CFF" fit results.
- • : VGG model.
- − : KM fit.

Guidal and Moutarde, Eur. Phys. J. **A42**, 7 (2009)

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Overview of current extraction methods.

Problems: Model dependence ? Degrees of freedom ? Extrapolations ?

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

where $a = \mathcal{O}(Q^{-1}), \quad b = \mathcal{O}(Q^{-4}), \quad c = \mathcal{O}(Q^{-1}),$
$d = \mathcal{O}(Q^{-2}), \quad e = \mathcal{O}(Q^{-5}).$

Overview of current extraction methods.

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Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

- **Underconstrained** problem (8 fit parameters : real and imaginary parts of 4 CFFs $\mathcal{H}$, $\mathcal{E}$, $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$).

Overview of current extraction methods.

Problems: Model dependence ? Degrees of freedom ? Extrapolations ?

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin \phi + b \sin 2\phi}{1 + c \cos \phi + d \cos 2\phi + e \cos 3\phi}$$

- **Underconstrained** problem.
- Need other asymmetries on **same** kinematic bin to allow extraction of **all CFFs** (or **add** $\simeq$ 5-10 % **systematic uncertainty**).

Overview of current extraction methods.

Problems: Model dependence ? Degrees of freedom ? Extrapolations ?

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- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

- **Underconstrained** problem.
- Need other asymmetries on **same** kinematic bin to allow extraction of **all CFFs**.
- Add physical input? **dispersion relations**, etc.

Kumericki *et al.* , arXiv:1301.1230

Guidal *et al.* , Rept. Prog. Phys. **76**, 066202 (2013)

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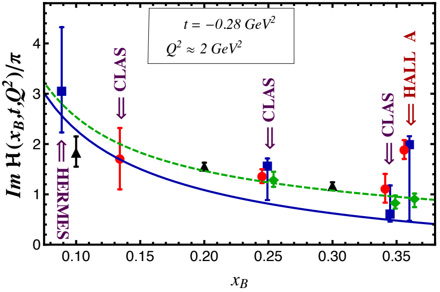

Kumericki and Müller, Exclusive 2010

- Without Hall A data.
- With Hall A data.
- △ : neural network.
- □ : "7-CFF" fit results.
- ◇ : "$\mathcal{H} - \tilde{\mathcal{H}}$".
- ○ : hybrid fits.

Overview of current extraction methods.

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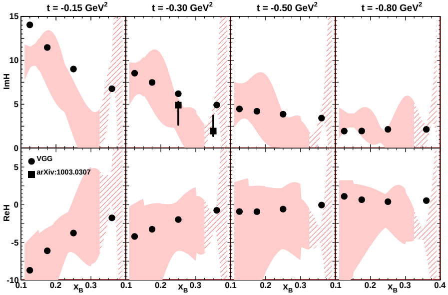
Hybrid : Local / global fit

Start from local fits and add smoothness assumption.



- Comparison to VGG model on JLab Hall B kinematics.
- Loss of information during the extraction.

Moutarde, Phys. Rev. **D79**, 094021 (2009)

Overview of current extraction methods.

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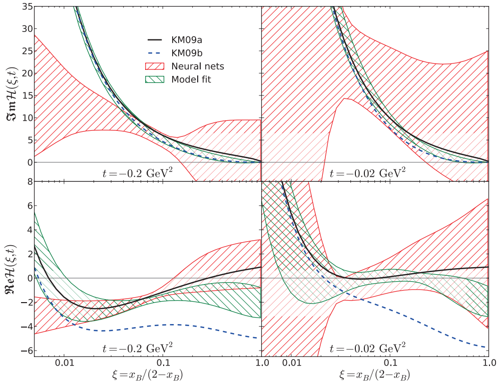

Kumericki *et al.* , JHEP **1107**, 073 (2011)

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