

Hadron interactions, color and QCD partons

3. Probing non-perturbative phenomena with perturbative tools



Yuri Dokshitzer

LPTHE Jussieu, Paris
& *PNPI, St Petersburg*

Ecole Joliot-Curie
September-October 2013

Screwing Non-Perturbative QCD with Perturbative Tools

About 15 years ago first theoretical attempts have been made to quantify genuine non-perturbative effects in perturbatively calculable (CIS) observables

The test case : the total cross section of e^+e^- annihilation into hadrons.

To predict $\sigma_{\text{tot}} \rightarrow \text{hadrons}$ one calculates instead the cross sections of quark and gluon production, $(e^+ e^- \rightarrow q \bar{q}) + (e^+ e^- \rightarrow q \bar{q} + g) + \text{etc.}$, where quarks and gluons are being treated *perturbatively* as real (un-confined, flying) objects.

The *completeness* argument provides an apology for such a brave substitution :

Once instantaneously produced by the electromagnetic (electroweak) current, the quarks (and secondary gluons) have nowhere else to go but to convert, *with unit probability*, into hadrons in the end of the day.

This *guess* looks rather solid and sounds convincing, but relies on two hidden assumptions:

1. The allowed hadron states should be *numerous* as to provide the quark-gluon system the means for “**regrouping**”, “**blanching**”, “**fitting**” into hadrons.
2. It implies that the “**production**” and “**hadronization**” stages of the process can be separated and treated independently.

1. To comply with the first assumption (“*regrouping*”) the annihilation energy has to be taken large enough, $\mathbf{s} \equiv \mathbf{Q}^2 \gg \mathbf{s}_0$. In particular, it fails miserably in the resonance region $\mathbf{Q}^2 < \mathbf{s}_0 \sim 2M_{\text{res}}^2$.

Thus, the point-by-point correspondence between hadron and quark cross sections, $\sigma^{\text{tot}}_{\text{hadr}}(\mathbf{Q}^2) \stackrel{?}{=} \sigma^{\text{tot}}_{\text{qq}}(\mathbf{Q}^2)$, cannot be sustained except at very high energies.

It can be traded, however, for something more manageable.

Invoking the dispersion relation for the photon propagator (causality = analyticity) one can relate the *energy integrals* of $\sigma^{\text{tot}}(\mathbf{s})$ with the correlator of electromagnetic currents in a deeply Euclidean region of large *negative* \mathbf{Q}^2 .

The latter corresponds to small space-like distances between interaction points, where the perturbative approach is definitely valid.

Expanding the answer in a formal series of local operators, one arrives at the structure in which the corrections to the trivial unit operator generate the usual perturbative series in powers of α_s (logarithmic corrections), whereas the vacuum expectation values of dimension-full (Lorentz- and colour-invariant) QCD operators provide non-perturbative corrections suppressed as powers of \mathbf{Q} .

This is the realm of the famous “**ITEP sum rules**” which proved to be successful in linking the parameters of the low-lying resonances in the *Minkowsky space* with expectation values characterising a non-trivial structure of the QCD vacuum in the *Euclidean space*.

ITEF “NP physics” = non-singular long range gluon fields

Bloch-Nordsieck theorem

The leaders among them are the gluon condensate $\langle \alpha_s \mathbf{G}_{\mu\nu} \mathbf{G}^{\mu\nu} \rangle$ and the quark condensate $\langle \psi \bar{\psi} \rangle$ which contribute to the total annihilation cross section, symbolically, as

$$\sigma_{\text{hadr}}^{\text{tot}}(Q^2) - \sigma_{q\bar{q}+X}^{\text{tot}}(Q^2) = c_1 \frac{\alpha_s G^2}{Q^4} + c_2 \frac{\langle \psi \bar{\psi} \rangle^2}{Q^6} + \dots$$

2. Validating the second assumption (“*production vs. hadronization*”) also calls for large Q^2 . To be able to separate the two stages of the process, it is *necessary* to have the production time of the quark pair Q^{-1} to be much smaller than the time $t_1 \sim \mu^{-1} \sim 1 \text{ fm}/c$ when the first hadron appears in the system. Whether this condition is *sufficient*, is another valid question. And a tricky one.

As we know, due to the gluon bremsstrahlung the perturbative production of secondary gluons and quark pairs spans an immense interval of time, ranging from a very short time, $t_{\text{form}} \sim Q^{-1} \ll t_1$, all the way up to a macroscopically large time $t_{\text{form}} \sim Q/\mu^2 \gg t_1$.

This accompanying radiation is responsible for formation of **hadron jets**.

It does not, however, affect the **total cross section**. It is the rare hard gluons with large energies and transverse momenta, $\sim Q$, that only matter.

This follows from the celebrated **Bloch-Nordsieck theorem** which states that the logarithmically enhanced (divergent) contributions due to real production of *collinear* and *soft* quanta cancel against the corresponding virtual corrections :

$$\sigma_{q\bar{q}+X}^{\text{tot}} = \sigma_{\text{Born}} \left(1 + \frac{\alpha_s}{\pi} [\infty_{\text{real}} - \infty_{\text{virtual}}] + \dots \right) = \sigma_{\text{Born}} \left(1 + \frac{3 C_F \alpha_s(Q^2)}{4 \pi} + \dots \right)$$

Can the Bloch-Nordsieck result hold *beyond* perturbation theory?

Looking into this problem produced an extremely interesting result that has laid a foundation for the development of perturbative techniques aimed at analysing non-perturbative effects.

V. Braun, M. Beneke and V. Zakharov have demonstrated that the real-virtual cancellation actually proceeds *much deeper* than was originally expected.

[*Phys.Rev.Lett.* **73** (1994) 3058]

- Introduce into the calculation of the radiative correction *gluon mass* m as an IR cutoff.

- Study the dependence of the answer on m .

A CIS quantity, by definition, remains finite in the limit $m=0$. This does not mean, however, that it is totally *insensitive* to the modification of gluon propagation.

In fact, the m -dependence provides a handle for analysing the *small transverse momenta* inside Feynman integrals. It is this region of integration over parton momenta where the QCD coupling gets out of control and the genuine NP physics comes onto the stage.

- Then, the sensitivity of a given CIS observable to the *infrared domain* is determined by the first non-vanishing term *non-analytic* in m^2 at $m=0$.

Bloch-Nordsieck theorem extended

In the case of one-loop analysis of the total annihilation cross section that we are discussing, one finds that in the sum of real and virtual contributions not only the terms singular at $m=0$,

$$\ln^2 m^2 \text{ and } \ln m^2,$$

cancel, as required by the *Bloch-Nordsieck theorem*,

but that the cancellation extends also to the whole tower of *finite terms*

$$m^2 \ln^2 m^2, m^2 \ln m^2, m^2, m^4 \ln^2 m^2, m^4 \ln m^2.$$

The first *non-analytic term* appears at the level of m^6 :

$$\frac{3 C_F \alpha_s}{4 \pi} \left(1 + 2 \frac{m^6}{Q^6} \ln \frac{m^2}{Q^2} + \mathcal{O}(m^8) \right)$$

It signals the presence of the non-perturbative Q^{-6} correction, which is equivalent to that of the ITP *quark condensate*.

The *gluon condensate* contribution emerges in the next order in α_s

Why “*gluon mass*”? What is it and how does it serve as a large-distance probe ?

$$\begin{array}{ccc} \text{wave function renormalization} & \xrightarrow{\text{yellow arrow}} & \text{running coupling} \\ G(k^2) = \frac{1}{-k^2 - i0} \xrightarrow{\text{green arrow}} \frac{Z(k^2)}{-k^2 - i0} = \frac{\alpha_s(-k^2)}{-k^2 - i0} & & \frac{Z(-k^2)}{Z(-\mu_R^2)} = \frac{\alpha(k^2)}{\alpha(\mu_R^2)} \end{array}$$

We want this identification to make sense in the entire k^2 plane

We know sufficiently well how $\alpha_s(-\mathbf{k}^2)$ behaves in the Euclidean region, at large negative \mathbf{k}^2 while we know next to nothing about the small- \mathbf{k}^2 region.

However, whatever the function α_s is, it had better respect **causality**.

Therefore we suppress the formal PT *tachion* (“Landau singularity”) and choose the “**physical cut**” alone, $0 < \mathbf{k}^2$, as a support for the **dispersive relation** :

$$\alpha_s(q^2) = \int_0^\infty \frac{dm^2 q^2}{(m^2 + q^2)^2} \alpha_{\text{eff}}(m^2)$$

$$\frac{d}{d \ln \mu^2} \alpha_{\text{eff}}(\mu^2) = -\frac{1}{2\pi i} \text{Disc} \{ \alpha_s(-\mu^2) \}$$

It can be formally inverted as an operator relation : $\alpha_{\text{eff}}(\mu^2) = \frac{\sin(\pi \mathcal{P})}{\pi \mathcal{P}} \alpha_s(\mu^2)$, $\mathcal{P} = \mu^2 \frac{d}{d\mu^2}$

We are ready now for a “**heavy gluon**” :

$$\frac{\alpha_s(-k^2)}{-k^2 - i0} = \int_0^\infty \frac{dm^2}{m^2} \alpha_{\text{eff}}(m^2) \cdot \frac{-d}{d \ln m^2} \frac{1}{m^2 - k^2 - i0}$$

Substitute into the Feynman diagram, and integrate first over the gluon 4-momentum :

$$V(Q^2, x) = \int_0^\infty \frac{dm^2}{m^2} \alpha_{\text{eff}}(m^2) \dot{\mathcal{F}}_V(\epsilon, x)$$

$$\epsilon = m^2 / Q^2 \quad \dot{\mathcal{F}} \equiv \frac{-d\mathcal{F}}{d \ln \epsilon}$$

\mathbf{F}_V - a “**Characteristic Function**” for the observable V

At this point there is no difference with the usual PT answer.

The CIS nature of the observable V guarantees convergence of the m^2 integration :

$\dot{\mathcal{F}}$ vanishes as a power of ϵ (ϵ^{-1}) in the $\epsilon \rightarrow 0$ ($\epsilon \rightarrow \infty$) limit.

Therefore the distribution $\dot{\mathcal{F}}$ **has a maximum** at some $\epsilon = C_V(x) = \mathcal{O}(1)$, and the integral is dominated by the large-momentum region $m^2 \sim Q^2$.

Approximating $\alpha_{\text{eff}}(m^2) \simeq \alpha_s(Q^2)$ we reproduce the one-loop PT answer :

$$V(Q^2, x) \simeq \alpha_s(Q^2) \mathcal{F}_V(0, x)$$

Using the *observable-dependent position of the maximum* of the m^2 -distribution as the scale for the coupling, $\alpha_s(C_V(x) \cdot Q^2)$, does a **better job** since it minimizes higher order effects.

The dispersive technology in this respect is close to the idea of “*commensurate scales*” (*Brodsky et al*).

At the one loop level we may substitute $\alpha_{\text{eff}}(m^2) = \alpha_s(m^2)$, develop the geometric series

$$\alpha_{\text{eff}}(m^2) \simeq \alpha_s \sum_{k=0}^{\infty} \left(\frac{\beta_0 \alpha_s}{4\pi} \ln \frac{Q^2}{m^2} \right)^k, \quad \alpha_s \equiv \alpha_s(Q^2)$$

and look for higher order perturbative corrections to our observable :

$$V(Q^2, x) - \alpha_s(Q^2) \mathcal{F}_V(0, x) \simeq \alpha_s \sum_{k=1}^{\infty} \left(\frac{\beta_0 \alpha_s}{4\pi} \right)^k R_k \quad \text{with} \quad R_k = \int_0^{\infty} \frac{d\epsilon}{\epsilon} \left(\ln \frac{1}{\epsilon} \right)^k \dot{\mathcal{F}}_V(\epsilon)$$

In the IR region :

$$\dot{\mathcal{F}}_V(\epsilon) \simeq \epsilon^p f_V(\ln \epsilon) \quad \longrightarrow \quad R_k^{\text{IR}} = \int_0^1 \frac{d\epsilon}{\epsilon} \left(\ln \frac{1}{\epsilon} \right)^k \epsilon^p f(\ln \epsilon) \sim p^{-k} k!$$

**non-Borel-
summable
series !**

Attempts to ascribe meaning to such a nasty series give rise to unphysical complex contributions at the level of Q^{-2p} terms : **INFRARED RENORMALON** problem.

This is generally interpreted as an intrinsic uncertainty in the summation of the perturbative series.

In fact, **infrared renormalons** are a purely **perturbative** phenomenon and have no direct relation to the presence of the “**Landau singularity**” in the running coupling !

The problem is of a physical nature and cannot be resolved by formal mathematical manipulations alone. It requires genuinely new physical input to obtain a sensible answer.

Let's introduce

$$\alpha_s(k^2) = \alpha^{\text{PT}}(k^2) + \alpha^{\text{NP}}(k^2)$$

It should be made clear that such a splitting is symbolic :

it represents the coupling not in terms of two **functions** but rather of two **procedures**.

Having met α^{PT} under the integral we are advised to calculate it **perturbatively**, that is in terms of (*not too long*) a series at the point $k^2 \sim Q^2$ that our integral is “sitting” around. *At the same time we are supposed not to worry about the PT-coupling being sick in the IR region.*

On the contrary, integrals with α^{NP} are determined by that very same **IR** region and **converge** :

$$\int_0^\infty \frac{dk^2}{k^2} \alpha_s^{\text{NP}}(k^2) \cdot k^{2p} = (\text{few } 100\text{s MeV})^{2p} \quad (\text{ITEF picture !})$$

Convergence of these integrals translates into vanishing of (*first few*) integer moments of α_{eff} :

$$\lim_{k \rightarrow \infty} k^{2p} \alpha^{\text{NP}}(k^2) = 0 \iff \int_0^\infty \frac{dm^2}{m^2} m^{2p} \alpha_{\text{eff}}^{\text{NP}}(m^2) = 0 \quad \left(\begin{array}{l} \text{small size instantons} \\ \mathbf{p} \leq \beta_0 \sim \mathbf{9} \end{array} \right)$$

thus explaining that mystery of “**non-analyticity**” in m^2 necessary to *trigger on* “**large distances**”

The non-analyticity of ϵ , necessary to generate NP power correction, is typically of two kinds.

- In the first case an **integer** power ϵ^p is accompanied by **logarithm(s)** of ϵ .

This is the case of **DIS** structure functions, the Drell-Yan “**K-factor**”, the width of hadronic **tau-lepton** decay, the total **e+e- annihilation** cross section :

$$\lim_{m \rightarrow 0} \dot{\mathcal{F}}_{\text{DIS}} = a(x) \cdot \frac{m^2}{Q^2} \ln \frac{Q^2}{m^2} + \dots$$

- Secondly, one may have a **half-integer p**.

This is the case for many so-called **jet-shape observables** that characterise, in a CIS manner, the structure of final states produced in hard processes.

Thrust (**T**), invariant jet masses, **C**-parameter, jet broadening (**B**), energy-energy correlation (**EEC**) etc. belong to the **p=1/2** class : they embody **1/Q** power effects due to confinement physics.

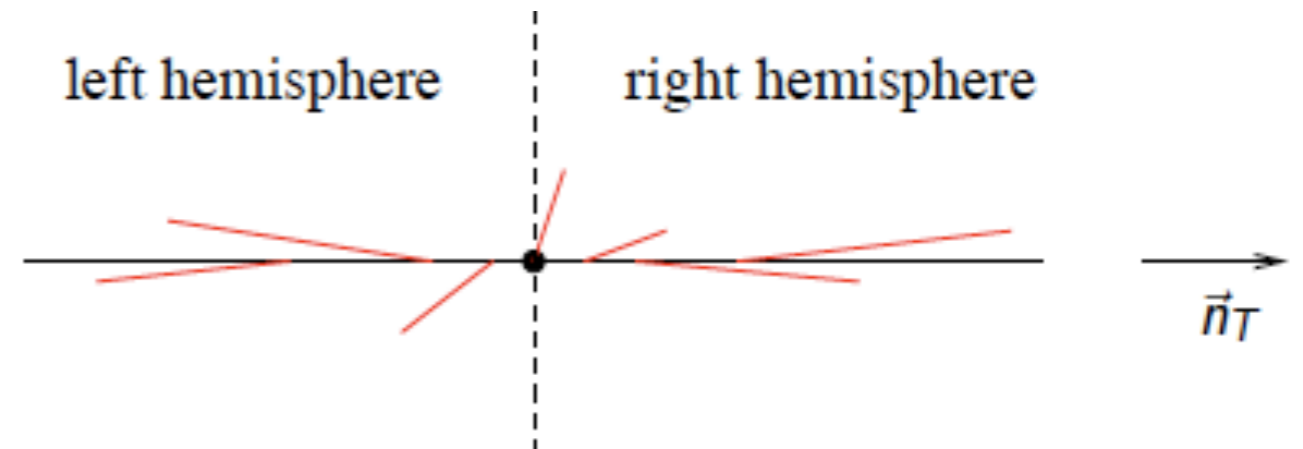
$$\lim_{m \rightarrow 0} \dot{\mathcal{F}}_V = a_V \cdot \sqrt{\frac{m^2}{Q^2}} + \dots$$

jet shape observables

thrust $T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$

C-param. $C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$

jet-mass $\rho = \frac{\left(\sum_{i \in \text{hemisphere}} p_i\right)^2}{(\sum_i E_i)^2}$



broadening $B_T = \frac{\sum_i p_{Ti}}{\sum_i |\vec{p}_i|}$

They are formally calculable in pQCD (being collinear and infrared safe) but possess **large non-perturbative $1/Q$ -suppressed corrections.**

T is obviously a CIS observable : unaffected by either collinear parton splitting or soft gluon radiation.

Two back-to-back particles produced in the c.m.s. of **e^+e^-** annihilation correspond to **$T=1$** .

Thrust deviates from unity for two reasons.

One is PT gluon bremsstrahlung : $\langle 1 - T \rangle^{(\text{PT})} = \mathcal{O}(\alpha_s)$

Another reason is pure hadronization physics :

PT radiation switched off, two outgoing quarks are believed to produce two narrow jets of **hadrons**

Hadrons are uniformly distributed in rapidity and have limited transverse momenta with respect to the jet axis (Field-Feynman hot-dog, or a Lund string).

Take a simplified “tube model” with an exponential inclusive distribution of hadrons :

$$\frac{dN}{d\eta dk_{\perp}} = \mu^{-1} \vartheta(\eta_m - |\eta|) e^{-k_{\perp}/\mu}, \quad \mu = \langle k_{\perp} \rangle$$

Total energy :

$$\sum_i |\vec{p}_i| = 2 \int d\eta dk_{\perp} \frac{dN}{d\eta dk_{\perp}} k_{\perp} \cosh \eta = 2\mu \sinh \eta_m = Q$$

z-momentum projections :

$$\sum_i |p_{zi}| = 2 \int d\eta dk_{\perp} \frac{dN}{d\eta dk_{\perp}} k_{\perp} \sinh \eta = 2\mu (\cosh \eta_m - 1)$$

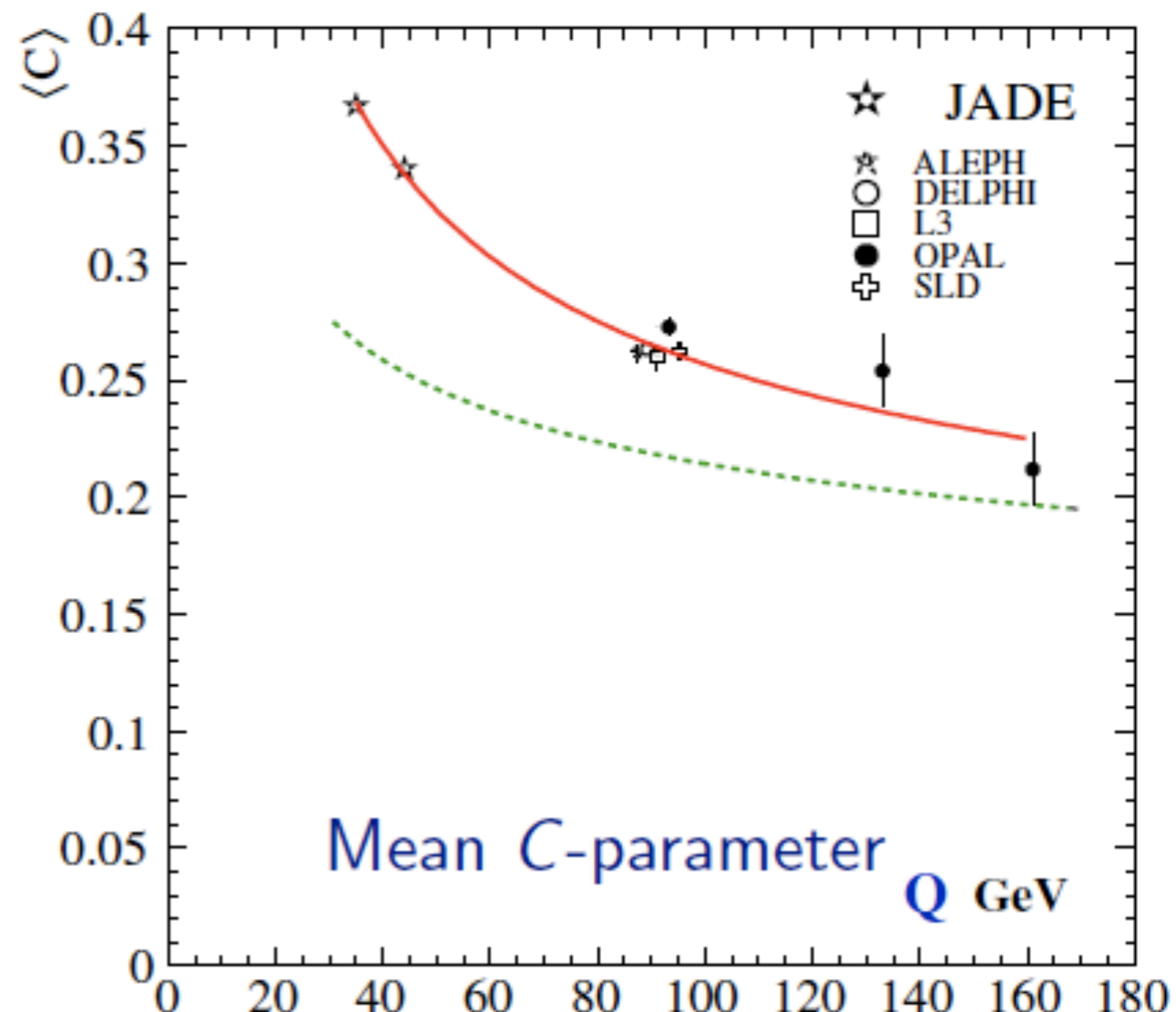
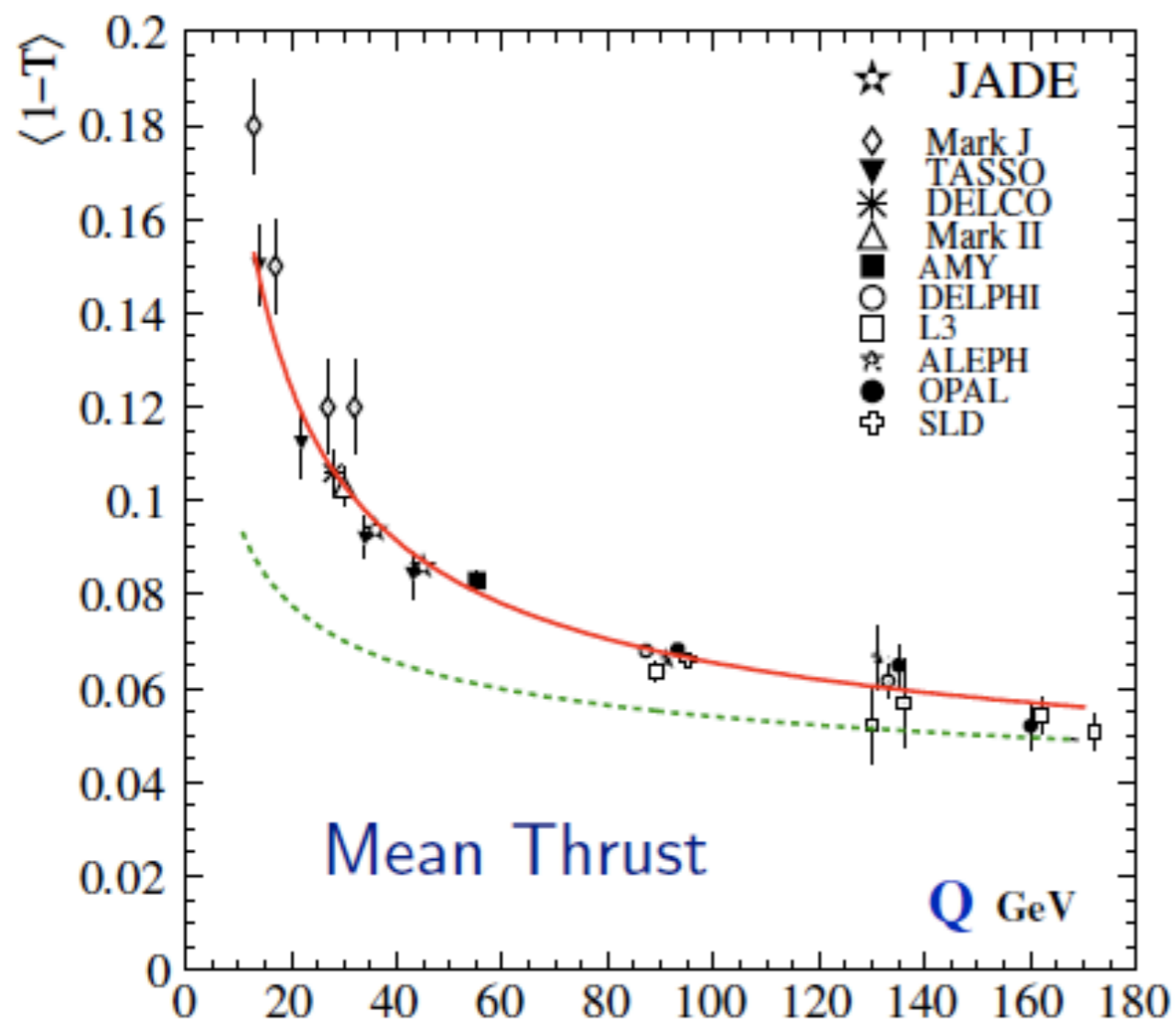
Constructing the ratio,

$$T = \frac{\cosh \eta_m - 1}{\sinh \eta_m} = 1 - \frac{2\mu}{Q} + \mathcal{O}\left(\frac{\mu^2}{Q^2}\right)$$

$$\langle 1 - T \rangle^{(\text{NP})} \simeq \frac{2 \langle k_{\perp} \rangle}{Q}$$

Negative $1/Q$ *hadronization correction* !

It is from the study of hadronization models that the $1/Q$ effects first came into focus (*Webber, 1995*)



“theory” :

$$\langle 1-T \rangle_{\text{hadron}} \approx \langle 1-T \rangle_{\text{parton}} + 1 \text{ GeV}/Q$$

$$\langle C \rangle_{\text{hadron}} \approx \langle C \rangle_{\text{parton}} + 4 \text{ GeV}/Q$$

$$\left\{ \begin{array}{l} 4 \\ 1 \end{array} \right. \implies \frac{3\pi}{2}$$

coupling in the IR

The PT approach normally would not provide us with such a **dimensional** parameter: gluon transverse momenta are broadly (*logarithmically*) distributed which results in the mean $\langle k_{\perp} \rangle \propto \alpha_s Q$

However now we have a PT-handle on the large-distance physics : the “**gluon-mass**” trigger.

Contribution to **thrust** from a single gluon with momentum \mathbf{k} reads, in terms of Sudakov variables,

$$\delta(1 - T) = \min\{\alpha, \beta\} \quad k = \alpha p + \beta \bar{p} + \mathbf{k}_{\perp},$$

$$\mathcal{F}_T \simeq \frac{C_F}{\pi} \int \frac{d\alpha}{\alpha} \frac{d\beta}{\beta} dk_{\perp}^2 \delta(\alpha\beta Q^2 - k_{\perp}^2 - m^2) \cdot \min\{\alpha, \beta\} = \frac{2C_F}{\pi Q} \int_0^{Q^2} \frac{dk_{\perp}^2}{\sqrt{k_{\perp}^2 + m^2}}$$

Evaluating the logarithmic derivative of the characteristic function,

$$\dot{\mathcal{F}} \equiv -m^2 \frac{d\mathcal{F}}{dm^2} \simeq \frac{2C_F}{\pi} \frac{m}{Q}$$

Thus, for non-perturbative correction to **mean values** of jet shape observables we get the integral

$$\langle V \rangle^{\text{NP}} = \frac{a_V}{Q} \cdot \frac{C_F}{2\pi} \int_0^{\infty} \frac{dm^2}{m^2} \sqrt{m^2} \alpha_{\text{eff}}^{\text{NP}}(m^2) \quad \text{where the coefficients } \mathbf{a}_V \text{ are simple numbers having a clear geometric origin.}$$

The magnitude of the first two moments (*jet shapes and DIS*) :

$$\mathcal{A}_1 \equiv \frac{C_F}{2\pi} \int_0^{\infty} \frac{dk^2}{k^2} \cdot k \alpha_s^{\text{NP}}(k^2) \simeq 0.2-0.25 \text{ GeV} \quad \mathcal{A}_2 \equiv \frac{C_F}{2\pi} \int_0^{\infty} \frac{dk^2}{k^2} \cdot k^2 \alpha_s^{\text{NP}}(k^2) \sim 0.2 \text{ GeV}^2$$

Parametrization of the answer in terms of the **full coupling** :

$$\int_0^{\mu_I} dk \alpha^{\text{NP}}(k^2) = \int_0^{\mu_I} dk \alpha_s(k^2) - \int_0^{\mu_I} dk \alpha^{\text{PT}}(k^2) = \mu_I \cdot \left[\alpha_0 - \left(\alpha_s(Q^2) + \beta_0 \frac{\alpha_s^2(Q^2)}{2\pi} \ln \frac{Q}{\mu_I} + \dots \right) \right]$$

The characteristic non-perturbative parameter - the average of the coupling over the IR region :

$$\alpha_0(\mu_I) \equiv \frac{1}{\mu_I} \int_0^{\mu_I} dk \alpha_s(k^2)$$

Non-perturbative corrections to **mean values** of jet shapes

$$\langle V \rangle = \langle V \rangle^{\text{PT}}(\alpha_s) + a_V \cdot \mathcal{P}$$

with

$$\mathcal{P} = \frac{4C_F \mathcal{M} \mu_I}{\pi^2 Q} \cdot \left\{ \alpha_0(\mu_I) - \left[\alpha_s + \beta_0 \frac{\alpha_s^2}{2\pi} \left(\ln \frac{Q}{\mu_I} + 1 + \frac{K}{\beta_0} \right) + \dots \right] \right\}$$

the so-called “**Milan factor**” takes care of next-to-leading PT effects in the leading NP power correction

Interestingly, the same NP parameter enters the **differential distributions** of jet shapes :

$$\frac{d\sigma}{dV}(V) = \frac{d\sigma^{(\text{PT})}}{dV}(V - a_V \mathcal{P})$$

Perturbatively calculable “**geometrical**” **coefficients** entering the jet shapes :

$V =$	$1 - T$	C	M_T^2	M_H^2
$a_V =$	2	3π	2	1

broading drama

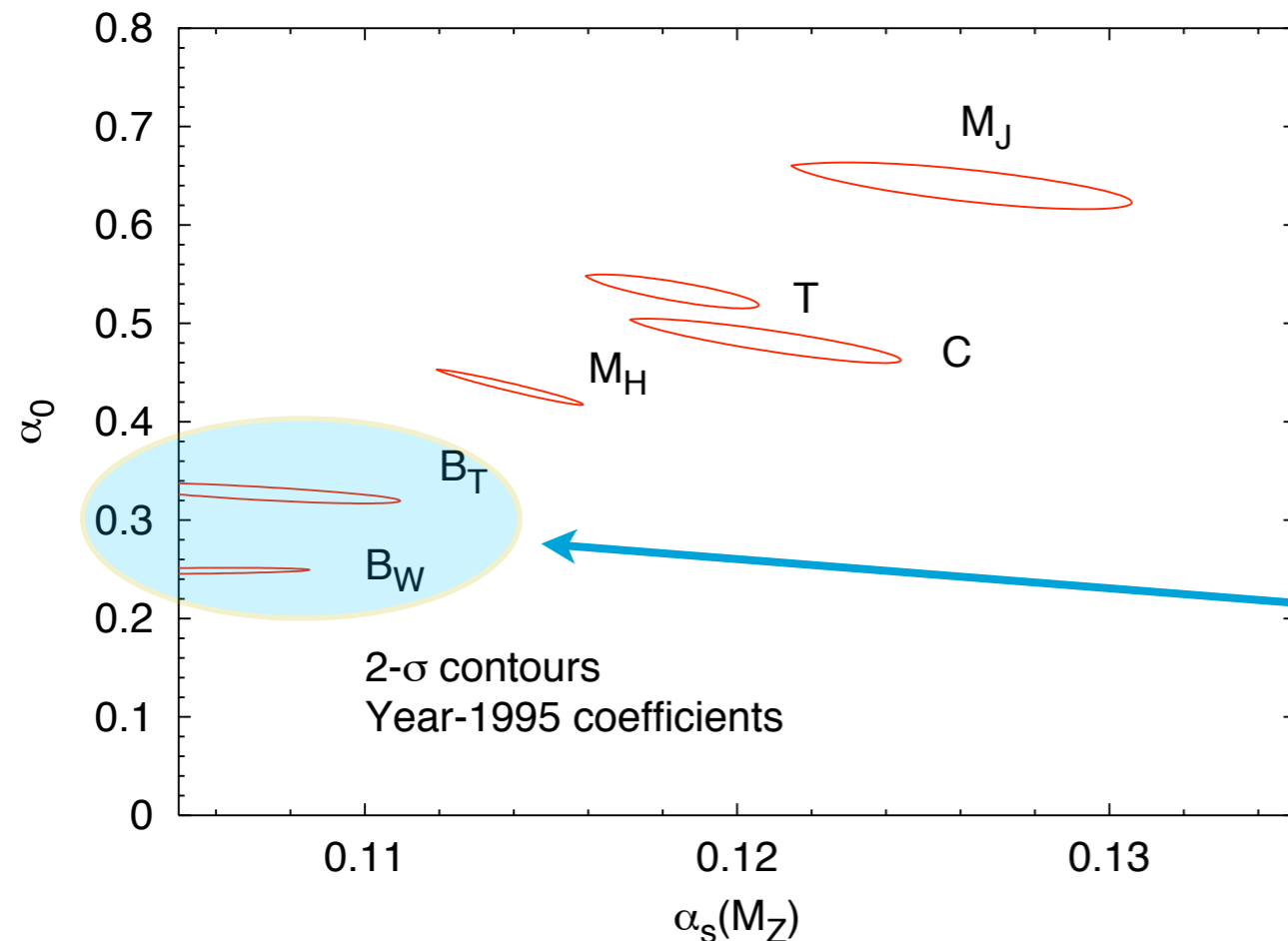
The phenomenology of power-suppressed contributions to jet shapes had a troubled childhood.

Only thrust and C-parameter remained unaffected by theoretical misconceptions...

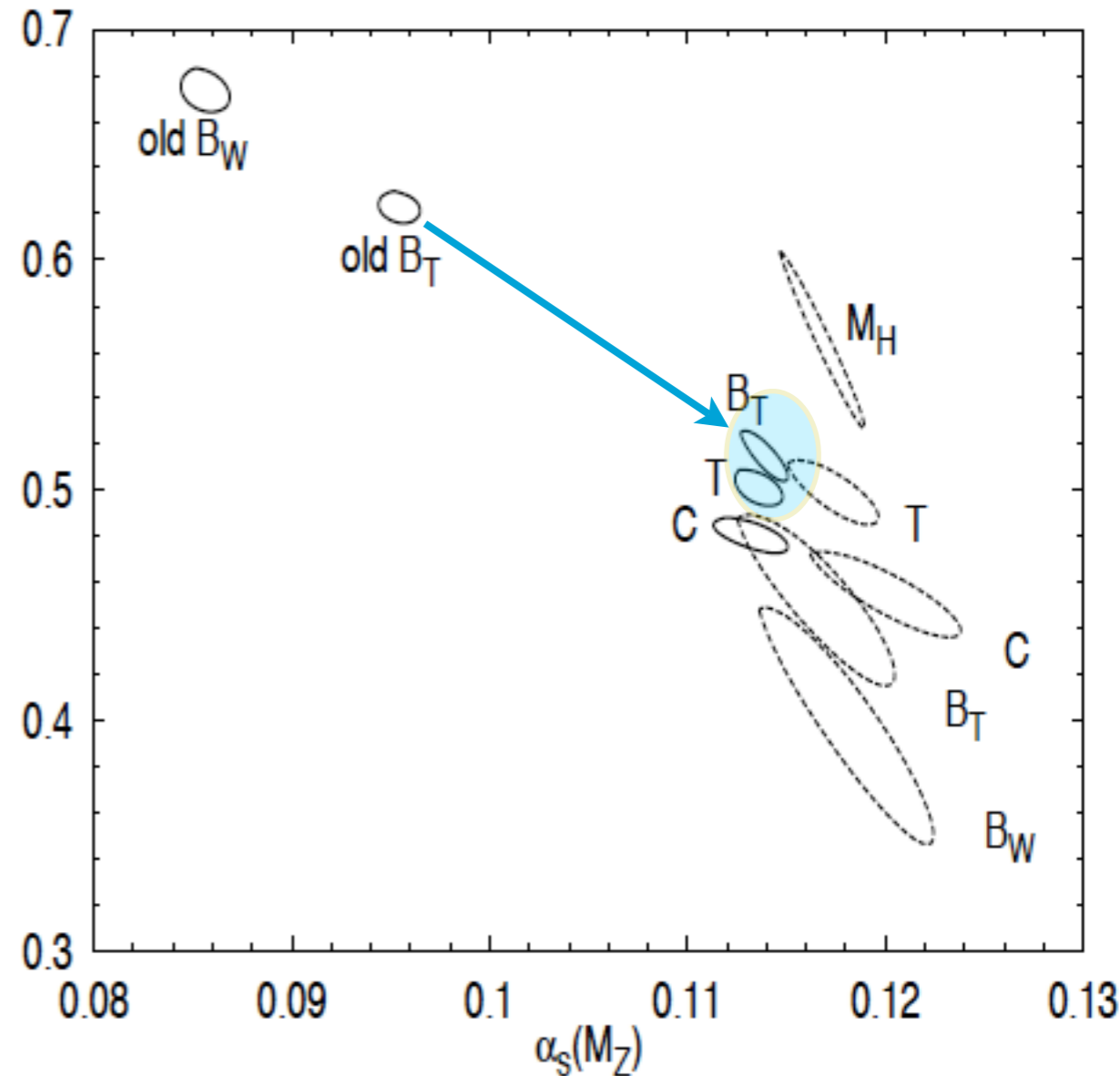
By 1997 a ball-park value of $\alpha_0 \sim 0.5$ was repeatedly emerging from the analyses of jet shapes in e^+e^- and DIS current jets in the Breit frame.

A typical resume ran like : “*The concept of a universal Power Correction parameter α_0 in DIS ep scattering and e^+e^- annihilation is supported*”.

Montpellier 1998 QCD conference : “*inconsistent results for the total and wide-jet **broadinging distributions***”



α_0



Correspondingly, the **mean values** of Broadenings (**T**otal and **W**ide-jet) were found off-mark too...

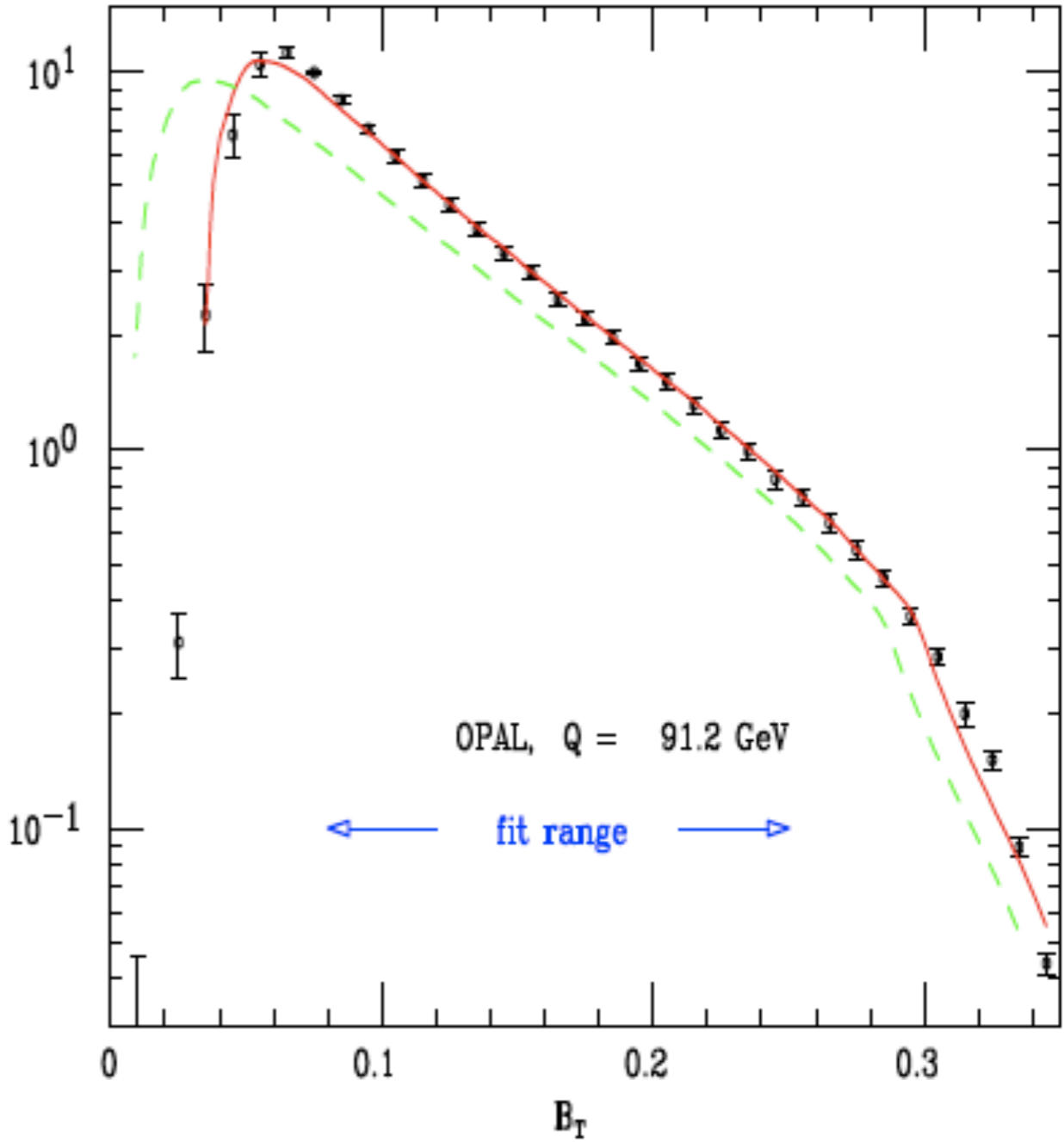
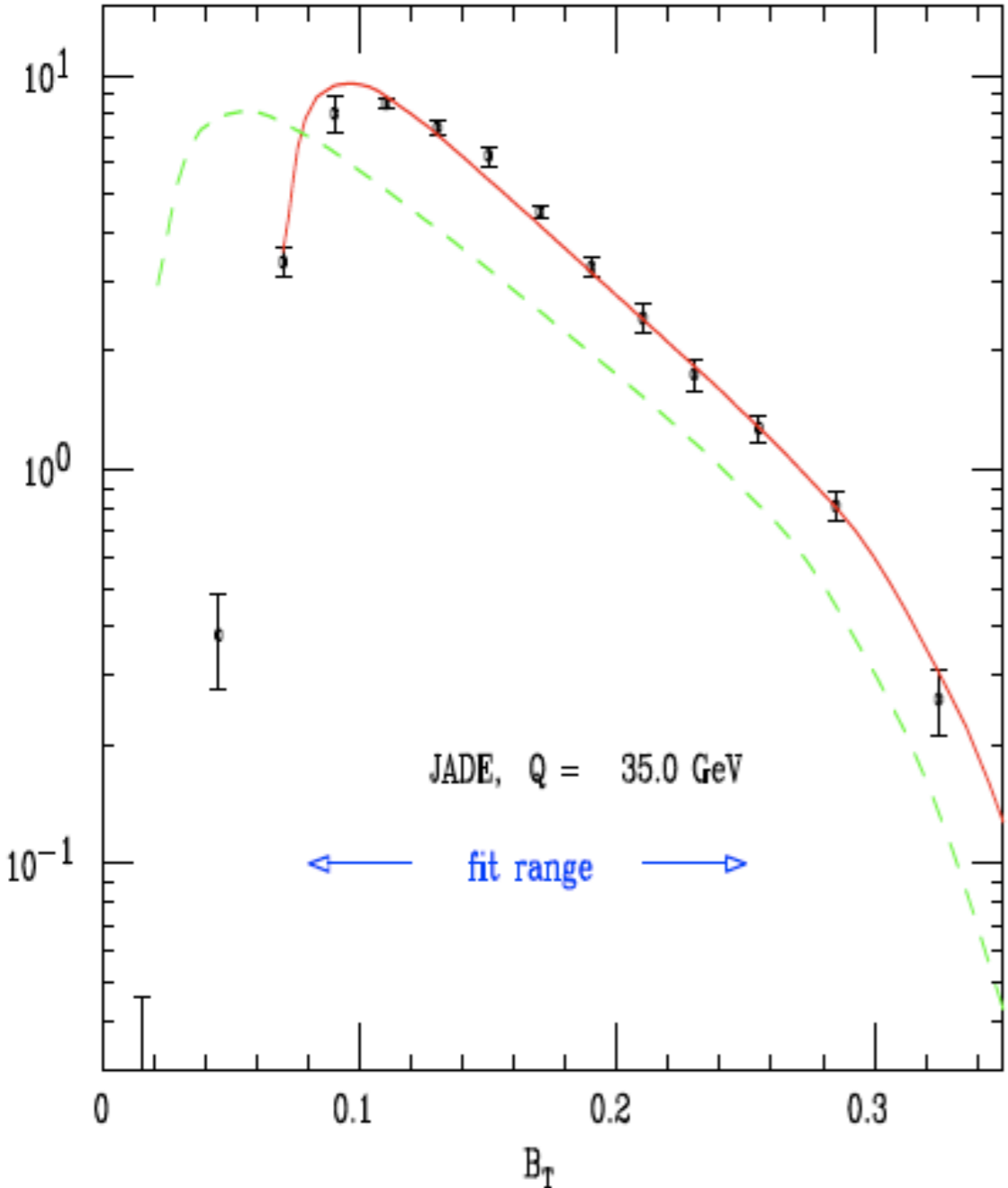
Theoretical revisiting of the Broadening measure taught an **important lesson** and has finally calmed the game ...

The broadening was put under scrutiny by the resurrected **JADE** collaboration.

Not only have they observed the discrepancy, but also have clarified what was going on !

They showed that hadronization effects in broadening not only *shift* the distribution to larger **B** values (as it is the case for *1-T* and **C**) but also *squeeze* it. **A bizarre observation !..**

How can it be that when you *smear* the distribution (moving from *partons* to *hadrons*) it actually becomes *sharper* !?



the broadening escape

The B distribution was found to have a rich structure exhibiting $\ln B/Q$ and $\ln Q/Q$ NP effects.

It was soon realized that one essential phenomenon was overlooked in the original NP-treatment of broadening, namely an *interplay* between NP and PT phenomena.

The effects produced by gluers in the presence of normal PT gluons are different from the effects of NP-radiation inferred from a pure first-order analysis, when the PT-radiation is “switched off”.

The simplest example : the story of the **Jet Mass** observables.

To trigger the NP-contribution we are advised to add to the parton system a *soft gluer*.

When do so at the Born level, that is add a *gluer* to the quark-antiquark system as the third and only secondary parton, we find a $1/Q$ confinement contribution to the squared mass of the *quark-gluer* system - the “**heavy jet**”. Meanwhile, the opposite “**lighter**” jet containing a lonely quark gets none :

$$a_T = a_{M_T^2} = a_{M_H^2}, \quad a_{M_L^2} = 0$$

$$a_T = a_{M_T^2} = 2a_{M_H^2} = 2a_{M_L^2}$$

There are always normal PT gluons in the game which are responsible for the bulk of the jet mass : it's not gluer's business to decide which jet is going to be *heavier*. Confinement effects are shared *equally*.

Now we are ready to address the *squeezed broadening* issue.

The feature that $1-T$ and C have in common is that the dominant NP-contribution is determined by radiation of gluers at *large angles*. This radiation is insensitive to the tiny mismatch $\Theta_q = \mathcal{O}(\alpha_s)$ between the quark and thrust axis directions which is due to PT gluon radiation.

Therefore the *quark momentum direction* can be identified with the *thrust axis*.

the broadening escape

The broadening, **on the contrary**, accumulates contributions which do not depend on rapidity, so that the **mismatch** between the **quark** and the **thrust** axis matters both in the B -means and distributions.

Having naively assumed that the quark direction coincides with that of the thrust axis, B accumulated NP-contributions from **gluons** with rapidities up to $\eta_i \leq \eta_{\max} \simeq \ln(Q/k_{ti})$.

In this case the shift in the B -spectrum would be logarithmically enhanced, $\Delta_B = a_B \mathcal{P} \cdot \ln \frac{Q}{Q_B}$

High-energy gluons are collinear to the **quark** rather than to the thrust axis and do not contribute to B .

$$\delta B_1^{(\text{NP})} \simeq a_1 \mathcal{P} \left\langle \ln \frac{1}{\Theta_q} \right\rangle$$

As a result, the NP correction to B comes out proportional to the **quark rapidity** !

For **mean values** of B observables this yields

$$\left\langle \ln \frac{1}{\Theta_q} \right\rangle \simeq \frac{\pi}{2\sqrt{C_F \alpha_s(Q)}}$$

It is the quark Sudakov form factor that describes the distribution of relative **quark - jet axis** angles.

How about the **distributions** in B ?

The shift in the **single jet** (**wide jet**) broadening is evaluated by averaging over the perturbative distribution in the **quark angle** while keeping the value of B fixed.

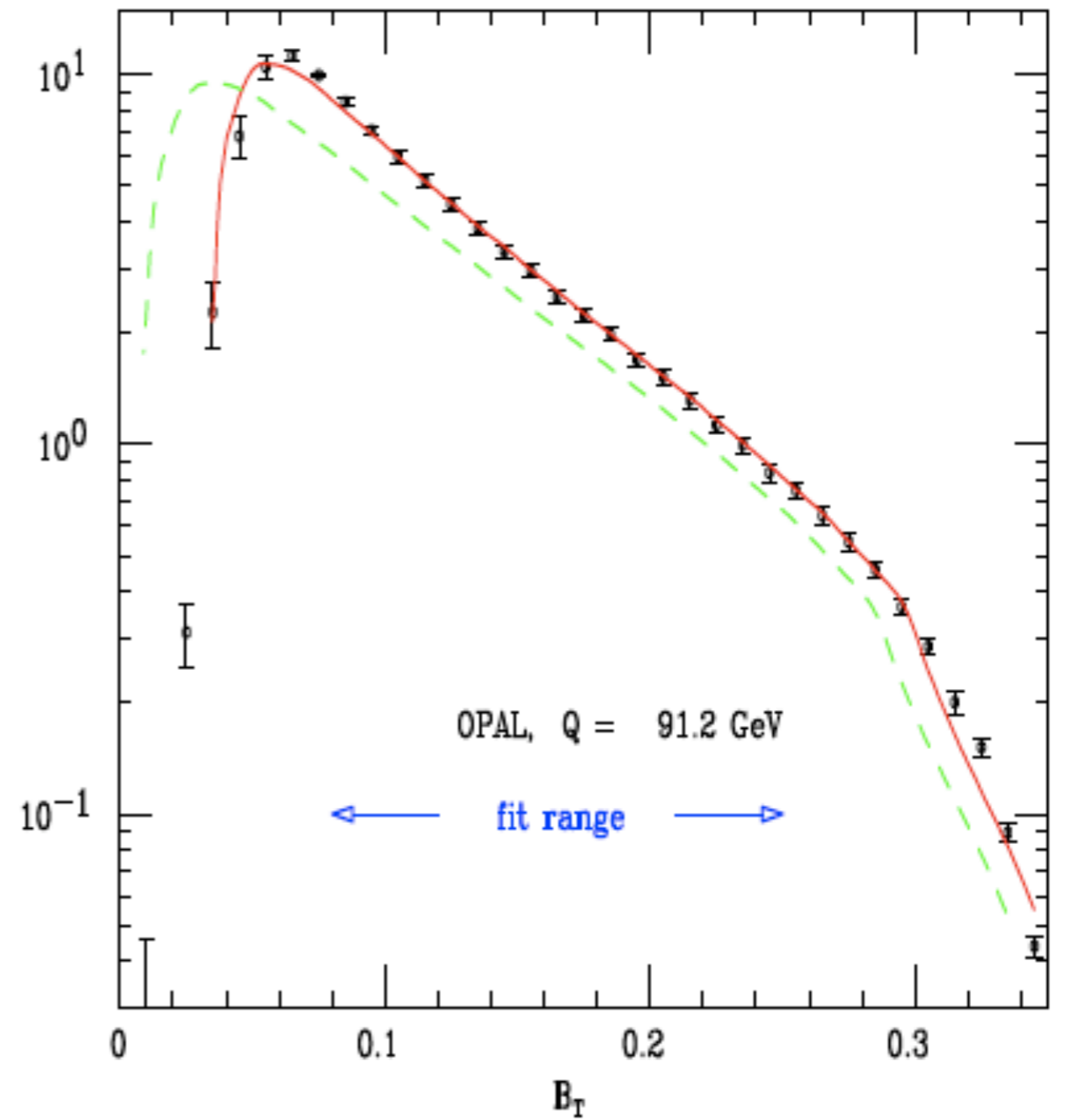
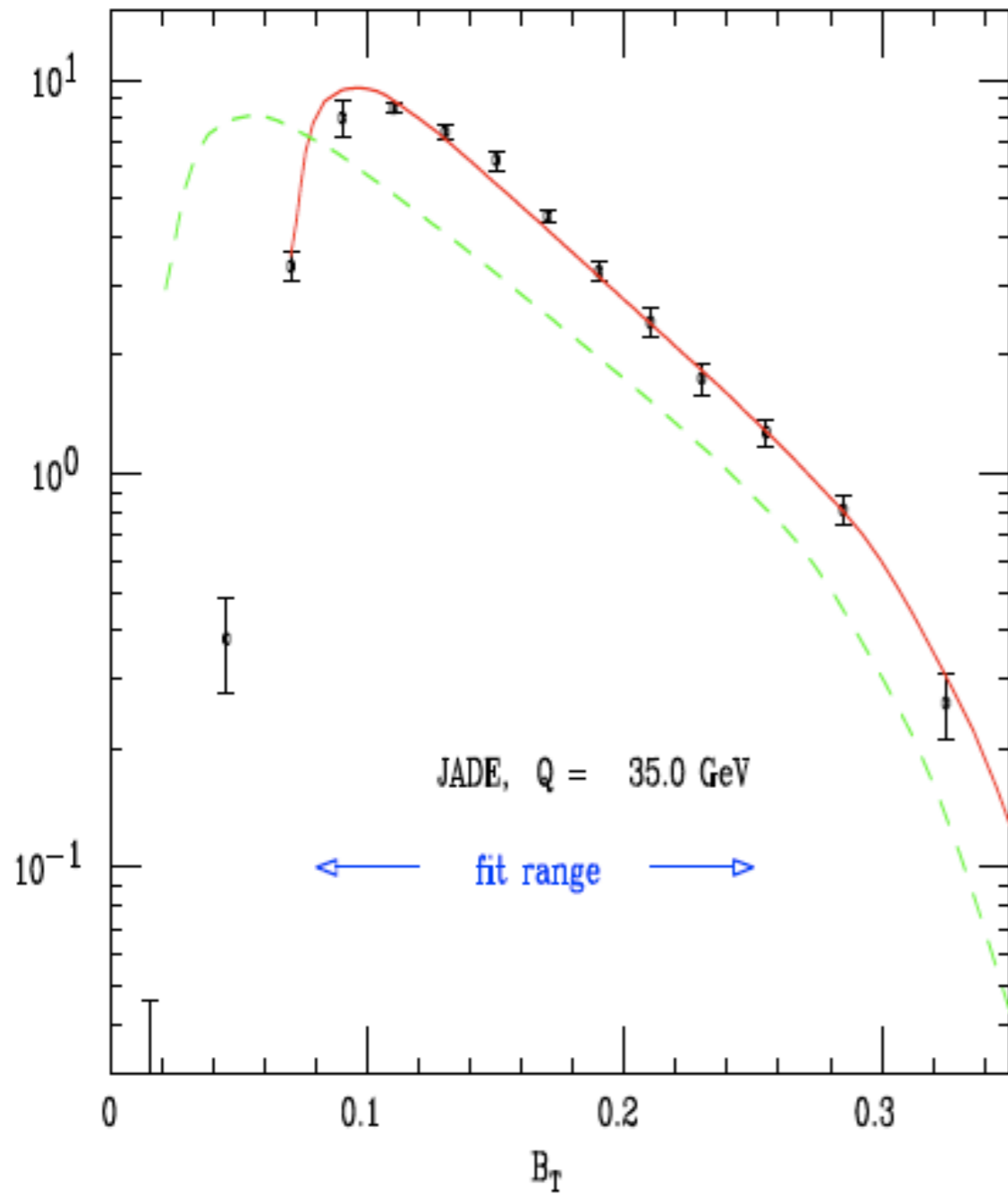
Since Θ_q is kinematically proportional to B 

$$\Delta_1(B) \simeq a_1 \mathcal{P} \cdot \ln \frac{B_0}{B}$$

(the B_T distribution has a somewhat more intricate structure ...)

The smaller is B , the larger the non-perturbative shift : **squeezing**

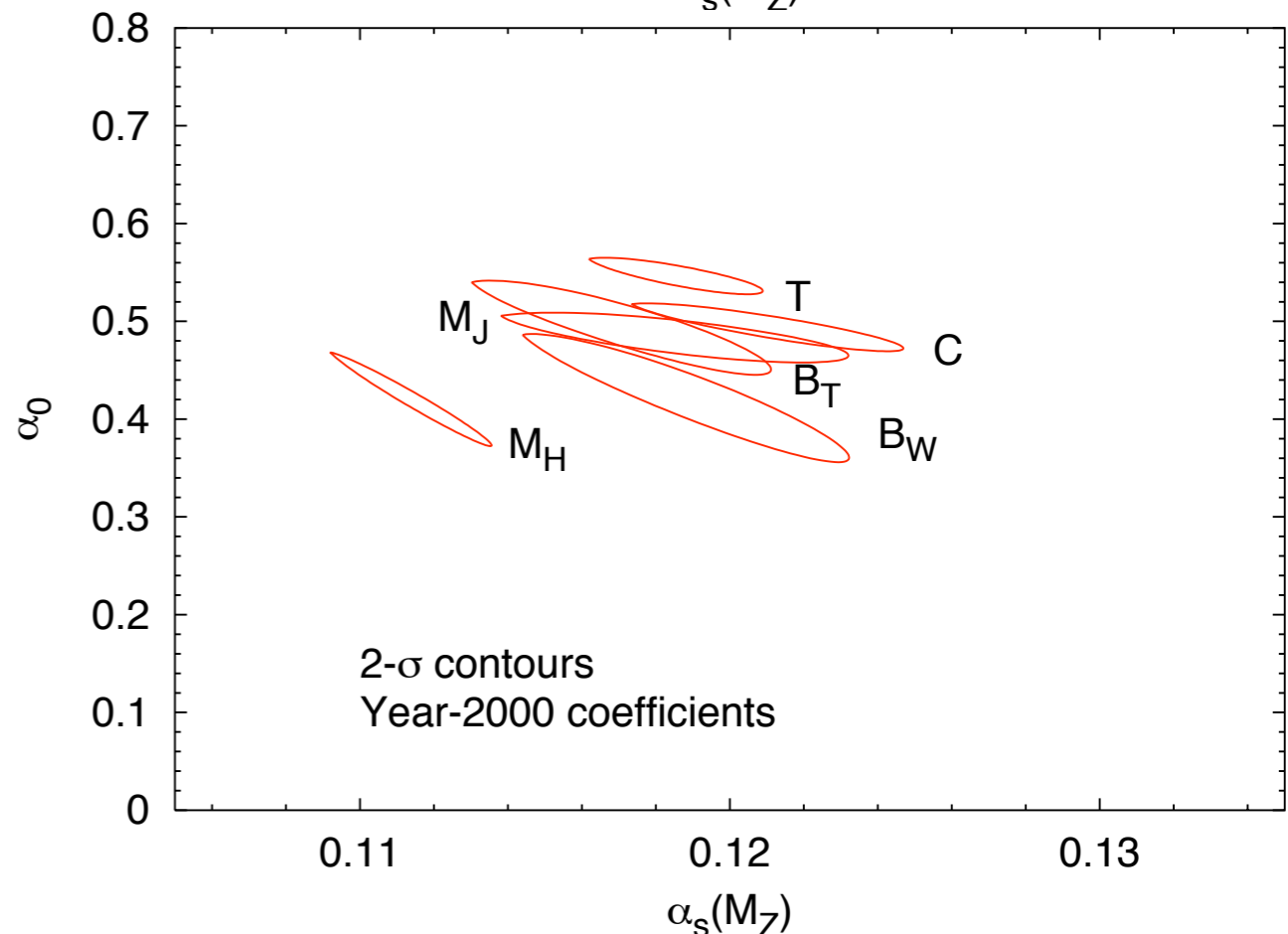
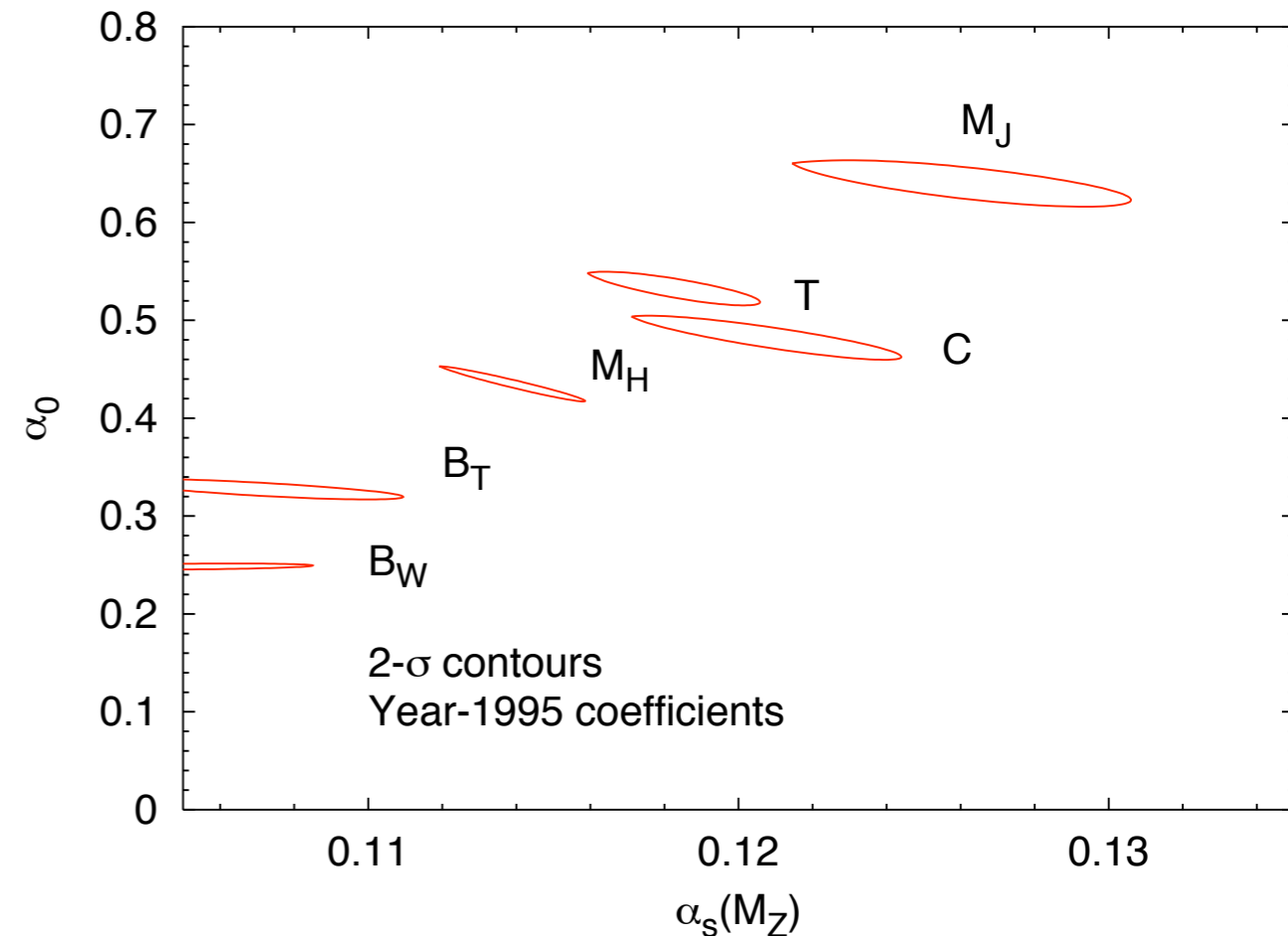
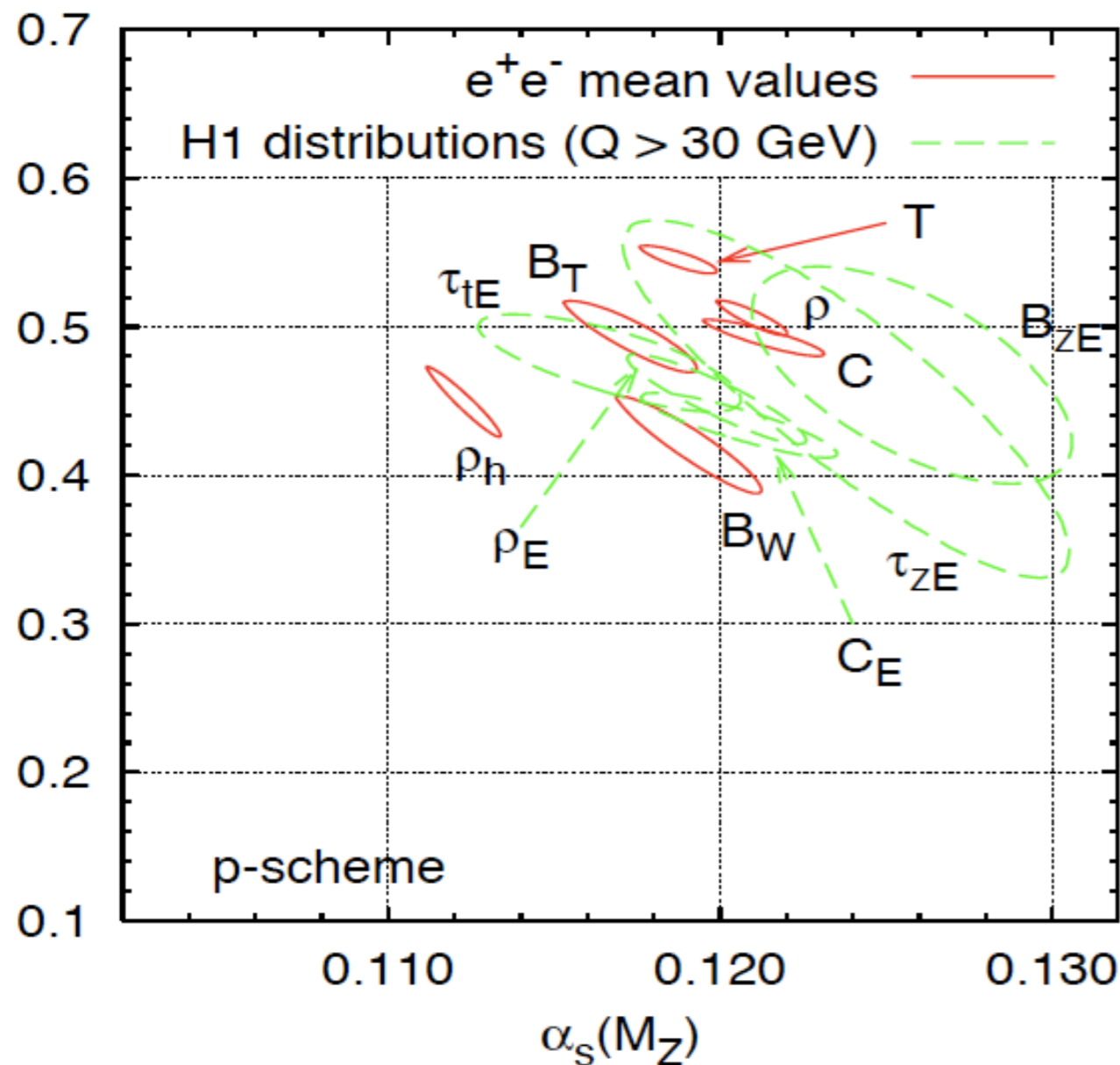
The hadronization effects in broadening not only *shift* the distribution to larger B values (as it is the case for $1-T$ and C) but also *squeeze* it.



NP effects in jet shapes

After the mis-concepts and errors have been fixed, the ensemble of jet shape measurements clustered :

Moreover, both **LEP** (+ **JADE**) and **HERA** have contributed to the scrutiny and to the convergence



Theory + Phenomenology of $1/Q$ effects in event shape observables, both in $e+e-$ annihilation and DIS systematically pointed at the **average** value of the infrared coupling

$$\alpha_0 \equiv \frac{1}{2 \text{ GeV}} \int_0^{2 \text{ GeV}} dk \alpha_s(k^2) \sim 0.5$$

The main features of this result are as follows : the average IR coupling is

- Universal

holds to within $\pm 15\%$

If not for the **universality**,

the whole game would have made no sense : it would have meant just trading **one unknown** - non-perturbative “smearing” effects in a given observable (like in MC event generators) - for **another unknown** function - the shape of the coupling in the infrared...

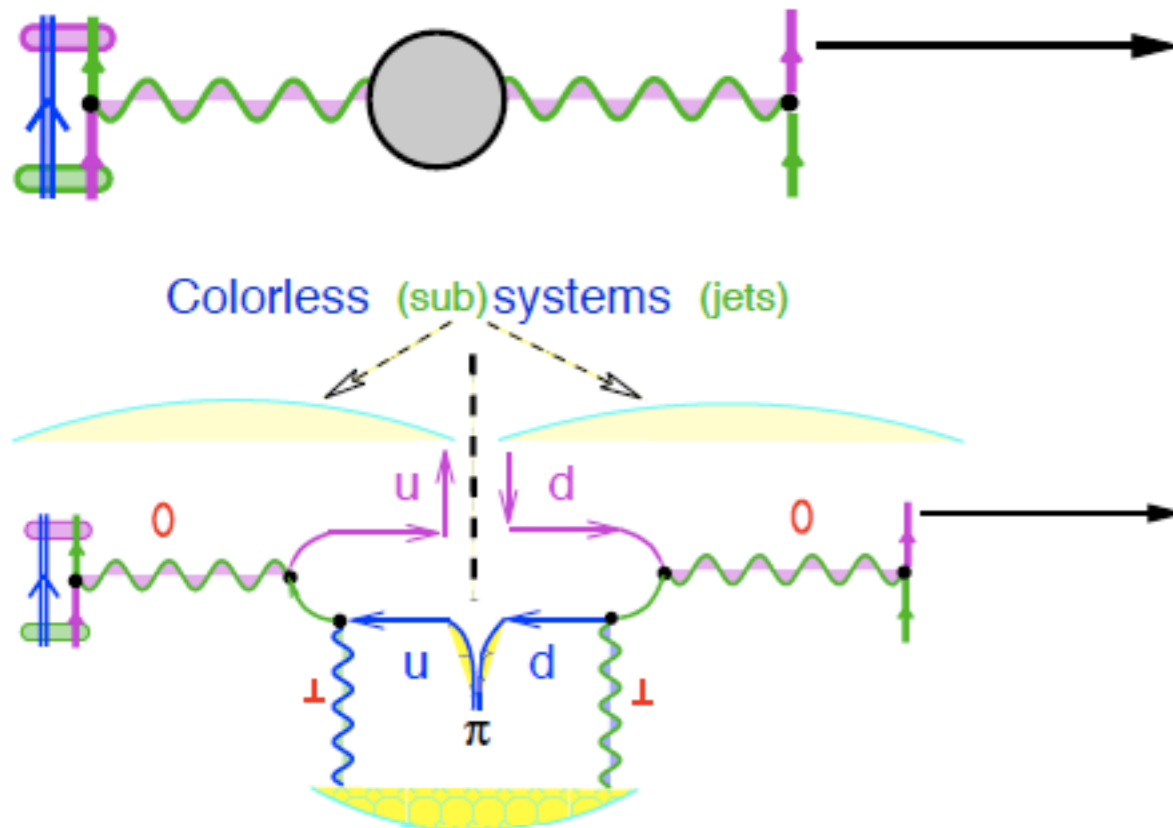
- Reasonably small (which opens intriguing possibilities ...)

- Comfortably **above** the Gribov's **critical value** ($\pi \cdot 0.137 \simeq 0.4$)

EXTRAS

A word about “*soft confinement*”

What happens with the Coulomb field when the sources move apart?



Bearing in mind that virtual quarks live in the background of gluons (zero fluctuations of A_{\perp} gluon fields) what we look for is a mechanism for binding (negative energy) *vacuum quarks* into *colorless hadrons* (positive energy physical states of the theory)

V.Gribov suggested such a mechanism — the supercritical binding of light fermions subject to a Coulomb-like interaction. It develops when the coupling constant hits a definite “critical value” (Gribov 1990)

$$\frac{\alpha}{\pi} > \frac{\alpha_{\text{crit}}}{\pi} = C_F^{-1} \left[1 - \sqrt{\frac{2}{3}} \right] \simeq 0.137$$

$$\left(C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3} \right)$$

An amazing success of the relativistic theory of electron and photon fields — quantum electrodynamics (QED) — has produced a long-lasting negative impact: it taught the generations of physicists that came into the business in/after the 70's to “not to worry”.

Indeed, today one takes a lot of things for granted :

- One rarely questions whether the alternative roads to constructing QFT — *secondary quantization*, *functional integral* and the *Feynman diagram* approach — really lead to the *same quantum theory* of interacting fields
- One feels ashamed to doubt an elegant powerful, *but potentially deceiving*, technology of translating the dynamics of quantum fields into that of statistical systems
- One takes the original concept of the “*Dirac sea*” — the picture of the fermionic content of the vacuum — as an anachronistic model
- One was taught to look upon the problems that arise with field-theoretical description of point-like objects and their interactions at very small distances (*ultraviolet divergences*) as purely technical : *renormalize it and forget it*.

QED : physical objects — *electrons and photons* — are in *one-to-one correspondence* with the fundamental fields that one puts into the local Lagrangian of the theory.

*The rôle of the QED Vacuum is “trivial”: it makes *e.m. charge* (and the electron mass operator) *run*, but does not affect the *nature* of the interacting fields.*

QCD : the Vacuum changes the bare fields *beyond recognition* ...

Gribov Confinement: setting up the Problem

- The question of interest is **the** confinement in real world (with 2 very light **u** and **d** quarks), rather than **a** confinement.
- No mechanism for binding massless **bosons** (gluons) seems to exist in Quantum Field Theory (QFT), while the Pauli exclusion principle may provide means for binding together massless **fermions** (light quarks).
- The problem of ultraviolet regularization may be more than a technical trick in a QFT with apparently **infrared-unstable dynamics** : the **ultraviolet** and **infrared** regimes of the theory may be tightly linked.
- The Feynman diagram technique has to be reconsidered in QCD if one goes beyond trivial perturbative correction effects.

Feynman's famous $i\epsilon$ prescription was designed for (*and applies only to*) quantum field theories with **stable perturbative vacua**.
- To understand and describe a physical process in a **confining theory**, it is necessary to take into consideration the **response of the vacuum**, which leads to essential modifications of the quark and gluon Green functions.

A known QFT example of such a violent response of the vacuum — screening of **super-charged ions** with **$Z > 137$** .

The expression for Dirac energy levels of an electron in a field created by the point-like electric charge Z contains $\epsilon \propto \sqrt{1 - (\alpha_{\text{e.m.}} Z)^2}$.

For $Z > 137$ the energy becomes *complex*. This means *instability*.

- Classically, the electron “*falls onto the centre*”.
- Quantum-mechanically, it also “*falls*”, but into the Dirac sea.

$$A_Z \implies A_{Z-1} + e^+, \quad \text{for } Z > Z_{\text{crit.}} \quad (\text{Pomeranchuk \& Smorodinsky 1945})$$

In the QCD context, the increase of the running quark-gluon coupling at large distances replaces the large Z of the QED problem.

Gribov generalized the problem of supercritical binding in the field of an ***infinitely heavy source*** to the case of two ***massless fermions*** interacting via ***Coulomb-like exchange***.

He found that in this case the supercritical phenomenon develops much earlier.

Namely, a ***pair of light fermions*** develops supercritical behaviour if the coupling hits a definite critical value

$$\frac{\alpha}{\pi} > \frac{\alpha_{\text{crit}}}{\pi} = 1 - \sqrt{\frac{2}{3}}$$

With account of the QCD color Casimir operator, the value of the coupling above which ***restructuring of the perturbative vacuum*** leads to ***chiral symmetry breaking*** and, *likely*, to ***confinement***, translates into

$$\frac{\alpha_{\text{crit}}}{\pi} = C_F^{-1} \left[1 - \sqrt{\frac{2}{3}} \right] \simeq 0.137$$