

Hadron interactions, colour and QCD partons

5. Exploring and exploiting N=4 SUSY :
QCD made simple(r) ((?))
+ entertainig puzzles



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September-October 2013

Parton Evolution

and

Higher Loops

Higher orders' headache

Recent years were/are marked by explosive progress in analytic calculations of multi-leg QFT amplitudes and multi-loop corrections

It took 3-4 years (1976 → 1980) to establish the next-to-leading (2-loop) parton Hamiltonian (splitting functions).

It took another 20+ years to calculate the DIS anomalous dimensions in three loops.

This is no doubt an enormous achievement which leaves, however, that air of deep sadness ...

Have a look at the **simplest** element of the an.dim. matrix corresponding to the **non-singlet** (valence quark) evolution

To remind you : in the 1st loop,

$$P_{qq}(x) = C_F \frac{1+x^2}{1-x}$$

$$\begin{aligned}
& +48H_{-1,-1,2} + 40H_{-1,0}\zeta_2 + 3H_{-1,0,0} - 22H_{-1,0,0,0} - 6H_{-1,2} - 4H_{-1,2,0} - 32 \\
& - \frac{3}{2}H_0\zeta_2 - 13H_0\zeta_3 - 14H_{0,0}\zeta_2 - \frac{9}{2}H_{0,0,0} + 6H_{0,0,0,0} + 6H_2\zeta_2 + 3H_3 + 2H_{3,0} \\
& + (1-x) \left[2H_{-3,0} - \frac{31}{8} + 4H_{-2,0,0} + H_{0,0}\zeta_2 - 3H_{0,0,0,0} + 35H_1 + 6H_1\zeta_2 - H_1, \right. \\
& + (1+x) \left[\frac{37}{10}\zeta_2^2 - \frac{93}{4}\zeta_2 - \frac{81}{2}\zeta_3 - 15H_{-2,0} + 30H_{-1}\zeta_2 + 12H_{-1,-1,0} - 2H_{-1,0} \right. \\
& - 24H_{-1,2} - \frac{539}{16}H_0 - 28H_0\zeta_2 + \frac{191}{8}H_{0,0} + 20H_{0,0,0} + \frac{85}{4}H_2 - 3H_{2,0,0} - 2H_3 \\
& \left. \left. - H_4 \right] + 4\zeta_2 + 33\zeta_3 + 4H_{-3,0} + 10H_{-2,0} + \frac{67}{2}H_0 + 6H_0\zeta_3 + 19H_0\zeta_2 - 25H_{0,0} \right. \\
& \left. - 2H_2 - H_{2,0} - 4H_3 + \delta(1-x) \left[\frac{29}{32} - 2\zeta_2\zeta_3 + \frac{9}{8}\zeta_2 + \frac{18}{5}\zeta_2^2 + \frac{17}{4}\zeta_3 - 15\zeta_5 \right] \right) \\
& - 24H_1\zeta_3 - 16H_{1,-2,0} + \frac{6}{9}H_{1,0} - 2H_{1,0}\zeta_2 + \frac{5}{2}H_{1,0,0} + 11H_{1,0,0,0} + 8H_{1,1,0,0}
\end{aligned}$$

2 × 2 anomalous dimension matrix occupies

1 st loop: 1/10 page

2 nd loop: 1 page

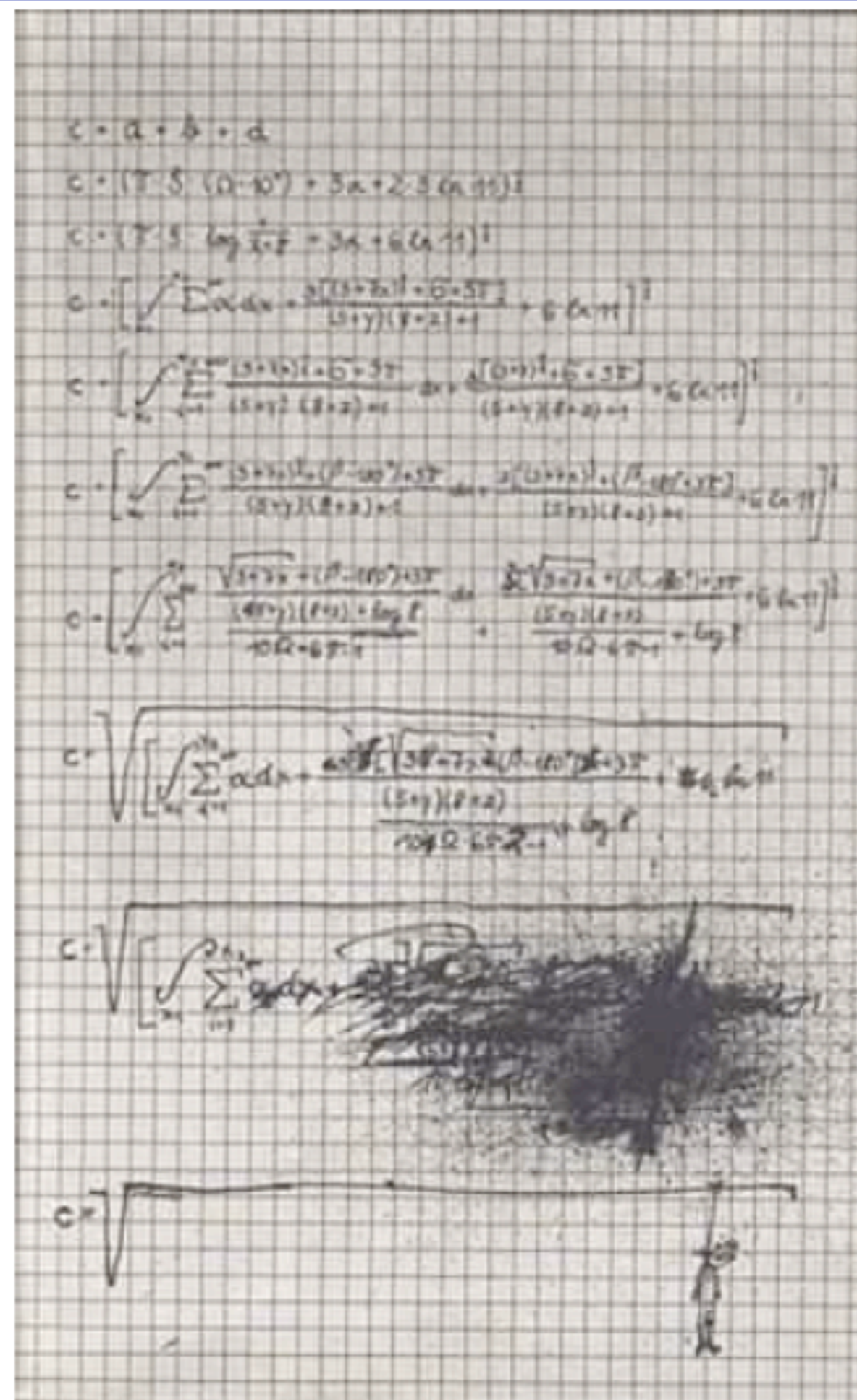
3 rd loop: 100 pages (200 K ascii)

Moch, Vermaseren and Vogt

[waterfall of results launched
March 2004, and counting]

$$V \sim \begin{cases} 10^{\frac{N(N-1)}{2}-1} \\ 10^{2^{N-1}-2} \end{cases}$$

not too encouraging a trend . . .



Could one fight complexity?

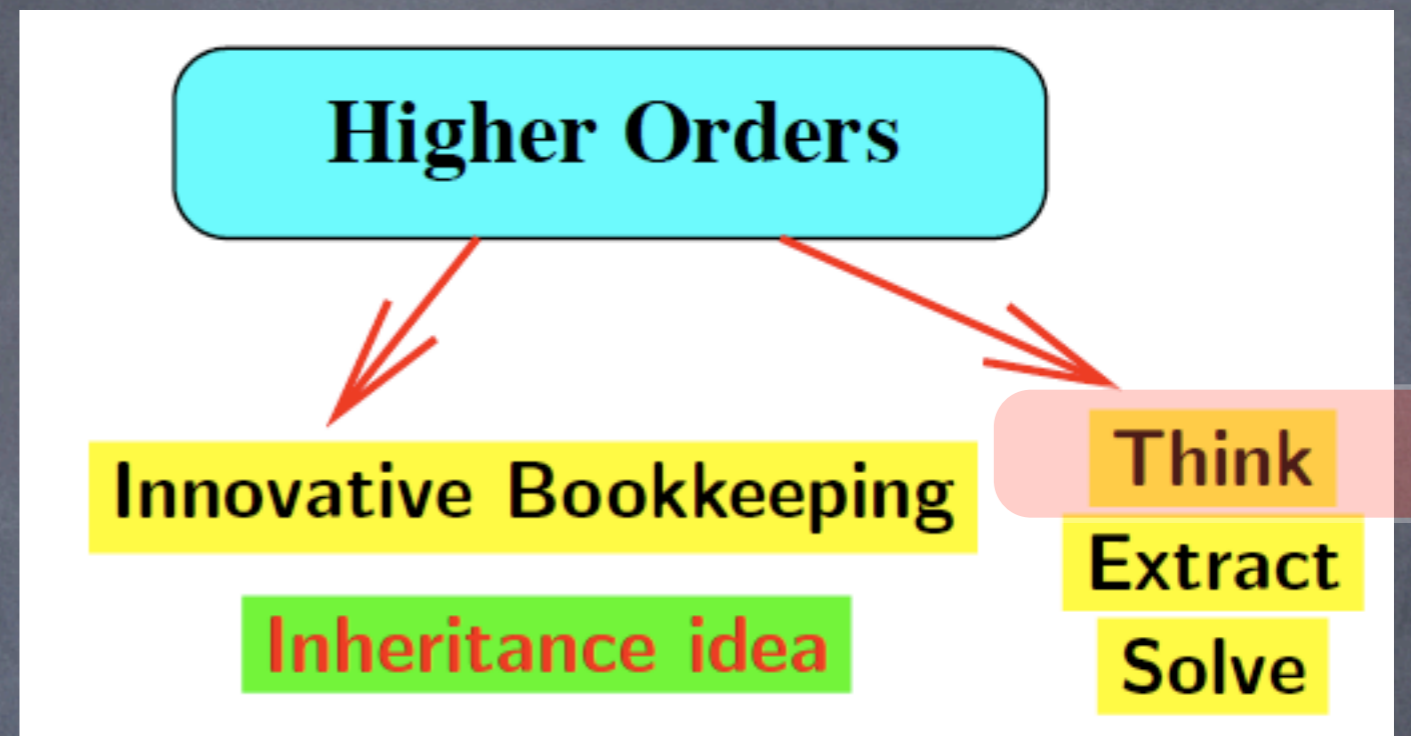
- ▶ complexity may be not *genuine* but *inherited* from the previous orders (*inheritance idea*)
- ▶ explore internal *hidden symmetries* of the problem (*fully solvable dynamics* ?)
- ▶ look into *puzzles*, with curiosity and respect, and try to reach a better understanding, while being ready for *surprises*



How to reduce complexity ?

Guidelines

- ✓ exploit internal properties :
 - ▶ Drell–Levy–Yan relation
 - ▶ Gribov–Lipatov reciprocity
- ✓ separate classical & quantum effects in the gluon sector



An essential part of gluon dynamics is **Classical**.

“Classical” does not mean “Simple”.

However, it has a good chance to be **Exactly Solvable**.

➡ A playing ground for theoretical theory: SUSY, AdS/CFT, ...

Different facets of - **supposedly simple** -
classical - “*soft*” - radiation physics :

- its basic teaching (LBK **no-free-lunch** theorem)
- its offerings (**$N=4$ SYM**)
- its **spectacular “failure”** (*soft photons in multiple hadroproduction*)

- pQCD, talking *quarks* and *gluons*, did the job it has been asked to perform
 - to measure quark and gluon *spins*
 - to establish $SU_c(3)$ as the true QCD gauge group
 - to verify Asymptotic Freedom.
- Moreover, comparing theoretical predictions concerning multiplication of partons, with production of hadrons in jets,
 - inclusive energy spectra of (relatively soft) hadrons **INSIDE** Jets, and
 - soft hadron multiplicity flows **IN-BETWEEN** Jets
 taught us an important lesson, or rather are sending us a hint, about non-violent nature of hadronization – “*Soft Confinement*”.
- First semi-quantitative understanding of the genuine **Non-Perturbative physics** of the Hard–Soft Interface has been gained.

The *Non-Perturbative* physics carries the features of its *Perturbative* counterpart :

The *Nature does respect* Its own *rules of engagement*.

An important example of such behavior - the “*no-free-lunch*” (**LBK**) wisdom

Celebrated soft bremsstrahlung theorem was formulated by Francis Low in 1956 for scalar charged particles and later generalized by Barnett and Kroll to fermions.

The very classical nature of **soft radiation** makes it **universal** with respect to intrinsic quantum properties of participating objects and the nature of the underlying scattering process

- it is only the **classical movement** of electromagnetic charges that matters.

$$d\sigma^{(1)}(p_i, \omega) \propto \frac{\alpha}{\pi} \frac{d\omega}{\omega} \left[\left(1 - \frac{\omega}{E}\right) \cdot \sigma^{(0)}(p_i) + \left(\frac{\omega}{E}\right)^2 \cdot \tilde{\sigma}(p_i, \omega) \right]$$

non-radiative (“Born”) **cross section**

Normally, for a particle production

$$|M|^2 \cdot \frac{d^3k}{\omega} \propto \omega d\omega \quad M = \mathcal{O}(1) \quad \text{in the } \omega \rightarrow 0 \text{ limit}$$

An enhanced matrix element, $M \propto \omega^{-1}$, characterizes **classical field** rather than **particle**

A dramatic consequence : **soft photons** “**don’t carry quantum numbers**”

If the non-radiative process is for some reason forbidden (*parity, C-parity, angular momentum*) the **veto cannot be lifted** by emitting a **soft photon** !

This “drama” turns into “tragedy” in the QCD context :

soft gluons “don't carry away no color” either

For many years pQCD practitioners were unable to describe the yields of heavy quarkonia in hadron collisions at Tevatron. For example, the measured yield of J/ψ at large transverse momenta was up to **50** times bigger than expected !

Pushed by this long-standing failure, theorists came up with a remedy :

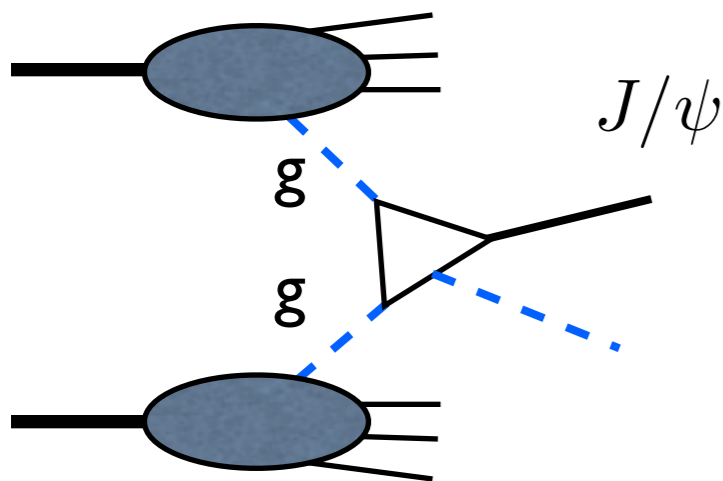
the “**color octet**” model for J/ψ **production**

J/ψ is an S-state of $C\bar{C}$: $^{2S+1}L_J = ^3S_1$ - a vector meson like photon : $P=-1, C=-1$

It can decay into 1(3) photon(s), photon and 2 gluons, into 3 gluons (*in color-symmetric state*)

two-body final state \longrightarrow **smallness** ...

... get **free lunch** by blaming **confinement** !



$g+g \longrightarrow \chi \longrightarrow J/\psi + \text{photon}$ (40% of the yield)

$g+g \longrightarrow J/\psi^{(8)} \longrightarrow J/\psi + \text{junk glue} : \omega \sim \Lambda_{\text{QCD}}$

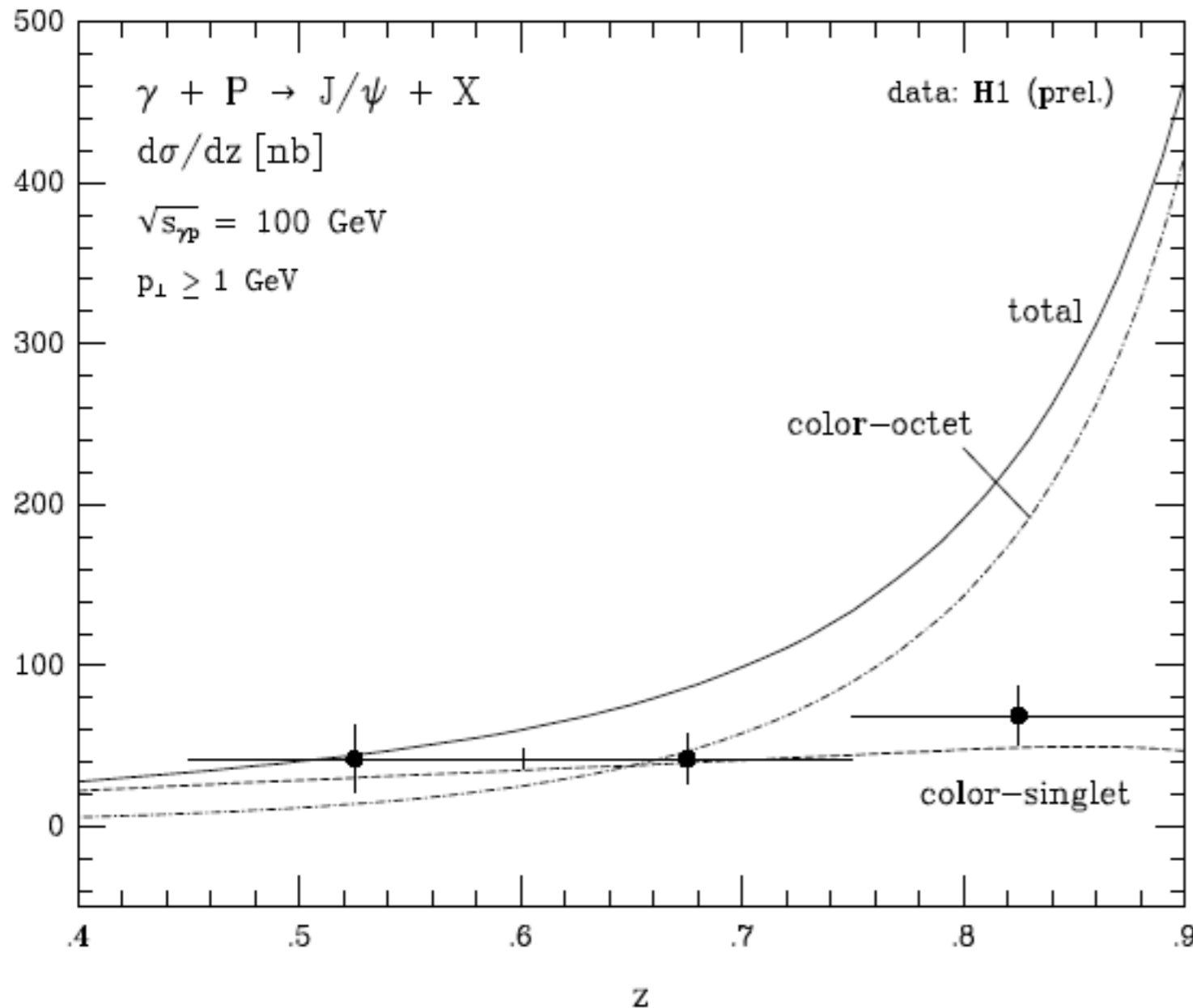
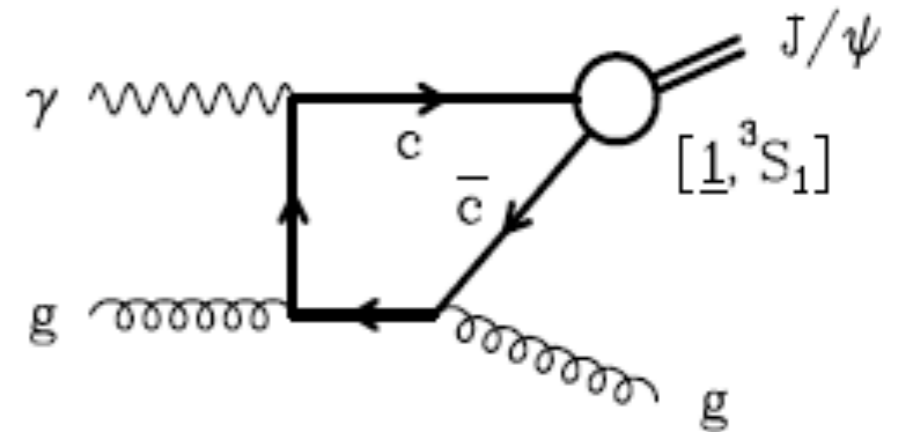
LBK : the price to pay for “**color evaporation**” =

$$\left(\frac{\Lambda_{\text{QCD}}}{M_c} \right)^2 \ll 1$$

A key test : photon fragmentation into J/ψ in e^-p collisions (HERA)

photon - gluon fusion :

IF the final state shaken-off glue could be “junky”
then the J/ψ spectrum would have **peaked** at **$z=1$**



Nowadays the “**color octet model**”
 is dying away, slowly and peacefully,
as it has to
 both
 from **photo-** and **electro-** production
 (HERA data)
 and from **hadron-hadron** collisions

*after M. Cacciari and P. Nason
 have explained the “mystery” of
 the “pQCD deficit” of heavy onia*

The J/ψ energy distribution $d\sigma/dz$ at the photon-proton centre of mass energy
 $\sqrt{s_{\gamma p}} = 100$ GeV integrated in the range $p_{\perp} \geq 1$ GeV.

It is instructive to see how the LBK wisdom shows up in the QCD parton dynamics

$$\frac{1+z^2}{1-z} \quad \xrightarrow{\text{yellow arrow}} \quad \tilde{\gamma}_{q \rightarrow q(x)+g} = \frac{C_F \alpha_s}{\pi} \left[\frac{x}{1-x} + (1-x) \cdot \frac{1}{2} \right]$$

$$\frac{1+z^4+(1-z)^4}{z(1-z)} \quad \xrightarrow{\text{yellow arrow}} \quad \tilde{\gamma}_{g \rightarrow g(x)+g} = \frac{C_A \alpha_s}{\pi} \left[\frac{x}{1-x} + (1-x) \cdot (x+x^{-1}) \right]$$

classical ← **gluons** → **quantum**

✗ Classical Field

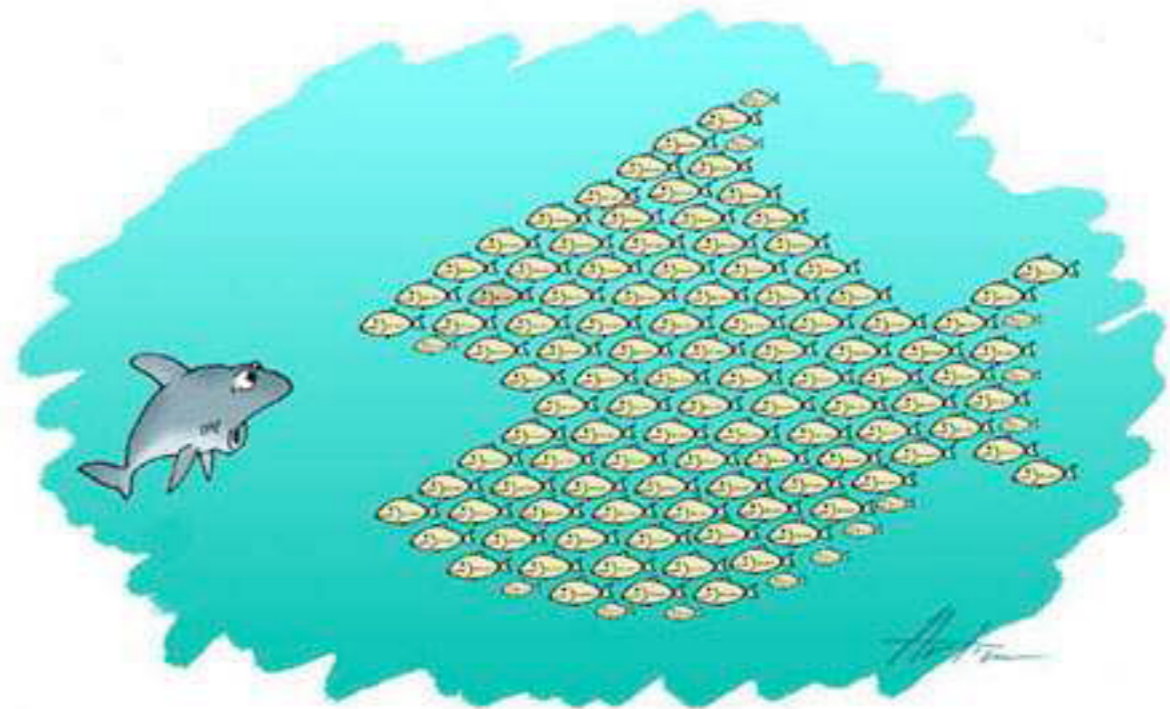
- ✓ infrared singular, $d\omega/\omega$
- ✓ define the physical coupling
- ✓ responsible for
 - ↳ DL radiative effects,
 - ↳ reggeization,
 - ↳ QCD/Lund string (gluons)
- ✓ play the major rôle in evolution

✗ Quantum d.o.f.s (constituents)

- ✓ infrared irrelevant, $d\omega \cdot \omega$
- ✓ make the coupling run
- ✓ responsible for conservation of
 - ↳ P -parity,
 - ↳ C -parity,
 - ↳ colour
 } in decays, production
- ✓ minor rôle

Classical nature of soft gluon radiation (*Francis Low*) may translate into **explicit solvability**

Hidden simplicity
goes hand-in-hand
with *hidden symmetries*



Recall a (*thirty+ years old*) saga of **hidden symmetries** of the parton dynamics

Drell-Levy-Yan relation :

$$P_{BA}^{(T)}(x) = \mp x \cdot P_{AB}^{(S)}(x^{-1}).$$

True in any QFT, it reflects the crossing and allows to link the two channels by analytic continuation, from $x < 1$ to $x > 1$:

Bukhvostov, Lipatov, Popov (1974)

Gribov-Lipatov reciprocity relation (GLR) :

$$P_{BA}^{(T)}(x_{\text{Feynman}}) = P_{BA}^{(S)}(x_{\text{Bjorken}})$$

$$x_B = \frac{-q^2}{2pq}, \quad x_F = \frac{2pq}{q^2}$$



$$P_{BA}(x) = \mp x P_{AB}(x^{-1})$$

Breaks down beyond the 1st loop **BUT can be resurrected in all orders** by returning to the **fluctuation time** as evolution variable !

Recall the first loop diagonal QCD anomalous dimensions:

$$\begin{aligned}\tilde{\gamma}_{q \rightarrow q(x)+g} &= \frac{C_F \alpha_s}{\pi} \left[\frac{x}{1-x} + (1-x) \cdot \frac{1}{2} \right], \\ \tilde{\gamma}_{g \rightarrow g(x)+g} &= \frac{C_A \alpha_s}{\pi} \left[\frac{x}{1-x} + (1-x) \cdot (x + x^{-1}) \right].\end{aligned}$$

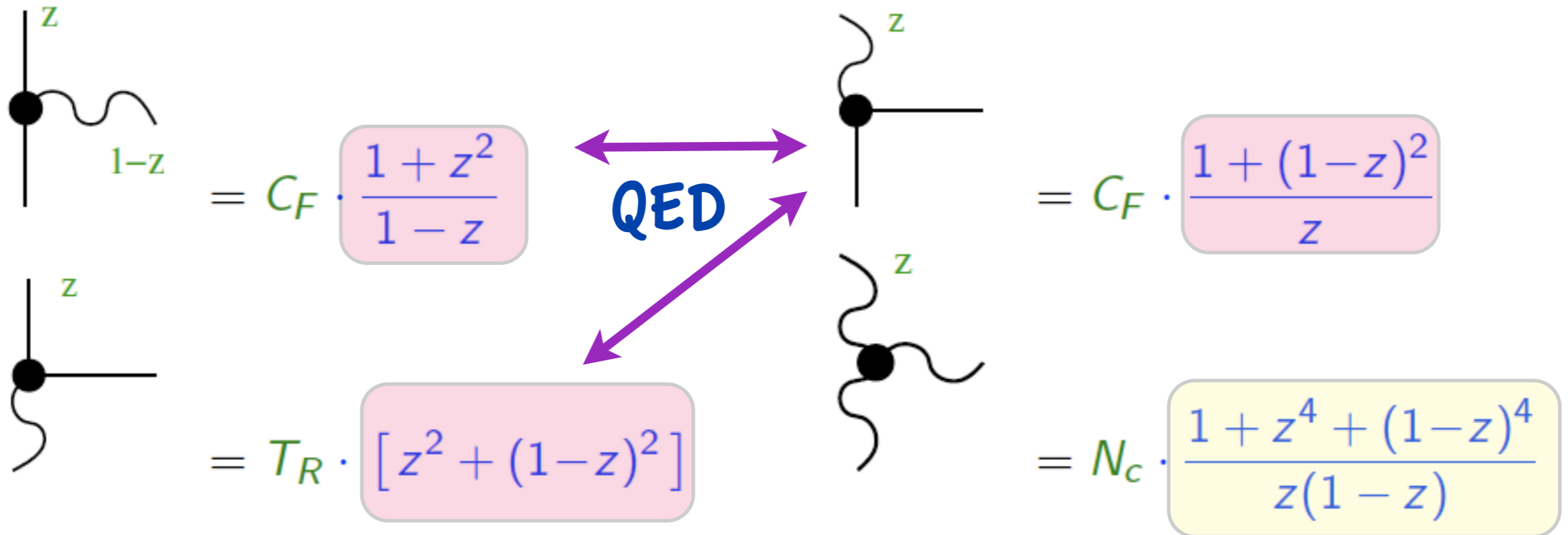
The first component is independent of the nature of the radiating particle — the **Low-Burnett-Kroll** classical radiation \implies “*clagons*”.

The second — “*quagons*” — is relatively suppressed as $\mathcal{O}((1-x)^2)$.

Classical and quantum contributions respect the GL relation, individually:

$$-xf(1/x) = f(x)$$

Apparent and Hidden symmetries of parton dynamics



- Exchange the decay products : $z \rightarrow 1 - z$
- Exchange the parent and the offspring : $z \rightarrow 1/z$

Three (QED) “kernels” are inter-related; gluon self-interaction stays put

- The story continues, however : All four are related !

$$w_q(z) = \frac{q[g](z)}{q} + \frac{g[q](z)}{q} = \frac{q[\bar{q}](z)}{g} + \frac{g[g](z)}{g} = w_g(z)$$

Color factors were excluded from the game ...

Super-Symmetric partner of QCD

+ infinite number of hidden invariants ! ..

$$C_F = T_R = C_A (=N_C)$$

Is a given infinity infinite enough as to make the theory solvable ?

In certain specific problems where one can identify QCD with its SUSY partner theory the integrability feature manifests itself !

- ✓ the Regge behaviour (large N_c)
Lipatov
Faddeev & Korchemsky (1994)
 - ✓ baryon wave function
Braun, Derkachov, Korchemsky,
Manashov; Belitsky (1999)
 - ✓ maximal helicity multi-gluon operators
Lipatov (1997)
Minahan & Zarembo
Beisert & Staudacher (2003)
- ✗ It is clagons which dominate in all the *integrability cases*
- ✗ Tree multi-clagon (Parke–Taylor) amplitudes are *known exactly*
Parke–Taylor (1986) = Bassetto–Ciafaloni–Marchesini (1983)

The higher the symmetry, the deeper integrability ...

N=4 SYM — the extreme case !

- ✗ Conformal theory $\beta(\alpha) \equiv 0$
- ✗ All order expansion for α_{phys}
- ✗ Full integrability via AdS/CFT

Beisert, Eden, Staudacher (2006)
 Maldacena; Witten,
 Gubser, Klebanov, Polyakov (1998)

What is so special about this theory ?

look at the **anomalous dimension** :
 (parton evolution “Hamiltonian”)

Maximally super-symmetric YM field model:
 Matter content = 4 Majorana fermions, 6 scalars;
 everyone in the adjoint representation.

$$\gamma \Rightarrow \frac{x}{1-x} + \text{no quagons !}$$

$$\frac{C_A^{-1}}{d \ln \mu^2} \frac{d}{d \ln \mu^2} \left(\frac{\alpha(\mu^2)}{4\pi} \right)^{-1} = -\frac{11}{3} + \frac{4}{2} \cdot \int_0^1 dx 2[x^2 + (1-x)^2] + \frac{6}{2!} \cdot \int_0^1 dx 2x(1-x) = \mathbf{0}$$

... makes one think of a **classical nature** (?) of the SYM-4 dynamics

Maximally super-symmetric N=4 YM allowed for a compact analytic solution of the GLR problem in 3 loops (all moments N)

D-r & Marchesini (2006)

4 loops

Beccaria & Fiorini (2009)

5 loops

Romuald Janik & Co (2010+)

... ALL loops ?

Let us examine what sort of functions the N=4 parton Hamiltonian is made of

In spite of having many states ($s = 0, \frac{1}{2}, 1$), the SYM-4 parton dynamics is built of a single “universal” anomalous dimension:

$$\gamma_+(N+2) = \tilde{\gamma}_+(N+1) = \gamma_0(N) = \tilde{\gamma}_-(N-1) = \gamma_-(N-2) \equiv \gamma_{\text{uni}}(N)$$

$$\gamma_{\text{uni}}^{(1)}(N) = -S_1(N) = - \int_0^1 \frac{dx}{x} (x^N - 1) \cdot \frac{x}{x-1} \equiv \mathbf{M} \left[\frac{x}{(1-x)_+} \right]$$

$$S_1(N) = \sum_{k=1}^N \frac{1}{k} = \psi(N+1) - \psi(1)$$

Euler's Harmonic Sum

Euler-Zagier harmonic sums

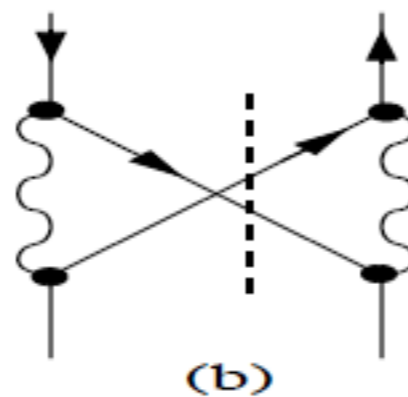
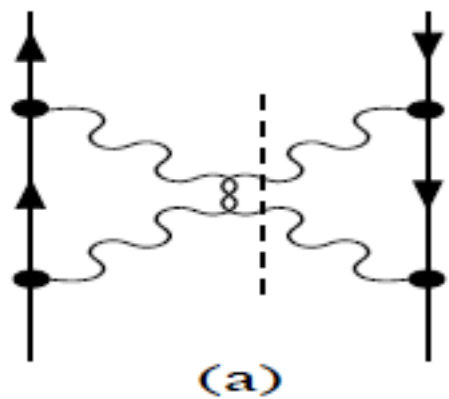
- In higher orders enter $m > 1$

$$S_m(N) = \sum_{k=1}^N \frac{1}{k^m} = \frac{(-1)^m}{\Gamma(m)} \int_0^1 dx x^N \frac{\ln^{m-1} x}{1-x} + \zeta(m)$$

- Starting from the 2nd loop, one encounters also *negative indices*

$$S_{-m}(N) = \sum_{k=1}^N \frac{(-1)^k}{k^m}$$

The origin of these *oscillating* sums — the $s \rightarrow u$ crossing:



$$(a) \leftrightarrow (b)$$

$$P \rightarrow -P$$

$$x \rightarrow -x$$

$$\frac{x}{1-x} \cdot \ln^2 x \rightarrow S_3(N)$$

- multiple indices — *nested sums*

$$S_{m, \vec{\rho}}(N) = \sum_{k=1}^N \frac{S_{\vec{\rho}}(k)}{k^m} \quad (\vec{\rho} = (m_1, m_2, \dots, m_i))$$

$$\gamma_1 = -S_1$$

$$\gamma_2 = \frac{1}{2}S_3 + S_1S_2 + \left(\frac{1}{2}S_{-3} + S_1S_{-2} - S_{-2,1}\right)$$

direct calculation by Kotikov & Lipatov, 2000

AK observation: γ_2 contains but the “most transcendental” structures !

Loop # 3 : since neither fermions nor scalars give rise to S_{2L-1} ,
pick out the *maximal transcendentality pieces* from the QCD an. dim.

$$\begin{aligned} \gamma_3 = & -\frac{1}{2}S_5 - \left[S_1^2S_3 + \frac{1}{2}S_2S_3 + S_1S_2^2 + \frac{3}{2}S_1S_4\right] \\ & - S_1 \left[4S_{-4} + \frac{1}{2}S_{-2}^2 + 2S_2S_{-2} - 6S_{-3,1} - 5S_{-2,2} + 8S_{-2,1,1}\right] \\ & - \left(\frac{1}{2}S_2 + 3S_1^2\right)S_{-3} - S_3S_{-2} + (S_2 + 2S_1^2)S_{-2,1} + 12S_{-2,1,1,1} \\ & - 6(S_{-3,1,1} + S_{-2,1,2} + S_{-2,2,1}) + 3(S_{-4,1} + S_{-3,2} + S_{-2,3}) - \frac{3}{2}S_{-5} \end{aligned}$$

“reciprocity respecting” harmonic functions

$$[\Gamma(a)]^{-1} \frac{x}{x-1} \ln^{a-1} \frac{1}{x} = \tilde{S}_a(x)$$

Let $\vec{m} = \{m_1, m_2, \dots, m_\ell\}$, and examine the recurrence relation

$$\tilde{\Phi}_{b, \vec{m}}(x) = -[\Gamma(b)]^{-1} \frac{x}{x-1} \int_x^1 \frac{dz (z+1)}{z^2} \ln^{b-1} \frac{z}{x} \cdot \tilde{\Phi}_{\vec{m}}(z)$$

$$\tilde{\Phi}_a(x) = \left(-x \tilde{\Phi}_a(x^{-1}) \right) \cdot (-1)^{a-1} \equiv \left(-x \tilde{\Phi}_a(x^{-1}) \right) \cdot (-1)^{w[a]}$$

For an arbitrary index vector (the *weight* $w \equiv \tau - \ell$)

$$\tilde{\Phi}_{\vec{m}}(x) = \left(-x \tilde{\Phi}_{\vec{m}}(x^{-1}) \right) \cdot (-1)^{w[\vec{m}]} \quad (-1)^{\# \text{ of negative indices}}$$

twist 2 RR kernels for N=4 SYM

Then, in terms of the physical coupling,

$$\mathbf{g}_{\text{ph}}^2 \equiv \frac{N_c \alpha_{\text{ph}}}{2\pi} = g^2 - \zeta_2 g^4 + \frac{11}{5} \zeta_2^2 g^6 - \left(\frac{73}{10} \zeta_2^3 + \zeta_3^2 \right) g^8 + \dots,$$

the perturbative series for the kernel, $\mathcal{P} = \sum_{n=1} \mathbf{g}_{\text{ph}}^{2n} \mathcal{P}_{\text{ph}}^{(n)}$, becomes

$$\mathcal{P}_{\text{ph}}^{(1)} = 4 \mathcal{S}_1$$

$$\mathcal{P}_{\text{ph}}^{(2)} = -4 \mathcal{S}_3 + 4 \Phi_{1,-2}$$

$$\mathcal{P}_{\text{ph}}^{(3)} = 8 \mathcal{S}_5 - 24 \Phi_{1,1,1,-2} - 8 \zeta_2 \mathcal{S}_3$$

$$-8 \mathcal{S}_1 \cdot [2 \hat{\Phi}_{1,1,-2} + \hat{\Phi}_{-2,-2} - \hat{\mathcal{S}}_{-4} + \zeta_2 \hat{\mathcal{S}}_{-2}].$$

Functions Φ — generalised harmonic sums, explicitly *reciprocity respecting* and “*algebraic*” in the $1/N \rightarrow 0$ limit.

why care ?

QCD and SUSY-QCD share the gluon sector !

Importantly, the maximal transcendentality (*clagon*) structures constitute **the bulk** of the QCD anomalous dimensions.

Employ $\mathcal{N}=4$ SYM to simplify the essential part of the QCD dynamics

$\mathcal{N}=4$ SYM dynamics is **classical**, in (un)certain sense

No truly quantum effects are being seen

(look at the β -function and/or the anomalous dimension)

If this is true, the goal would be

to derive a **one-line-all-orders** expression for γ from $\gamma^{(1)}$ in $\mathcal{N}=4$ SYM
and then to export it into QCD,
to cover “90%” of the small-distance parton dynamics

Soft Photon

Puzzle

Study of the Dependence of Direct Soft Photon Production on the Jet Characteristics in Hadronic Z^0 Decays

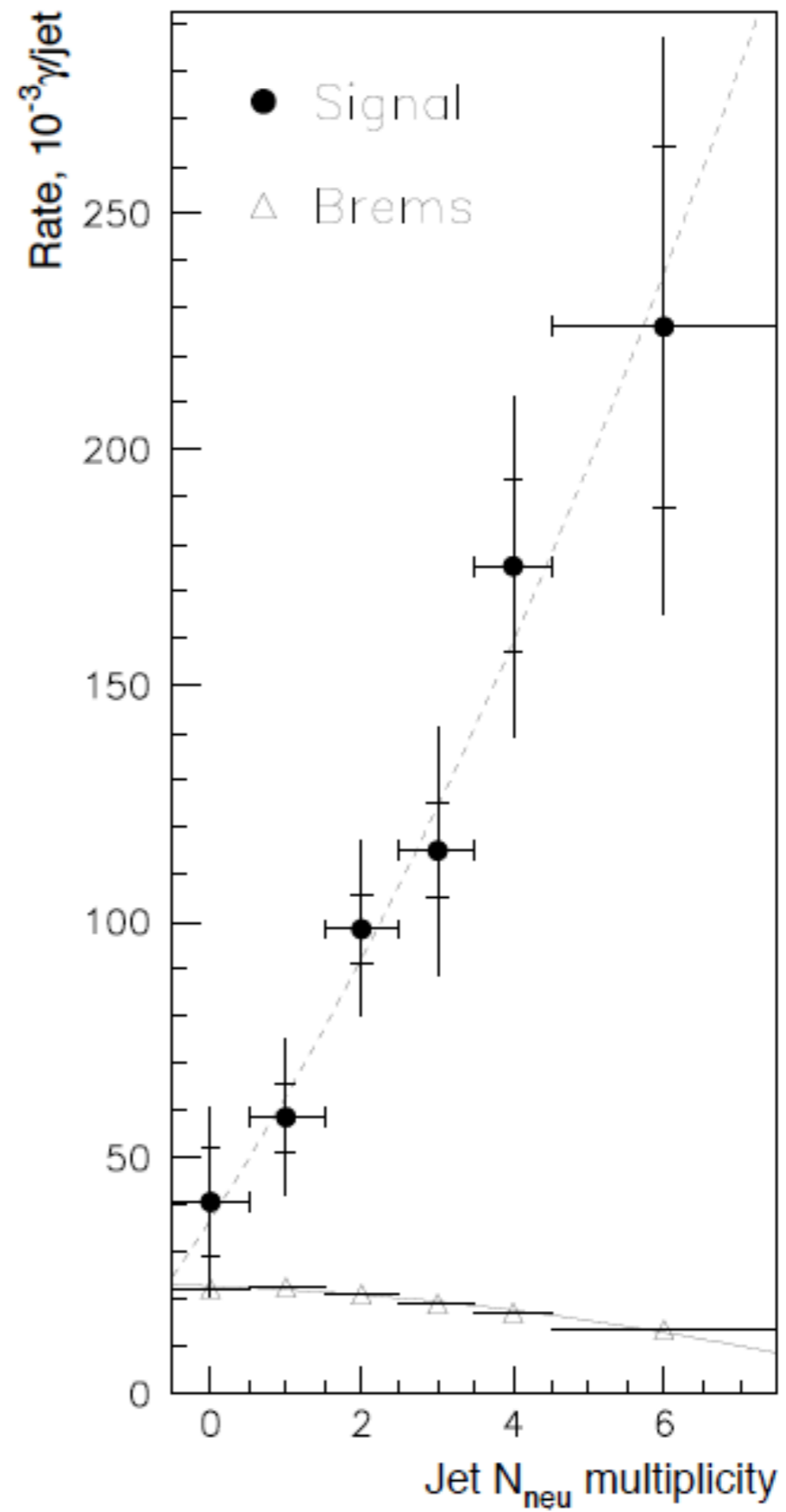
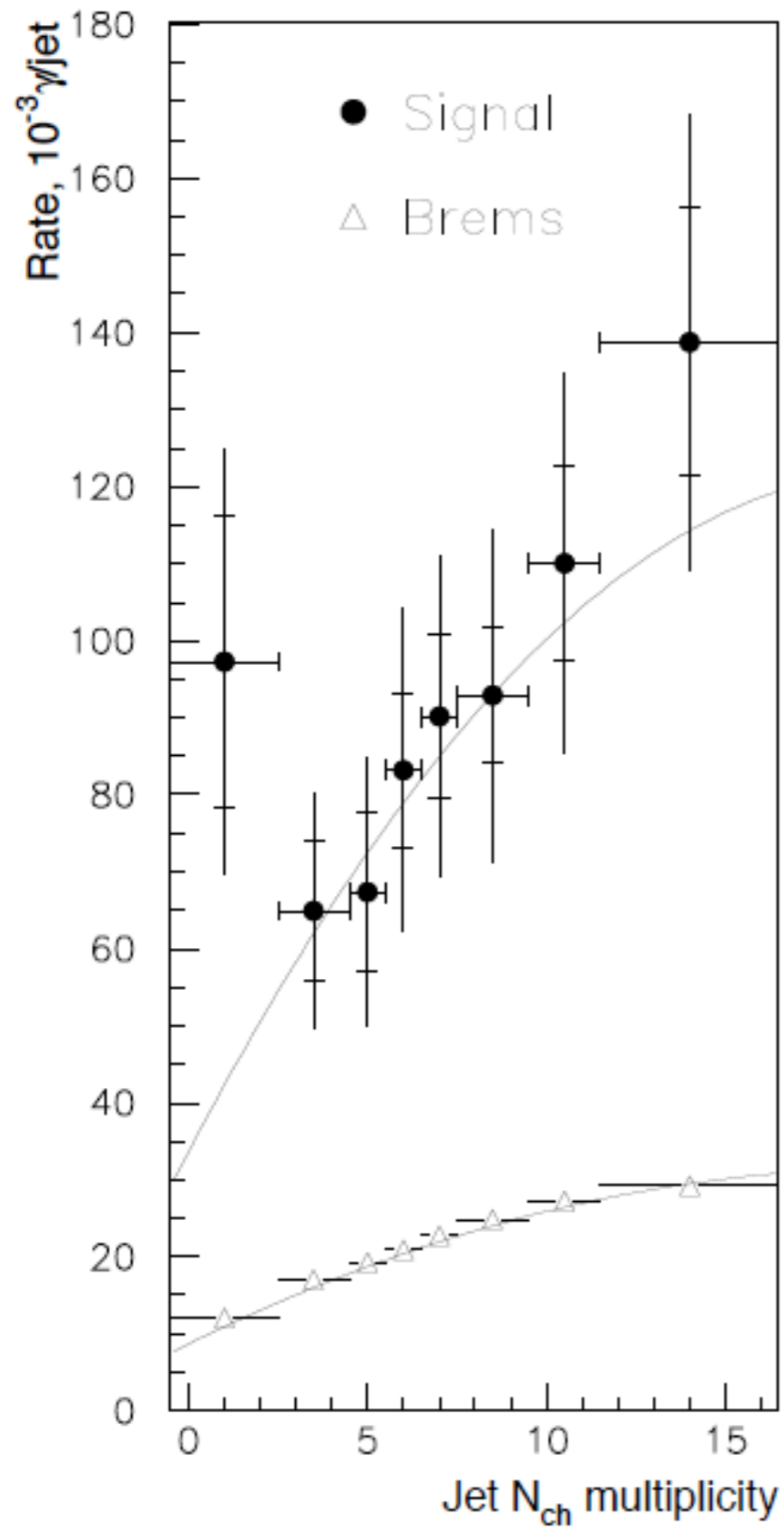
DELPHI Collaboration

$$\frac{dN_\gamma}{d^3\vec{k}} = \frac{\alpha}{(2\pi)^2} \frac{1}{E_\gamma} \int d^3\vec{p}_1 \dots d^3\vec{p}_N \sum_{i,j} \eta_i \eta_j \frac{(\vec{p}_{i\perp} \cdot \vec{p}_{j\perp})}{(P_i K)(P_j K)} \frac{dN_{hadrons}}{d^3\vec{p}_1 \dots d^3\vec{p}_N}$$

- *calculate*
- *compare with the data*
- *say: “oh-la-la...”*

$$\bullet 200 \text{ MeV} \leq E_\gamma \leq 1 \text{ GeV}$$

DELPHI photons vs. hadron multiplicity



- A steady progress in high order perturbative QCD calculations is worth accompanying by reflections upon the origin and the structure of higher loop correction effects
- Reformulation of parton cascades in terms of the Gribov–Lipatov reciprocity respecting evolution equations (RREE)
 - reduced complexity by (at least) an order of magnitude
 - improved perturbative series (less singular, better “converging”)
 - linked interesting phenomena in the DIS and $e+e-$ annihilation channels
- The Low theorem should be part of theor. phys. curriculum, worldwide
- A complete solution of the **N=4 SYM** QFT should provide us one day with a **one-line-all-order** description of the major part of QCD parton dynamics
- Understanding of the anomalous production of “soft photons” in lepton–hadron and hadron–hadron interactions should teach us a thing or two about confinement ...

instead of conclusions

- Physics of hadrons and their interactions has never been simple, and will never be.
- Certain simplifications occur at large momentum transfer (*asymptotic freedom*) as well as at large interaction energies (*TCAM, the “old theory”*)
- To progress, one needs to master and exploit both.

make sure that your library possesses the Book

Strong Interactions of Hadrons at High Energies

Gribov lectures on theoretical physics

CUP 2008

instead of conclusions

- On the pQCD side, we know how quarks and gluons form small-distance **initial state** fluctuations and how they multiply in the **final state** forming hadron jets.
- Moreover, we started to learn how to trigger and quantify, in a universal way, genuine **confinement effects** in perturbatively calculable observables.
- We inferred from these studies the effective large-distance interaction strength :

$$\left\langle \frac{\alpha_s}{\pi} \right\rangle_{IR} = 0.14 - 0.17$$

- High energy phenomena involving heavy nuclei - a rich source of **inspiration**, study **tools** and, nowadays, - **puzzles**
- Physics of **hadronization** - binding quarks/gluons into white hadrons - remains a challenge for QCD . . . **“SOFT CONFINEMENT”** . . .

Heat is building up, and QCD is about to undergo
a **faith transition**

We are getting ready to convince ourselves to talk
quarks and **gluons** down to, and into, the **InfraRed**

a missing piece

